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NEW HIGH SCHOOL ARITHMETIC

ACADEMIC ∵ INDUSTRIAL
COMMERCIAL

BY

WEBSTER WELLS, S.B.
AUTHOR OF A SERIES OF TEXTS ON MATHEMATICS

AND

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PREFACE

The high school course in arithmetic is unique in its educational possibilities. It is rich in its results in general mind and character training. For *all* pupils who take it, it develops skill in computation, and furnishes indispensable knowledge of business and industrial processes and facts that are not acquired in the lower grades or elsewhere in school. For *many* pupils, such skill and knowledge are distinctly necessary preparation for vocations which they enter later; the girl who becomes a teacher, the boy who becomes a farmer or mechanic, the boy or girl who enters any form of commercial work — these all find arithmetical skill and the information acquired in solving the problems invaluable.

The *New High School Arithmetic* furnishes material for such instruction. It is adapted to the needs of all the pupils, whether they take a general course, a normal course, an industrial vocational course, or a commercial course.

It is modern and varied in its appeal because of the use of a great variety of problem material drawn from current life. Observe the variety of material used in the problems on pages 10–12, 19–22, 41–43, 212–214, 216–220, 230–234; the abundance of problems having vocational interest for boys, such as 43–48 on page 61, 42–43 on page 65, 53–55 on page 73, 12–16 on page 105, all of Chapter VI, 33 on page 231, etc.; the domestic science problems such as 47–50 on page 70, 51–52 on page 73, 23–25 on page 77, etc.; the appeal to the interest of the country pupils as well as those in the city as in example 19 on page 76, 22 on page 77, 25–30 on page 121, etc.; the effort to arouse interest in national and local affairs as in examples 31–36 on page 98, 4–7 on page 104, § 144 on page 201, § 196 on page 292, etc.

It is accurate and entirely adequate in its teaching of business and industrial practices. Note, as examples, the tabulations of statistical material (pp. 14–16); bills on pages 34–37; estimating costs on pages 153, 154, 156, 179–188; and the full treatment of "commercial arithmetic" in Chapters VIII to XIV inclusive.

In its pedagogy this text is like the other texts of the Wells and Hart series. It is mathematically accurate; note particularly § 98 on page 145, § 100 on page 147, and § 113 on page 165. The thought processes involved in solutions are developed clearly in an inductive manner; see § 49 on page 67, § 50 on page 71, § 107 on page 158, etc. The processes are clearly and strikingly expressed in unique form in rules such as those on pages 30, 138, 255, etc. The model solutions set an example of clearness and order that cannot fail to impress pupils and result in more accurate computation by them; see pages 44, 45, 75, 108, 269, 308, 311, etc. New important terms are printed in striking manner (page 17) and are fully indexed. There is provided for drill purposes an abundance of examples and problems (pp. 5, 6, 8, 39, 58, 235) which are of the simple sensible sort met in real applications (pp. 55, 56, 57, 59, 63, 103). These examples can and should often be done mentally, and will be found to be properly graded in difficulty in the separate lists. Topics which are not strictly essential for all pupils are plainly marked as supplementary. These topics are in some cases interesting but not necessary (as Cube Root on page 142), and in other cases are important but can be included in a course only when local conditions render it possible (as Topic A on page 179).

How much and what to include in the course in arithmetic in any particular community depends upon the previous preparation of the pupils, upon the special aims of the course, and upon the time allowed for the study. In most cases some selection of topics as well as of examples and problems becomes necessary.

Chapters I to V, excepting the Supplementary Topics, are a fundamental part of any course. An effort should be made, however, to get over these chapters as speedily as is consistent with the development of a reasonable amount of skill in computation. An abundance of drill has been provided, with the expectation that some of it will be done orally and some of it will be used for special assignments for those pupils who are weak in computation. Obviously it is not necessary that all the examples and problems should be solved by any one class.

What use is made of Chapter VI will depend upon whether the course in arithmetic precedes or follows the course in geometry. If it follows, then much of this chapter between pages 145 and 168 should be omitted, although some of the problems should be solved. If time must be saved, pages 168 to 197, and 205 to 207 may also be omitted. If this chapter precedes the course in geometry, it will be found valuable preparation for that course.

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NEW
HIGH SCHOOL ARITHMETIC

I. INTEGERS

1. Writing and Reading Integers in the Hindu System. — Pupils are familiar with the manner of writing and reading small **Integers** or whole numbers in the Hindu System by means of the digits 0, 1, 2, 3, ···, 9.

NOTE. — Our system of writing numbers is often improperly called the Arabic System.

They should recall that a number like 2,643 is read "*two thousand, six hundred forty-three,*" not "*and* forty-three."

EXERCISE 1

1. Read 4,275; 3,560; 2,708; 6,029.
2. What are the digits?
3. Tell what each digit of the number 6,035 represents.
4. In numbers like *fourteen,* what does the syllable "teen" mean?
5. In numbers like *thirty,* what does the syllable "ty" mean?

2. For convenience in reading integers, the digits of a number are divided into groups of three, starting from the right. Each group is called a **Period.** The first three figures form the *units'* period; the second three, the *thousands'* period; the third three, the *millions'* period; the fourth three, the *billions'* period.

Thus, 20,793,106,452 is read *twenty billion, seven hundred ninety-three million, one hundred six thousand, four hundred fifty-two.*

NOTE. — The following table gives the names of each of the first fourteen periods. Only the first four of these periods are used in ordinary computation.

PERIOD	DESIGNATION	PERIOD	DESIGNATION
First	Units	Eighth	Sextillions
Second	Thousands	Ninth	Septillions
Third	Millions	Tenth	Octillions
Fourth	Billions	Eleventh	Nonillions
Fifth	Trillions	Twelfth	Decillions
Sixth	Quadrillions	Thirteenth	Undecillions
Seventh	Quintillions	Fourteenth	Duodecillions

EXERCISE 2

1. Read the following numbers:

a. 2,745,368
b. 13,609,432
c. 205,083,620
d. 3,682,749,063
e. 4,005,060,805
f. 46,375,082,73

2. Read the following figures, which give the total production of wheat in each of the countries named in the year 1915:

United States . . .	1,011,505,000 bushels
Canada	336,258,000 bushels
Europe	2,080,819,000 bushels
Asia	460,245,000 bushels
Africa	90,859,000 bushels

3. Read the following sums, which give the total taxable property in Chicago in the years indicated:

a. 1890, $ 219,354,368
b. 1895, $ 243,476,825
c. 1900, $ 276,565,880
d. 1905, $ 407,991,625
e. 1910, $ 848,994,536
f. 1915, $ 1,047,788,676

4. Read the amount appropriated by each of the following Congresses of the United States:

a. 58th Cong., 1905–1906, appropriated $1,600,053,544.80
b. 59th Cong., 1907–1908, appropriated $1,799,537,864.70
c. 60th Cong., 1909–1910, appropriated $2,052,411,841.79
d. 61st Cong., 1911–1912, appropriated $2,053,391,290.72
e. 62d Cong., 1913–1914, appropriated $2,118,060,671.12
f. 63d Cong., 1915–1916, appropriated $2,231,055,150.57

5. Read the following figures, which give the estimated value of the farm products of the United States in the years indicated.

a. 1910, $9,037,390,744
b. 1911, $8,819,174,959
c. 1912, $9,342,790,149
d. 1913, $9,849,512,511
e. 1914, $9,894,960,531
f. 1915, $10,501,686,375

3. In the **Roman System** of writing integers the following symbols are used:

I represents 1
V represents 5
X represents 10
L represents 50
C represents 100
D represents 500
M represents 1000

All other small integers are represented by writing two or more of these letters side by side, according to the following:

Rule I. — If a letter follows one of the same or greater value, the number represented is the sum of their values.

Rule II. — If a letter precedes one of greater value, the number represented is the difference between their values.

Thus, XVII $= 10 + 5 + 1 + 1 = 17$.

Thus, XC $= 100 - 10 = 90$.

The Roman System of writing numbers is seldom used now except for numbering chapters in books, hours on a clock face, dates on buildings or monuments, and the volumes of a set of books.

EXERCISE 3

Read the following numbers:

1. *a.* VIII *b.* XVIII *c.* XIX *d.* XXVII *e.* XL

2. *a.* XC *b.* CX *c.* MDC *d.* MCL *e.* DCXI

Write the following numbers in Roman numerals:

3. The numbers from 11 to 19.

4. The numbers 22, 32, 43, 48.

5. The numbers 50, 52, 55, 59.

6. The numbers 60, 72, 86, 89.

7. The numbers 90, 93, 95, 98.

8. The numbers 200, 450, 635, 872.

9. The numbers 1000, 1600, 1870, 1923.

10. Read the following:

a. MDCCXXII
b. MDCCLXXVI
c. MCDXCII
d. MDCCCXIX
e. MDCCCLXI
f. MCMXIV

11. Write in Roman numerals the number of the year in which you were born.

12. Write in Roman numerals the number of this year.

Addition of Integers

4. The integers arranged in the order 1, 2, 3, 4, etc., are said to form a **Sequence.**

The **Sum** of two integers, like 3 and 5, is the fifth integer beyond 3 in the sequence of integers.

Addition of Integers is the process of finding the sum when two or more integers are given. The integers added are called **Addends.**

5. Skillful addition depends upon accurate and rapid recognition of the sums of the following pairs of integers. These are all the possible pairs of two digits, omitting the digit zero.

1	2	4	3	5	8	3	9	4
$\underline{1}$	$\underline{4}$	$\underline{6}$	$\underline{7}$	$\underline{6}$	$\underline{9}$	$\underline{3}$	$\underline{1}$	$\underline{3}$
5	2	9	7	8	5	4	9	2
$\underline{1}$	$\underline{6}$	$\underline{4}$	$\underline{7}$	$\underline{6}$	$\underline{8}$	$\underline{4}$	$\underline{3}$	$\underline{2}$
4	2	6	4	7	6	8	5	8
$\underline{1}$	$\underline{5}$	$\underline{3}$	$\underline{8}$	$\underline{6}$	$\underline{9}$	$\underline{7}$	$\underline{9}$	$\underline{8}$
7	4	2	3	2	3	8	1	2
$\underline{9}$	$\underline{5}$	$\underline{8}$	$\underline{1}$	$\underline{3}$	$\underline{5}$	$\underline{1}$	$\underline{6}$	$\underline{7}$
8	2	1	5	4	1	6	7	9
$\underline{3}$	$\underline{9}$	$\underline{7}$	$\underline{5}$	$\underline{7}$	$\underline{2}$	$\underline{6}$	$\underline{5}$	$\underline{9}$

EXERCISE 4

Addition Drill Table

NOTE. — The following table can and should be used for a variety of drill. The following exercises are suggestive. It is not necessary that all be done at this time or that every pupil do all of them. On the other hand, any pupil who is inaccurate or slow in adding should be required to practice on this drill daily until he can do the exercises skillfully. From time to time, a few minutes can be spent profitably upon oral drill based on this table.

Do not copy the numbers from the book.

Give the results orally or write *only* the results, when possible.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
a.	16	56	86	46	76	66	26	36	96
b.	17	27	47	37	67	57	97	87	77
c.	18	38	48	28	58	78	68	88	98
d.	29	19	49	59	39	69	89	99	79
e.	38	17	46	53	75	64	85	92	24
f.	57	45	26	19	74	33	68	87	98
g.	16	37	58	27	49	67	76	95	86
h.	47	66	85	54	39	77	96	23	18
i.	78	93	48	64	36	24	59	15	88
j.	56	13	79	28	94	65	89	32	43

1. To each number of row *a*, add

a. 4 *b.* 3 *c.* 5 *d.* 9 *e.* 7 *f.* 6 *g.* 8

2–10. Add these same numbers successively in rows *b*, *c*, *d*, ⋯ *j*.

11–19. Add these same numbers successively to the numbers in columns *a*, *b*, *c*, ⋯ *i*.

20–29. Add 11 to the numbers in rows *a*, *b*, *c*, ⋯ *j*.

30–39. Add 12 to the numbers in rows *a*, *b*, *c*, ⋯ *j*.

6. Some Devices Employed in Adding.

a. **When adding columns, name results only.**

$$\begin{array}{r}6\\7\\5\\\underline{4}\\22\end{array}$$

Think: 9, 16, 22. *Write* 22.
Do not think: 4 *and* 5 are 9;
9 *and* 7 are 16;
16 *and* 6 are 22.

EXERCISE 5

Addition Drill Table

NOTE. — Read the note at the beginning of Exercise 4.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
a.	5	7	8	6	2	3	4	9	7	6
b.	3	4	5	7	6	5	8	3	2	9
c.	8	3	9	2	5	4	3	7	6	5
d.	4	5	6	8	7	6	5	8	4	3
e.	9	6	4	3	4	7	6	5	8	7
f.	2	8	3	9	8	9	7	6	3	8
g.	6	9	2	5	3	8	2	4	5	2
h.	7	2	7	4	9	2	9	2	9	4
i.	8	7	5	6	4	7	8	5	8	6
j.	3	6	8	7	6	5	4	6	5	5

1. Hold a slip of paper below row *b*. Give orally or write upon the paper the sum of the two numbers in each column that appear above the paper.

2–9. Similarly, give orally or write upon slips of paper the sum of the three numbers in each column that are in rows, *a*, *b*, and *c*; of the four numbers in each column that are in rows *a*, *b*, *c*, and *d*; etc.

10–18. Similarly, find the sums of the numbers in each column that are in the lower two rows, the lower three rows, etc.

NOTE.—By omitting row *a*, and following the directions for Examples 1–9, other examples can be made; also, by omitting row *j*, and following the directions for Examples 10–18, more examples can be provided; or, by adding a row at the top of the table, new examples are provided.

b. **When adding columns of integers, group and add at sight integers whose sum is 10.**

These numbers may not be consecutive in the column.

73
32
15
27 *Think:* 10, 18, 25, 35. *Write* 5 and carry 3.
98 *Think:* 7, 17, 27, 37. *Write* 37.
84
46
375

In the same manner, group and add at sight integers whose sum is readily recognized. Thus, if 5 and 3 appear in sequence in a column, instead of adding them separately, add 8 at once. Skill in making such combinations comes with practice and greatly increases the student's speed in adding.

EXERCISE 6

Add the following columns, grouping numbers whose sum is 10:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*
2	1	2	6	7	2	1	3	8	9	4	5	7	3	5	8
8	9	8	4	3	7	9	5	1	1	5	5	3	6	4	2
3	6	5	3	8	8	8	7	3	6	1	2	5	1	1	9
7	4	5	7	1	2	4	5	6	4	3	8	4	7	2	1
4	5	8	1	1	3	6	2	5	2	7	7	1	3	8	6
6	5	1	9	7	6	2	8	2	8	6	2	7	6	4	5
8	3	1	6	6	1	5	4	3	7	5	6	4	5	7	3
4	2	9	7	4	7	3	6	9	5	7	4	3	2	7	2
2	8	3	4	6	3	8	7	1	5	3	9	3	3	5	7
8	5	7	3	2	8	5	3	4	2	8	1	5	4	5	1
6	2	5	1	7	9	4	6	2	1	1	7	6	9	8	4
4	8	5	6	3	1	6	3	4	7	1	2	7	2	3	6

EXERCISE 7

Addition Drill

Read the note at the beginning of Exercise 4.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
a.	223	746	637	659	63	477	785	693	762	781
b.	452	863	708	794	872	536	439	265	574	626
c.	537	227	430	664	580	229	546	537	435	543
d.	365	308	546	573	85	306	757	729	918	567
e.	248	549	723	685	37	540	368	456	832	112
f.	374	802	862	746	709	907	472	862	767	346
g.	889	980	995	334	430	823	23	578	648	752
h.	496	679	726	628	667	321	94	384	953	374
i.	534	863	849	792	82	530	506	724	165	439
j.	263	421	741	873	280	865	729	932	718	326

1–20. See the examples suggested for Exercise 5, page 7.

21. Find the sum in each column of the numbers in rows *b*, *c*, *d*, and *e*; in rows *c*, *d*, *e*, *f*, and *g*; etc.

c. **When long columns are to be added, the units' column may be added and the sum written; then the tens' column may be added and the result written as in the following example;** etc.

67
835
243
506
789

The sum of the units = 30 units = 30
The sum of the tens = 21 tens = 210
The sum of the hundreds = 22 hundreds = 2200
The complete sum = 2440

NOTE.—If one who is adding in this manner is interrupted, he need not repeat the whole addition.

EXERCISE 8

Add upward the following columns, and verify your results by adding again downward:

1.	**2.**	**3.**	**4.**	**5.**
1789	5403	4529	7854	1827
6543	786	7992	6215	4329
2177	9230	467	9448	25070
915	1157	8920	4007	6118
6783	8980	3508	651	2522
325	7526	2463	5869	3909

6.	**7.**	**8.**	**9.**	**10.**
79856	45340	43765	58964	77167
35117	10087	89140	86723	63489
32949	76322	35174	48213	68791
18817	36450	64385	54876	74153
85622	78809	79160	83538	84375
16304	39713	23099	71766	97561

11. The prize cow of a certain herd produced milk as follows: Monday 37 lb.; Tuesday 36 lb.; Wednesday 40 lb.; Thursday 38 lb.; Friday 37 lb.; Saturday 39 lb.; Sunday 38 lb. What was the production for the week?

12. A grocer received on Monday $ 135.25, on Tuesday $ 84.40, on Wednesday $ 106.65, on Thursday $ 122.70, on Friday $ 93.62, and on Saturday $ 185.56. What were his total receipts for the week?

13. The following table gives the land and water areas of each of the ten largest states. What is the combined land area of these states, the combined water area, and the total area of each state?

STATE	LAND	WATER	TOTAL
Texas . . .	262,398	3498	
California . .	155,652	2645	
Montana . .	146,201	796	
New Mexico .	122,503	131	
Arizona . . .	113,810	146	
Nevada . . .	109,821	869	
Colorado . .	103,658	290	
Wyoming . .	97,594	320	
Oregon . . .	95,607	1092	
Utah	82,184	2806	
Total			

14. The immigration into the United States during the fiscal year 1916, by months, was: July, 21,504; August, 21,940; September, 24,513; October, 25,450; November, 24,545; December, 18,901; January, 17,293; February, 24,740; March, 27,586; April, 30,560; May, 31,021; June, 30,764.

Rule off a suitable form in which to tabulate these statistics (see Ex. 13); then tabulate the statistics, and find the total for the year.

15. The following table gives the total number of male and female inhabitants in the United States whose ages fell between certain years at the time of the 1910 census. Find the total number of inhabitants:

Under 5 years, 10,631,364; 5–9 yr., 9,760,632; 10–14 yr., 9,107,140; 15–19 yr., 9,063,603; 20–24 yr., 9,056,984; 25–29 yr., 8,180,003; 30–34 yr., 6,972,185; 35–39 yr., 6,396,100; 40–44 yr., 5,261,587; 45–49 yr., 4,469,197; 50–54 yr., 3,900,791; 55–59 yr., 2,786,951; 60–64 yr., 2,267,150; 65–69 yr., 1,679,503; 70–74 yr., 1,113,728; 75–79 yr., 667,302; 80–84 yr., 321,754; 85–89 yr., 122,818; 90–94 yr., 33,473; 95–99 yr., 7391; 100 yr. and over, 3555; age unknown, 169,055.

16. The total appropriations made by Congress in 1918 for the first year of the war with Germany and the estimates for the year 1919 were, by departments, as follows:

	ESTIMATES FOR 1919	TOTAL APPROPRIATIONS FOR 1918
Legislative	$ 15,220,628	$ 15,753,378
Executive	1,241,960	11,522,690
State department	7,349,331	6,568,706
Treasury department	736,621,972	7,635,827,932
Independent offices	920,040,891	1,203,434,878
District of Columbia	18,411,124	15,318,430
War department	10,102,599,149	7,526,337,716
Panama canal	23,171,624	23,593,299
Navy department	1,047,914,027	1,596,936,455
Interior department	211,530,122	217,574,484
Postoffice department proper .	1,825,340	1,943,260
Postal service, payable from postal service	331,818,345	331,851,170
Department of agriculture . .	49,193,551	56,889,546
Department of commerce . .	17,734,065	13,605,935
Department of labor	7,285,551	5,452,836
Department of justice . . .	12,400,254	11,349,716
Increase of compensation, all departments (estimated) . .		15,000,000

Find the total for each year.

17. The following bills were paid by a house owner for the construction and equipment of his home: carpenter, $3016.77; mason, $1500; plasterer, $674; tinsmith, $94.25; plumber, $825.75; heating contractor, $623.50; electrician, $174.88; hardware merchant, $195.00. Tabulate and find the total.

18. Of the total number of farmers in 1910, 3,948,722 were owners of farms, 58,104 were managers of farms, and 2,354,676 were tenants. Find the total number of farmers.

19. The number of manufacturing establishments in the United States in 1909 in the principal geographic divisions were: Middle Atlantic, 81,315; East North Central, 60,013; New England, 25,351; West North Central, 27,171; South Atlantic, 28,088; Pacific, 13,579; East South Central, 15,381; West South Central, 12,339; Mountain, 5254. Tabulate and find the total.

d. **Mental Addition.** — The following example illustrates a useful form of mental addition.

Example. — Add 34 and 59.

Solution. — Think $34 + 50 = 84$; then $84 + 9 = 93$.

EXERCISE 9

Find mentally the following sums:

1. $43 + 62$

2. $27 + 38$

3. $36 + 57$

4. $83 + 55$

5. $56 + 73$

6. $68 + 94$

7. $39 + 54$

8. $28 + 69$

9. $42 + 87$

10. $57 + 38$

11. $52 + 66$

12. $75 + 49$

13. $82 + 33$

14. $93 + 58$

15. $44 + 67$

16. $37 + 67$

17. $75 + 46$

18. $89 + 29$

e. **Horizontal Addition.** — If numbers to be added happen to be arranged horizontally, they should be added without taking time to rewrite them in a column.

Example. — Find the following sum:

$$55 + 36 + 24 + 47 + 63$$

Solution. — 1. Adding the units, *think* 5, 11, 15, 22, 25. Write 5 and carry 2.

2. Adding the tens, *think* 7, 10, 12, 16, 22. Write 22 in front of the 5.

3. $\therefore 55 + 36 + 24 + 47 + 63 = 225.$

EXERCISE 10

Find the following sums without copying the numbers:

1. $26 + 43 + 65 + 34$

2. $47 + 32 + 27 + 56$

3. $69 + 72 + 85 + 47$

4. $233 + 152 + 415 + 376$

5. $528 + 365 + 147 + 283$

6. $694 + 268 + 153 + 562$

7. The large naval vessels of the principal foreign countries built up to July 1, 1914, were as follows:

	Dreadnaughts	Battleships	Battle Cruisers	Armored Cruisers	Cruisers	Total
England . . .	20	40	9	34	74	
Germany . . .	13	20	4	9	41	
France	3	18	0	20	9	
Japan	2	13	2	13	13	
Russia	0	7	0	6	9	
Italy	3	8	0	9	6	
Austria-Hungary	3	6	0	2	5	

Find the total number of vessels of each country.

8. Ten lamp cleaners of a gas company cleaned the indicated numbers of lamps on the six days of a week. Find the total number cleaned by each during the week; also the

total for all men on each day; also determine a satisfactory check.

EMPLOYEES	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTAL FOR WEEK
John Ewing . .	50	54	53	60	58	55	
Alfred Bower . .	15	18	22	29	25	33	
Harry Smith . .	32	37	41	30	39	43	
James Swenson .	60	62	63	60	61	59	
Roy Crane . . .	40	45	43	49	51	47	
Tim Sullivan . .	57	52	46	40	39	50	
Lee Anderson . .	50	47	41	45	49	48	
Jacob Brown . .	80	85	70	82	75	50	
Paul Derby . .	12	21	30	35	41	32	
Andrew Ritz . .	27	35	41	47	39	42	
Total Daily							

9. At a milk condensery, certain farmers on the days of one week sold the number of pounds of milk indicated. Find the total each day for all the farmers and the total for the week for each farmer.

FARMER	MON.	TUES.	WED.	THURS.	FRI.	SAT.	SUN.	WEEK TOTAL
A. . .	125	128	123	130	126	121	123	
B. . .	880	875	882	879	876	883	877	
C. . .	231	232	235	227	229	236	230	
D. . .	240	235	236	241	243	237	238	
E. . .	270	260	265	273	262	268	271	
F. . .	1162	1158	1156	1160	1163	1159	1155	
Total								

10. In a campaign to raise funds for a public benefaction, five teams obtained the following sums during one week:

TEAM	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTAL
A. . . .	$205	$187	$175	$210	$230	$255	
B. . . .	314	285	270	283	300	325	
C. . . .	108	92	76	87	101	98	
D. . . .	524	465	423	386	672	509	
E. . . .	132	143	107	196	184	165	
Totals							

Determine the daily totals and the totals for each team for the week. Check the results.

11. The number of men and women teachers employed in the United States in the years 1910–1914 inclusive were:

YEAR	MEN	WOMEN	TOTAL
1910	110,481	412,729	
1911	110,328	423,278	
1912	114,559	432,730	
1913	113,342	451,118	
1914	114,662	465,396	

Find the total number in each year.

12. The following table gives data of manufacturing establishments in 1909 in the United States and certain of its possessions. Find the totals.

ITEMS	U. S. PROPER	ALASKA	HAWAII	PORTO RICO	TOTAL
No. of establishments	268,491	152	500	939	
Persons engaged . .	7,678,578	3,479	7,572	18,122	
Capital invested . .	18,428,270,000	13,060,000	23,875,000	25,544,000	
Expenses .	18,454,090,000	9,454,000	31,753,000	31,139,000	
Value of products .	20,672,052,000	11,340,000	47,404,000	36,750,000	

13. Household expenditures of a family. Find the total for each month and the yearly total for each item.

MONTHS	COAL	LIFE AND FIRE INSURANCE	TAXES	DOCTORS AND DRUGGISTS	HOUSE ACCOUNT	CLUBS	RECREATION	TOTAL
Jan.	$124.50	$15.05	$53.95	$64.42	$ 85.00	$ 7.50	$11 50	
Feb.		85.63		4.00	90.19	2.50	7.25	
Mar.	16.50		9.50		97.04		8.00	
Apr.		15.05		5.00	87.15		6.00	
May		27.00		2.85	83.17	15.00	8.30	
June				13.75	98.45		4.25	
July		15.05		7.67	83.75		9.00	
Aug.					118.44	25.00	5.35	
Sept.		39.40		5.00	85.77		55.00	
Oct.		15.05			87.93		5.75	
Nov.				6.75	90.25	2.70	9.40	
Dec.				3.63	100.65	5.00	15.00	
Total								

14. The bookkeeper of A. C. Cole prepared the following summary of the year's business. Find the totals.

MONTH	TOTAL SALES OF					
	COAL	WOOD	ICE	LIME	CEMENT	TOTAL
January . . .	$12,500	$245	$ 250	$150	$ 25	
February . .	14,785	320	275	85	78	
March . . .	11,690	165	440	246	164	
April	10,850	185	1062	580	275	
May	7,270	200	2480	625	583	
June	4,340	115	5675	840	962	
July	7,675	125	8342	972	995	
August . . .	8,940	195	9730	863	1145	
September . .	10,845	250	8650	985	1067	
October . . .	10,420	275	3240	745	965	
November . .	13,680	305	1035	585	1084	
December . .	17,690	265	469	239	215	
Total						

Subtraction of Integers

7. In **Subtraction,** two numbers are given, the **Subtrahend,** and the **Minuend.** Subtraction is the process of finding the number, called the **Difference,** which must be added to the subtrahend to give the minuend.

Thus, to subtract 8 from 12 means to find the number which must be added to 8 to give 12. This is 4.

Subtraction is also thought of as *a process of taking away* and pupils are doubtless more familiar with it in that form.

In actual business computations, both meanings of subtraction find application. The following example illustrates a solution in accordance with the first meaning.

Example 1. — Subtract 483 from 758.

Minuend	758
Subtrahend	483
Difference	275

Think $3 + 5 = 8$. *Write* 5 in the units' place.

Think $8 + 7 = 15$. *Write* 7 in the tens' place.

Think 1, carried, $+ 4 = 5 + 2 = 7$. *Write* 2 in the hundreds' place.

Check: The sum of the subtrahend and difference equals the minuend.

Example 2. — Subtract 436 from 859 by the *taking away* method and explain the solution.

Whatever the method used, skill in subtraction, like skill in addition, depends upon accurate and rapid recognition of the difference between (*a*) any integer less than ten and a greater integer to ten inclusive, or (*b*) between any such integer and the integer formed by adding ten to a smaller integer.

EXERCISE 11

Find the differences indicated in the last sentence of the preceding paragraph. There are eighty-one such differences.

EXERCISE 12

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
a.	21	43	37	66	58	74	88	79	92
b.	44	95	24	35	89	76	62	53	18
c.	87	64	93	25	46	71	38	13	57
d.	72	26	47	94	55	32	17	63	86
e.	31	15	67	83	22	45	96	54	95
f.	85	73	56	49	16	65	97	28	34
g.	59	33	84	78	14	27	48	99	61
h.	77	98	81	52	39	12	23	69	46
i.	11	82	75	36	68	91	51	42	29

Read the note at the beginning of Exercise 4.

1. From each number of row *a*, subtract:

a. 3 *b.* 7 *c.* 4 *d.* 8 *e.* 5 *f.* 6 *g.* 9

2–9. Use rows *b*, *c*, *d*, etc., in like manner.

10–18. Subtract the numbers given in Example 1 from columns *a*, *b*, *c*, etc.

EXERCISE 13

Read the note at the beginning of Exercise 4.

	a	*b*	*c*	*d*	*e*
a.	82765	49038	76209	53782	69458
b.	75638	48527	72589	51683	67465
c.	72947	43782	70693	49785	63746
d.	68755	39476	69837	45846	60983
e.	65682	33852	64967	41762	57876
f.	57765	28080	58925	36547	52897
g.	52325	27863	45786	24763	50079
h.	49765	26793	42687	21875	48869
i.	43689	19847	37786	16954	38745
j.	32976	10756	29989	7653	10594

1. (Parts *a, b, c, d, e.*) Place a strip of paper below row *b*. Upon it write the results of subtracting the numbers in row *b* from those in row *a*.

2. (Parts *a, b, c, d, e.*) Write similarly the differences between the numbers of row *c* and those of row *b*.

3. Subtract the numbers of any row from those in the preceding row.

4. Subtract the numbers of any row from those of any preceding row: *e.g.*, those of row *e* from those of row *a*, etc.

5. A dentist's clerk submits to him the following record of the accounts of patients whose payments are overdue.

PATIENT	AMT. DUE ON FIRST OF YEAR	AMT. PAID DURING YEAR
A	$ 37.75	$12.00
B	18.50	3.75
C	132.00	35.00
D	97.25	23.50
E	62.50	15.00
F	85.40	22.75
G	124.60	33.50
H	106.75	29.25

Determine:

a. the total amount due at the beginning of the year;

b. the total amount paid during the year;

c. the amount still due from each patient;

d. a satisfactory method of checking the accuracy of the work.

6. The income and expenditures of the United States Post-office for the years 1911–1915 inclusive were:

	INCOME	EXPENDITURES
1911	$ 237,879,823.60	$ 237,648,926.68
1912	246,744,015.88	248,525,450.08
1913	266,619,525.65	262,067,540.33
1914	287,934,565.67	283,558,102.62
1915	287,248,165.27	298,546,026.42

Determine for each year the amount earned or the amount lost by the Post-office Department.

7. From the data of Example 6, determine the annual increase in expenditures in the years 1912 to 1915 inclusive.

8. In 1914, Chicago spent for lighting its streets the following amounts: gas lights, $ 257,680.47; gasoline lights, $ 149,793.80; rented electric lights, $ 113,682.95; municipal electric lights, $ 614,941.94. In 1915 the amounts were: gas lights, $ 228,919.39; for gasoline lights, $ 132,016.36; for rented electric lights, $ 134,227.29; for municipal electric lights, $ 718,808.62.

Arrange these results in tabular form, providing space for the totals for each year, and a column for increases or decreases; insert the totals and the increase or decrease.

9. In 1900 the value of farm property in the north was $ 14,455,452,476; in the south, $ 4,269,854,719; and in the west, $ 1,714,593,969. In 1910 the value of farm property in the north was $ 27,481,267,056; in the south, $ 8,972,126,889; in the west, $ 4,538,055,145. Arrange these items in tabular form, making a place for the totals for the United States for both years, and for the increase in each section during the 10-year period. Find the totals and the increases.

10. In 1916 there were 7579 national banks, with deposits of $ 10,877,087,000, loans of $ 7,685,335,000, and reserve cash of $ 1,295,706,000; in 1917 there were 7604 national banks, with deposits of $ 12,771,833,000, loans of $ 8,967,297,000, and reserve cash of $ 1,573,295,000. Tabulate, including a column for increases. Insert the increases in the column.

11. A heating contractor has the following incomplete stock record at the close of the year. Complete it by filling in the incomplete portions.

KIND OF PIPE	ON HAND JAN. 1	BOUGHT DURING YEAR	TOTAL	USED DURING THE YEAR	ON HAND AT END OF YEAR
½ in.	1325 ft.	2505		1672	
¾ in.	3740 ft.	6006		4357	
1 in.	2338 ft.	3014		3265	
1¼ in.	1876 ft.	3009		2433	
1½ in.	1755 ft.	3057		2870	
2 in.	1976 ft.	2004		2287	
2½ in.	1285 ft.	2512		1645	
3 in.	648 ft.	810		853	
4 in.	375 ft.	408		325	
5 in.	268 ft.	200		252	
6 in.	1431 ft.	100		98	

12. A lumber company has the following stock record for a certain month. Complete the record.

	BALANCE PREVIOUS MONTH	RECEIPTS	TOTAL	SALES DURING MONTH	BALANCE CLOSE MONTH
Lumber (in feet) . .	756,841	193,547		141,970	
Lath in bundle . .	2,208			328	
Shingles, in bunches	406	680		231	
Posts, number . . .	631	32		81	
Doors, number. . .	315	35		170	
Windows, number .	680	122		213	

13. The population of the Continental United States at each census from 1790 was as follows:

1910, 91,972,266; 1900, 75,994,575; 1890, 62,947,714; 1880, 50,155,783; 1870, 38,558,371; 1860, 31,443,321; 1850, 23,191,876; 1840, 17,069,453; 1830, 12,866,020; 1820, 9,638,453; 1810, 7,239,881; 1800, 5,308,483; 1790, 3,929,214.

Tabulate, providing a column for the increases.

Determine the increase in population at each census.

14. Complete the following monthly stock record of a coal dealer.

Kind of Coal	On Hand Mar. 1st	Received During Month	Total Supply During Month	Sold During Month	On Hand End of Month
	Tons	Tons	Tons	Tons	Tons
Anthracite					
Egg	53	182		147	
Stove . . .	112	245		310	
Pea	93	176		215	
Bituminous					
Lump . . .	253	1245		1162	
Pocahontas .	173	648		579	

8. The usual practice in **Making Change** is based upon the adding up method of subtraction.

Example.—Find the change to be given when a $ 5.00 bill is offered in payment for purchases amounting to $ 1.63.

Solution.—$1.63 and 2¢ = $1.65; $1.65 and 10¢ = $1.75; $1.75 and 25¢ = $2.00; $2.00 and $2.00 = $4.00; $4.00 and $1.00 = $5.00.

Hence the change consists of two pennies, a dime, a quarter, a $1.00 bill and a $2.00 bill; altogether $3.37.

In practice, the procedure is as follows:

$1.63 and 2¢ is $1.65; and 10¢ is $1.75; and 25¢ is $2.00, and $2.00 is $4.00; and $1.00 is $5.00.

EXERCISE 14

Determine the coins and bills of largest denomination with which to "make change" when a $ 5.00 bill is offered as payment for purchases of:

1. 33¢
2. 72¢
3. 94¢
4. $ 1.06
5. 67¢
6. 23¢
7. 85¢
8. $ 1.15
9. $ 4.45
10. $ 3.87
11. $ 2.78
12. $ .22
13. $ 1.69
14. $ 1.35
15. $ 1.94
16. $ 3.62

Similarly, when $ 10.00 is offered after the purchase of goods that cost:

17. $ 2.56
18. $ 4.75
19. $ 6.89
20. $ 9.27
21. $ 8.32
22. $ 3.43
23. $ 2.71
24. $ 5.29
25. $ 4.58
26. $ 7.86

Determine the amount and kind of change when the buyer offers:

27. $ 5.00 after he makes purchases amounting to 18¢, 37¢, and $ 1.00.

28. $ 2.00 after he makes purchases amounting to 13¢, 67¢, and 70¢.

29. $ 10.00 after he makes purchases amounting to $ 1.25, $ 2.60, $ 3.15, and 45¢.

30. $ 5.00 after he makes purchases amounting to $ 3.17, 14¢, 29¢, and 83¢.

31. $ 20.00 after he makes purchases amounting to $ 9.89, $ 2.57, and $ 1.98.

32. $ 10.00 after purchases amounting to 76¢, $ 2.53, and 45¢.

33. $ 5.00 after purchases amounting to 98¢, 63¢, and 24¢.

34. $ 10.00 after purchases amounting to $ 1.35, $ 2.65, and $ 5.20.

35. \$2.00 after purchases amounting to 28¢, 37¢, and 46¢.

36. \$20.00 after purchases amounting to \$12.69, \$3.46, 82¢, and \$1.78.

Multiplication of Integers

9. In **Multiplication,** a **Multiplier** and a **Multiplicand** are given, and a **product** is to be determined.

The **Multiplicand** is the number multiplied; the **Multiplier** is the number by which the multiplicand is multiplied.

When the multiplier and multiplicand are integers, the product is obtained by using the multiplicand as an addend as many times as there are units in the multiplier.

The symbol for multiplication is ×. It will be read "*times*" in this book. Hence 3×5 means $5 + 5 + 5$ or 15.

10. If a number of things *of one kind* be multiplied by an integer, the product is things *of the same kind.*

Thus, 6 times 7 books is 42 books.

It is agreed that the multiplier is always a pure or abstract number. Thus 5 cannot be multiplied by 4 ft., nor can 6 bu. be multiplied by 7 bu.

11. **Parentheses,** ( ), placed about two or more numbers which are connected by mathematical symbols, means that the result obtained by performing the computation within the parentheses is to be treated as a single number in further computation.

Thus, *a.* $3 + (5 + 6)$ means that $5 + 6$ or 11 is to be added to 3.

b. $4 \times (7 + 8)$ means that $7 + 8$ or 15 is to be multiplied by 4.

12. **Three Fundamental Laws of Multiplication.**

a. Since $3 \times 5 = 5 + 5 + 5 = 15$,

and $5 \times 3 = 3 + 3 + 3 + 3 + 3 = 15$,

clearly $3 \times 5 = 5 \times 3$.

This is an illustration of

The Commutative Law of Multiplication

The multiplier and multiplicand in a product may be interchanged without changing the value of the product.

b Since $(3 \times 2) \times 5 = 6 \times 5 = 30$,
and $3 \times (2 \times 5) = 3 \times 10 = 30$,
clearly $(3 \times 2) \times 5 = 3 \times (2 \times 5)$.

This is an illustration of

The Associative Law of Multiplication

To obtain the product of three integers, the third may be multiplied by the product of the first two, or the product of the last two may be multiplied by the first.

c. Since $3 \times (4 + 5) = 3 \times 9$ or 27,
and $(3 \times 4) + (3 \times 5) = 12 + 15$ or 27,
clearly $3 \times (4 + 5) = (3 \times 4) + (3 \times 5)$.

This is an illustration of

The Distributive Law of Multiplication

To multiply the sum of two (or more) numbers by a number, multiply each of them by the multiplier and add the products so obtained.

Example 1.—$5 \times (2 + 4 + 7) = (5 \times 2) + (5 \times 4) + (5 \times 7)$

$$= 10 + 20 + 35$$

$$= 65.$$

Another form of the distributive law is illustrated in the following example.

Example 2.—$(3 + 7) \times 4 = (3 \times 4) + (7 \times 4)$

$$= 12 + 28$$

$$= 40.$$

EXERCISE 15

1. Prove that $5 \times 4 = 4 \times 5$.

Of what law is this an example?

2. Find in two ways the value of:

a. $5 \times (3 + 4)$
b. $6 \times (2 + 5)$
c. $7 \times (3 + 2)$
d. $8 \times (5 + 4)$
e. $10 \times (6 + 4)$
f. $15 \times (8 + 5)$

3. Find in two ways the value of:

a. $5 \times 3 \times 4$
b. $2 \times 4 \times 6$
c. $3 \times 4 \times 7$
d. $6 \times 2 \times 5$
e. $8 \times 3 \times 4$
f. $9 \times 4 \times 5$

13. **Skill in Multiplication** depends upon accurate and rapid recognition of the products of the forty-five pairs of integers on p. 5. Ability to give also the product when such integers as 12, 13, 14, and 15 are multiplied by any of the integers 1, 2, 3, ⋯ 9, is well worth securing, and ambitious pupils will endeavor to acquire it.

EXERCISE 16

Multiplication Drill Table

	a	*b*	*c*	*d*	*e*	*f*	*g*
a.	13	67	38	82	45	97	74
b.	26	83	65	92	66	29	38
c.	504	708	320	623	786	509	275
d.	3025	4068	5409	6725	8392	7653	8207
e.	5746	7295	6327	4008	8659	4767	9724

Read the note at the beginning of Exercise 4.

1. Multiply each number of row *a* by:

a. 2 *b.* 3 *c.* 4 *d.* 5 *e.* 6 *f.* 7 *g.* 8 *h.* 9

2–5. Use rows *b*, *c*, *d*, and *e* in the same manner.

6–12. Multiply all the numbers of the table by 13, 24, 35, 46, 57, 68, and 98 respectively.

13. Give the result of multiplying each number in row *a* by:

a. 10 *b.* 100 *c.* 1000

14. Give a rule for multiplying an integer by:

a. 10 *b.* 100 *c.* 1000

15. Multiply each of the numbers

3000 240 5300 650 8200 7600

by:

a. 2 *b.* 7 *c.* 30 *d.* 50 *e.* 80

14. The Multiplication Process is based upon the Distributive **Law of Multiplication.** (§ 12.)

Example 1. — Multiply 462 by 7.

The actual process is:

$$\begin{aligned} 7 \times 462 &= 7 \times (400 + 60 + 2) \\ &= 2800 + 420 + 14 \\ &= 3234. \end{aligned}$$

The solution is usually arranged thus:

$$\begin{array}{r} 462 \\ \underline{7} \\ 3234 \end{array}$$

7 × 2 units is 14 units, or 1 ten and 4 units; 4 is written in the units' place, and 1 ten is added to the next product.

7 × 6 tens is 42 tens. Adding 1 ten gives 43 tens, or 4 hundreds and 3 tens. 3 is written in the tens' place, and 4 hundreds are added to the next product.

7 × 4 hundreds is 28 hundreds. Adding 4 hundreds gives 32 hundreds or 3 thousand, 2 hundred. These are written in their proper places.

Example 2. — Multiply 65 by 27.

The actual process is

$$\begin{aligned} 27 \times 65 &= (20 + 7) \times 65 \\ &= (20 \times 65) + (7 \times 65) \\ &= 1300 + 455 \\ &= 1755. \end{aligned}$$

The solution is usually arranged thus:

$$\begin{array}{r} 65 \\ \underline{27} \\ 455 \\ \underline{130} \\ 1755 \end{array}$$

EXERCISE 17

Multiply the following:

1. 873 by 956.
2. 2600 by 3950.
3. 487 by 8009.
4. 4067 by 997.
5. 3476 by 625.
6. 6872 by 599.
7. 60507 by 3784.
8. 15,063 by 9874.
9. 54,189 by 7998.
10. 7677 by 4912.
11. 2946 by 5335.
12. 82,821 by 7269.
13. 93,247 by 2461.
14. 35,895 by 6927.

15. Find the value of a hay stack estimated to contain 35 tons of hay, if hay sells at $ 22 per ton.

16. A farmer has 145 bushels of potatoes from a 1-acre field. What is their value if potatoes are selling at 85 ¢ per bushel?

17. What is the value of the corn harvested from a 20-acre field if the average yield is 32 bushels per acre, and the crop sells for 87 ¢ per bushel?

18. What is the value of the wheat harvested from a 20-acre field if the average yield is 19 bushels per acre and the crop sells for 95 ¢ per bushel?

19. What is the value of a crop of oats harvested from a 20-acre field if the average yield is 45 bushels per acre and the crop sells for 39 ¢ per bushel?

20. What is the cost of furnishing each of 650,000 soldiers one pair of shoes at $ 4.85 per pair?

21. What is the cost of furnishing each of 375,000 soldiers a rifle at $ 24.50 each?

22. A certain electric light company makes a charge of 26 ¢ per foot for extensions of its line. What will be the cost for running a line four miles?

23. A farmer has fifteen cows, each of which yields, on the average, 6200 pounds of milk per year. What is his income from the sale of milk if he gets $2.23 per hundred pounds?

24. A florist has the following plants on hand to be sold at the indicated average prices during the Christmas holidays.

500 plants to sell at $2.50
500 plants to sell at $1.75
800 plants to sell at $1.25
1250 plants to sell at $ .65

What will he receive if he sells all of them?

25. In a certain city, 77 pensioners received payments from the firemen's pension fund, as follows:

29 widows who received $20 a month each;
28 minors who received $6 a month each;
13 disabled firemen who received $50 a month each;
5 retired firemen who received $40 a month each;
2 dismissed firemen who received $30 a month each.

What was the total payment per month from the fund?

Short Methods of Multiplication

15. There are several **Short Methods of Multiplication** that can be used so often as to justify an effort to learn them.

a. Multiplying by 9 is the same as multiplying by (10 – 1).

Thus:

$$\begin{aligned} 9 \times 56 &= (10 - 1) \times 56 \\ &= (10 \times 56) - (1 \times 56) \\ &= 560 - 56 \\ &= 504. \end{aligned}$$

The solution may be arranged thus:

$$\begin{array}{r} 56 \\ \underline{9} \end{array} = \begin{array}{r} 560 \\ \underline{56} \\ 504 \end{array}$$

Rule. — To multiply any number by nine, annex a zero to it and from the result subtract the given number.

Example. — Multiply 463 by 9.

Solution.

$$\begin{array}{r}463\\ \underline{9}\end{array} = \begin{array}{r}4630\\ \underline{463}\\ 4167\end{array}$$

b. Multiplying by 99, 999, etc., may be performed in similar manner; *e.g.*

Rule. — **To multiply any number by 99, annex two zeros to it (*i. e.* multiply by 100) and then subtract the given number.**

Example. — Multiply 234 by 99.

Solution.

$$99 \times 234 = \begin{array}{r}23400\\ \underline{234}\\ 23166\end{array}$$

c. Multiplying by 50 is the same as multiplying by $\frac{1}{2}$ of 100. Hence,

Rule. — **To multiply any number by 50 annex two zeros to it (*i. e.* multiply by 100) and divide the result by 2.**

Example. — Multiply 436 by 50.

Solution. — $436 \times 50 = \frac{43600}{2} = 21800.$

d. Multiplying by 25 is the same as multiplying by $\frac{1}{4}$ of 100. Hence,

Rule. — **To multiply any number by 25, annex two zeros to it and then divide by 4.**

Example. — Multiply 789 by 25.

Solution. — $789 \times 25 = \frac{78900}{4} = 19725.$

e. Multiplying by 15 is the same as multiplying by $(10 + 5)$. But the product of any number by 5 is one half its product by 10. Hence,

Rule. — **To multiply any number by 15, first annex a zero to the number and then to the result add one half itself.**

Example.—Multiply 234 × 15.

Solution.—
$$\begin{array}{r l} 234 \times 15 = & 2340 = 10 \times 234 \\ & \underline{1170} = \tfrac{1}{2} \text{ of } 2340 \\ & 3510 \end{array}$$

f. Multiplying by 75 is the same as multiplying by (50 + 25). But the product of any number by 25 is one half its product by 50. Hence,

Rule.—To multiply any number by 75, annex to it two zeros and divide by 2. (See Rule (*c*)), and then add to this result one half itself.

Example.—Multiply 378 by 75.

Solution.—
$$\begin{array}{r l} 378 \times 75 = 2 \,| & \!\!\underline{37800} \\ & 18900 = 50 \times 378 \\ & \underline{\ \ 9450} = \tfrac{1}{2} \text{ of } 18900 \\ & 28350 \end{array}$$

g. Rules (*a*) and (*b*) are only special cases of the device illustrated in the following examples:

Example 1.—Multiply 76 by 103.

Solution.—Since 103 = 100 + 3,
$$\begin{aligned} 103 \times 76 &= (100 + 3) \times 76 \\ &= (100 \times 76) + (3 \times 76) \\ &= 7600 + 228 \\ &= 7828 \end{aligned}$$

The solution should be arranged thus:
$$\begin{array}{r l} 103 \times 76 = & 7600 = 100 \times 76 \\ & \underline{\ \ 228} = 3 \times 76 \\ & 7828, \text{ adding} \end{array}$$

Example 2.—Multiply 237 × 98.

Solution.— 98 = 100 − 2.
$$\therefore 98 \times 237 = (100 \times 237) - (2 \times 237).$$

The solution should be arranged thus:
$$\begin{array}{r l} 98 \times 237 = & 23700 = 100 \times 237 \\ & \underline{\ \ \ \ 474} = \ \ 2 \times 237 \\ & 23226, \text{ subtracting} \end{array}$$

This form of multiplication can be employed to advantage whenever the multiplier is a number a little more or a little less than 10, 100, 1000, etc.

h. Familiarity with these rules and careful scrutiny of the multipliers will suggest other short methods of multiplication.

EXERCISE 18

1. Multiply each of the following numbers by 9:

a. 27
b. 46
c. 78
d. 94
e. 263
f. 375
g. 496
h. 547
i. 624
j. 938
k. 463
l. 758

2. Multiply each of the numbers given in Example 1 by 99.

3. Multiply each of the numbers given in Example 1 by 999.

4. Multiply each of the numbers given in Example 1 by 50.

5. Multiply each of the numbers given in Example 1 by 25.

6. Multiply each of the numbers given in Example 1 by 15.

7. Multiply each of the numbers given in Example 1 by 75.

8. Multiply the first six numbers in Example 1 by 97.

9. Multiply the first six numbers in Example 1 by 102.

10. Multiply the first six numbers of Example 1 by 98.

11. Multiply the first six numbers of Example 1 by 105.

12. Find the cost of 75 caps at 99¢ each.

13. Find the cost of 325 bushels of corn at $1.04 per bushel.

14. Find the cost of 137 bushels of potatoes at 93¢ per bushel.

15. Find the total length of 76 boards, each 25 inches long.

16. Find the total weight of 130 bags of sugar, each weighing 50 pounds.

17. Find the cost of 56 window shades at 75¢ each.

18. Find the cost of 23 pounds of beef at 25¢ per pound.

19. Find the cost of 35 heads of lettuce at 15¢ per head.

20. Find the cost of three dozen shovels at $1.50 each.

21. Find the cost of 54 bags of cement at 50¢ per bag.

22. If concrete curbing and gutter on a street cost 65¢ per running foot, what is the cost for the curbing and gutter on both sides of a city block 625 feet long?

23. Find the cost of 745 bushels of corn at $1.10 per bushel.

24. Find the cost of excavating 1345 cubic yards of dirt at 25¢ per yard.

25. How much should a farmer receive for 78 pounds of spring chicken at 25¢ per pound?

26. How much pay is due an employee who has worked 54 hours at the rate of 50¢ per hour? At the rate of 25¢ per hour? At the rate of 75¢ per hour? At the rate of 15¢ per hour?

27. The cashier in a lunch room takes in three 15¢ checks, seventy-four 25¢ checks, and thirty-three 50¢ checks. How much money was taken in?

28. The ticket seller at a game sold 428 tickets at $1.25; 642 at $1.50; and 233 at $1.75. How much cash should he have?

29. What is the cost of 13 yards of lace at 94¢ per yard?

30. How much should a fruit grower receive for 37 cases of strawberries at $2.10 per case?

31. In making a 16-ft. concrete highway, 100 bags of cement were used for each 50 feet. What is the total number of bags of cement required for 1 mile of highway?

Bills

16. An itemized statement of articles or goods sold and their cost, or a statement showing the amount of work done by one person or firm for another, together with the amount of remuneration earned, is called a **Bill**.

Example 1.

NORTHTOWN, ILLINOIS.
Feb. 18, 1918.

Johnson Coal Company
Two Lakes, Wisconsin.

Bought of **The Northern Illinois Mining Company.**

TERMS — Cash in 30 days.

2/15/16	*37 T*	*#2 Lump Coal*	*@ $2.35*	*86*	*95*		
2/17/16	*65 T*	*Mine Run*	*@ $1.89*	*122*	*85*	*209*	*80*

NOTE. — The symbol @ is read "at."

Example 2.

N. B. Warner Co.
Seedsmen
Chicago, Illinois.

SOLD TO Bender Floral Co.
Washington, Indiana.

TERMS — 60 days DATE *3/23/18*

2000	*Narcissus Emperor*	*@ $14.50*	*29*	*00*		
1500	*Narcissus Empress*	*@ $14.00*	*21*	*00*	*50*	*00*

Received Payment
Oct. 12, 1917
W. B. Warner Co.

EXERCISE 19

Prepare bills for the following examples:

1. On Sept. 23, 1917, the Davis Fur Co. sold to A. M. Johnson Co., Owasee, Ill., the following items:

3 pair gloves #4345 @ $5.25; 1 pair gloves #4475 @ $6.75; 12 pair gloves #4375 @ $2.60; 8 pair boys' mittens #3450 @ $.79; and 9 pair gloves #9354 @ $4.20.

2. On March 15, 1918, the Jenkins Saw Co. of Indianapolis, Indiana, sold to the Marshville Hardware Co., Marshville, Indiana, the following goods:

½ doz. 26″ ripsaws #8 m 320 @ $1.65; ¼ doz. 28″ ripsaws #8 m 320 @ $1.75; ⅓ doz. 26″ handsaws #8 m 315 @ $1.58; ½ doz. 24″ handsaws #8 m 330 @ $1.95; 1 doz. 22″ panel saws #8 m 332 @ $1.83; 1 doz. 26″ handsaws #8 m 340 @ $2.15; ⅔ doz. 28″ ripsaws #8 m 342 @ $2.45; and ⅓ doz. hacksaw outfits #8 m 350 @ $1.78.

3. The Central Book Co. of Detroit, Mich., on Aug. 24, 1917, sold to J. C. Carpenter, Stationer and Bookseller, Detroit, Mich., the following books:

24 copies Corn Cookery by Cade @ 60¢; 8 copies Modern Cook Book by Jewett @ $1.35; 15 copies Simple Cookery by Tyson @ $1.00; 25 copies Meals Without Meat by James @ $1.80; 36 copies Wheatless Meals by Morgan @ 63¢.

4. The States Auto Supply Co. of Chicago, Ill., sold on April 17, 1918, the following goods to the Northern Indiana Garage, Crown Point, Indiana:

1 doz. wheel pullers @ 22¢ each; 2 doz. rear tire holders @ $1.75 each; 3 doz. sets cork gaskets @ 50¢ per set; ½ doz. steel autolocks @ $1.70 each; 1½ doz. pump connections @ 25¢ each; 4 doz. boxes repair patches @ $8.40 per dozen; 8 Merkel vulcanizers @ 75¢ each; 15 Ford timers @ 85¢ each; 8 doz. Chic spark plugs @ 35¢ each; 18 two-cylinder pumps @ $1.35 each.

5. John Dahl, Carpenter, Maumee, Ill., on May 17, 1918, submitted the following bill for labor and materials furnished for a repair job for Andrew Cameron:

400 ft. oak flooring @ $85 per 1000 ft.; 400 ft. hemlock flooring @ $40 per 1000 ft.; 2 doors @ $5.35; 100 ft. finishing oak @ $120 per 1000 ft.; 5 (8 hr.) days, master carpenter @ 60¢ per hr.; 5 days journeyman carpenter @ 45¢ per hr.; 5 days apprentice @ 30¢ per hr.

6. The Browning Shoe Co., East St. Louis, Ill., sold to A. M. Jeffrey, Pardee, Ill., the following goods, on May 15, 1918:

10 pr. shoes #1507 @ $3.73; 8 pr. shoes #1208 @ $3.56; 15 pr. shoes #1397 @ $4.00; 5 pr. shoes #1505 @ $6.70; 20 pr. shoes #1061 @ $3.20; 25 pr. oxfords #1303 @ $2.45; 15 pr. oxfords #6241 @ $2.90; 24 pr. slippers #1393 @ $1.35; 18 pr. brown shoes #1340 @ $4.45; and 6 pr. brown shoes #1807 @ $4.25.

7. On Feb. 25, 1918, the Payson Brush Co. of Cleveland, Ohio, sold to Morgan Bros., Interior Decorators, the following goods:

½ doz. 8″ whitewash brushes #131 @ $2.50; ⅓ doz. 8¼″ whitewash brushes #131 @ $2.95; ¾ doz. 7″ kalsomine brushes #310 @ $1.25; ⅔ doz. 8″ kalsomine brushes #310 @ $1.43; ⅓ doz. 7″ kalsomine brushes #311 @ $1.72; ¼ doz. 8″ kalsomine brushes #311 @ $2.12; 1½ doz. 8″ paste brushes #331 @ $2.32; ⅔ doz. 10″ smoothing brushes #335 @ $1.97; 1⅓ doz. 12″ smoothing brushes #335 @ $2.43; 1¼ doz. dusting brushes #321 @ $.63.

8. The Western Table Co. of Grand Rapids, Mich., sold on March 10, 1918, to Kellog and Burgess, Furniture Dealers of Kalamazoo, Mich., the following goods:

25 kitchen tables #L 240 @ $2.15; 6 porcelain-top tables #L 243 @ $6.78; 18 drop-leaf tables #L 241 @

$ 7.42; 5 extension tables # L 250 @ $ 11.78; 12 extension tables # L 253 @ $ 14.55; 3 dining tables # L 248 @ $ 17.67; 4 dining-room tables # L 257 @ $ 23.87.

9. Martin and Lewis, Wholesale Clothiers, New York City, sold on Jan. 30, 1918, to the Hub Clothing Store of Jersey City, New Jersey, the following goods:

15 boys' suits # 3700 @ $ 10.70; 18 boys' suits # 1300 @ $ 10.85; 24 boys' suits # 1031 @ $ 9.75; 25 boys' suits # 9300 @ $ 9.85; 12 boys' suits # 6300 @ $ 12.35.

10. Mr. Otto Niemann built for Mr. William Harvey on his estate a drive, and on Sept. 17, 1917, submitted a bill for the following items:

Grading — 3 men, 2 days each @ $ 2.75 per day; 1 man, 2 days @ $ 4.00 per day; 2 teams, 2 days @ $ 5.50 per day. Stone, and spreading stone — 96 cu. yd. crushed stone @ 85¢ per yd.; 42 cu. yd. screenings @ 75¢ per yd. Labor — 2 men 3 days each @ $ 2.75 per day; 1 man 3 days @ $ 4.00 per day. Steam roller — 1 day @ $ 10.00 per day.

Division of Integers

17. In **Division** two numbers are given, the **Dividend** and the **Divisor.** Division is the process of determining the number, called the **Quotient,** by which to multiply the divisor so as to give the dividend as product. The symbol for division is $\div$.

Thus, $15 \div 3$ is 5, for $5 \times 3 = 15$.

The dividend is the number divided; the divisor is the number by which the dividend is divided.

If the dividend and divisor are integers and the quotient is also an integer, the division is exact; then,

$$\textit{dividend} = \textit{quotient} \times \textit{divisor}.$$

When the dividend and divisor are integers, and no integer can be found which will serve as quotient, then the division is not exact, and there is a **Remainder.** This remainder is the difference between the dividend and nearest smaller integer which does exactly contain the divisor.

Thus, 17 does not exactly contain 3. 15 is the next smaller integer which does exactly contain 3. 17 – 15 or 2 is the remainder when 17 is divided by 3. Notice that

$$17 = 5 \times 3 + 2.$$

In general, *dividend* = *quotient* × *divisor* + *remainder.*

18. The following forms for division are suggested:

Example 1. — Divide 756 by 3.

$$\begin{array}{r} 252 \\ 3\,\underline{\overline{)756}} \end{array}$$

Explain this solution.

Example 2. — Divide 22236 by 68.

$$\begin{array}{rr} & 327 \\ 68 & \overline{)22236} \\ & \underline{204} \\ & 183 \\ & \underline{136} \\ & 476 \\ & \underline{476} \end{array}$$

68 is contained in 222 three times with a remainder. Place 3 in the quotient above the 2 in the hundreds' place. 3 × 68 = 204. Subtracting 204 from 222 gives 18. Annex 3 to 18, forming 183. 183 contains 68 twice with a remainder. Place 2 in the quotient in the tens' place. 2 × 68 = 136. Subtracting 136 from 183 gives 47. Annex 6 to 47, forming 476. 476 contains 68 seven times. Place 7 in the quotient in the units' place. 7 × 68 = 476. Subtracting 476 from 476, the result is zero. Hence the quotient is 327, and the remainder is zero.

Example 3. — Divide 80791 by 386.

$$\begin{array}{rrl} & 209 & \\ 386 & \overline{)80791} & \\ & \underline{772} & \\ & 3591 & \\ & \underline{3474} & \\ & 117 & = \textit{Remainder} \end{array}$$

In this solution, after subtracting 3474, there is the remainder 117.

EXERCISE 20

Division Drill

	a	*b*	*c*	*d*	*e*	*f*	*g*
a.	56	78	104	93	187	216	1302
b.	48	97	233	75	128	463	3522
c.	39	86	326	85	209	527	2674
d.	66	45	448	67	382	307	4849
e.	96	59	267	44	175	438	5367

Read the note at the beginning of Exercise 4, p. 5.

1. Give the quotient and the remainder when each number of row *a* of the foregoing table is divided by:

a. 2 *b.* 3 *c.* 4 *d.* 5 *e.* 6 *f.* 7 *g.* 8 *h.* 9

2–5. Use rows *b*, *c*, *d*, and *e* in the same manner.

NOTE.—Other drill examples can be based upon this table. For example, divide the numbers of the table by 11, 12, 13, etc.

EXERCISE 21

Division Drill Table

	a	*b*	*c*	*d*	*e*
a.	8557	11266	12212	14749	21414
b.	10260	16059	22032	31698	38826
c.	10792	18164	27968	37012	56088
d.	24795	39498	49068	60204	74472
e.	29406	47236	61247	73138	85940

1. Divide the numbers of row *a* by 43.

2. Divide the numbers of row *b* by 54.

3. Divide the numbers of row *c* by 76.

4. Divide the numbers of row *d* by 87.

5. Divide the numbers of row *e* by 98.

NOTE.—Other drill examples can be made by changing the divisors. For example, divide every number in the given table by 215.

19. In applications of division, two types of problems occur.

If any number of things of one kind be divided by another number of things of the same kind, the quotient is an abstract number.

Example 1. — A train travels 35 miles an hour. In how many hours will it travel 105 miles if it travels always at the same rate?

Solution. — 1. 105 miles $\div$ 35 miles $= 3$.
2. Hence it will take 3 hours.

If any number of things of one kind be divided by an abstract number, the quotient will be a number of things of the same kind as the dividend.

Example 2. — A train travels 120 miles in three hours. How far does it travel in one hour if it travels always at the same rate?

Since it travels 120 miles in 3 hours, it will travel in one hour as many miles as 3 is contained times in 120, or 40 miles.

$$120 \div 3 = 40 \text{ miles.}$$

In applications of division, the remainder frequently has no meaning and must be ignored.

Example 3. — An autoist stopping to buy gasoline finds that he has just $1.67. If gasoline is selling for 22¢ per gallon, how many gallons may he ask for?

Solution. — $\$1.67 \div \$.22 = 7^{+}$.

That is, he has enough money for 7 gallons, but not enough for 8 gallons. He buys 7 gallons and has 13¢ left over.

$$\begin{array}{r|r} & 7 \\ \cline{2-2} 22 & 167 \\ & 154 \\ \cline{2-2} & 13 \end{array}$$

This example illustrates one of the most common forms in which division is used. The division is performed with the expectation that there will be a remainder, the desire being to find the largest possible integral quotient.

EXERCISE 22

1. How many windows can be glazed with 144 panes of glass if four panes are required for each window?

2. How many carloads of 40 tons each will be required to deliver 33,600 tons of coal?

3. Electric light companies place poles between 100′ and 120′ apart. Assuming that the poles are placed 110′ apart, how many poles are required for one mile of electric light line? If the poles cost $1.75 each, what is the total cost for poles?

4. How many bed sheets 99 inches long can be cut from a bolt of muslin containing 50 yards, if 4 inches must be allowed on each sheet for hems?

5. In 1915, 221,537 short tons of coal briquets were made, valued at $1,035,716. What was the value of coal briquet per ton?

6. A long ton of American coal occupies about 43 cubic feet. How many tons in round numbers can be carried in a vessel having a storage capacity of 180,000 cubic feet?

7. How many kitchen towels about 30″ long can be cut from a 10-yard bolt of toweling, no allowance being made for hemming?

8. How many window curtains each 68″ long can be cut from 36 yards of curtain material, if 4″ are allowed on each curtain for the hem at bottom and top?

9. How many pillow slips 36″ long can be cut from a bolt of pillow-slip tubing containing 10 yards, if 4″ are allowed for hems on each slip?

10. A man decides to spend about $200 for coal. If coal costs $11.50 per ton, how many full tons of coal may he order?

11. If oats are selling for $.83 per bushel, how many bushels may a man order who cannot spend more than $10.00? How much change will he receive from his $10.00 bill?

12. How many shrubs at $.35 each may a man order who can afford to spend at most $15.00?

13. If a maid can save $3.75 per week, how many weeks will it take her to save enough to buy a suit costing $22.50?

14. A worker folding Red Cross surgical dressings, folded 75 in one hour. What was the average time for folding each dressing?

15. There were approximately 260,000 miles of railroads in the United States in 1917. They earned net about $1,022,741,992. What was the net income per mile in dollars? (That is, ignore the cents in the result.)

The operating expenses were $2,943,458,440. Find the operating cost per mile in dollars.

16. In 1900, there were 5,737,372 farms in the United States, embracing 838,591,774 acres, and valued at $20,439,901,164; in 1910, there were 6,361,502 farms, embracing 878,798,325 acres, valued at $40,991,449,090. Find the approximate size in acres and the approximate value in dollars of a farm in each of these years.

Arrange the given data and your results in a table.

17. The exports of wheat in 1913–1914 were 92,393,755 bushels, valued at $87,953,456; in 1914–1915, 259,642,533 bushels, valued at $333,552,226; in 1915–1916, 173,274,015 bushels, valued at $215,532,681.

Tabulate these statistics, providing a column for the value of the wheat per bushel. Determine the value in dollars and cents, and insert it in the proper column.

18. The following ration, approximately, is used by a certain agricultural college for feeding its dairy cows:

Silage, 3 lb. per day per 100 lb. live weight
Hay, 1 lb. per day per 100 lb. live weight
Beets, 1 lb. per day per 100 lb. live weight
Concentrated feed, 1 lb. per day per $3\frac{1}{2}$ lb. of milk produced per day

How many tons, approximately, of silage, hay, beets, and concentrated feed are required for a herd of 10 Guernsey cows whose average weight is 1050 pounds and which produce on the average 7000 pounds of milk per year per cow?

19. In 1914, there were 9,045,081 ordinary life insurance policies in force in the United States, amounting to $17,425,501,137. What was the average value of each policy?

20. If an automobile travels one mile in 45 seconds, how many miles would it travel in an hour at that rate?

Factoring Integers

20. A **Prime Integer** is one that can be divided exactly only by itself and unity; as, 1, 2, 3, 5, 7, 11, etc.

A **Composite Integer** is one that can be divided exactly by other integers as well as itself and unity; as, 4, 6, 8, 9, 10, etc.

21. Every integer by which a number can be divided exactly is a **Factor** of that number.

Thus, 3, 8, 6, 12, etc., are factors of 24.

22. An **Even Number** is one that can be divided by 2.

23. The **Prime Factors** of an integer are the prime integers whose product is the given integer.

Thus, the prime factors of 24 are 2, 2, 2, and 3, for

$$2 \times 2 \times 2 \times 3 = 24.$$

24. **To Factor** an integer means to find its prime factors.

Rule. — To factor an integer:

1. Divide the integer by one of its prime factors.

2. Divide the quotient by one of its prime factors.

3. Continue in this way until a prime quotient is obtained.

4. The complete set of factors consists of the various divisors and the final quotient.

Example. — Factor 1092.

Solution. — 1. $1092 = 2 \times 546$

2. $= 2 \times 2 \times 273$ (for $546 = 2 \times 273$)

3. $= 2 \times 2 \times 3 \times 91$ (for $273 = 3 \times 91$)

4. $= 2 \times 2 \times 3 \times 7 \times 13$ (for $91 = 7 \times 13$)

5. $\therefore$ the factors of 1092 are 2, 2, 3, 7, and 13.

In applying this rule, always start with the smallest prime integer that appears to be a factor. Also, *use the form of solution* employed in the illustrative example.

25. The following **Rules of Divisibility** aid in selecting the factors of a number.

I. A number is divisible by 10 if it ends in zero.

II. A number is divisible by 5 if it ends in zero or five.

III. A number is divisible by 2 if it ends in zero or an even number.

IV. A number is divisible by 3 if the sum of its digits is divisible by three.

Thus, 384 is divisible by 3 since $3 + 8 + 4$ or 15 is divisible by 3.

The proofs of these rules appear in § 30, p. 46.

EXERCISE 23

1. What are the prime numbers between 1 and 100?

Find the prime factors of the following numbers:

2. 26
3. 38
4. 42
5. 54
6. 63
7. 68
8. 75
9. 84
10. 96
11. 110
12. 128
13. 145
14. 212
15. 237
16. 256
17. 264
18. 275
19. 294
20. 310
21. 345
22. 382
23. 444
24. 526
25. 612
26. 738
27. 828
28. 885
29. 962
30. 1000
31. 1256

26. A **Common Divisor** or **Common Factor** of two or more integers is a factor of each of them.

Thus, 2 is a common divisor of 18, 24, and 38.

27. Two numbers that do not have a common divisor except 1 are **prime to each other.**

Thus, 8 and 11 are prime to each other.

28. The **Greatest Common Divisor** (**G. C. D.**) of two or more integers is the greatest integer that is a divisor of each of them.

Thus, 6 is the G. C. D. of 18, 24, and 42.

29. Finding the G. C. D. of Given Integers.

Example. — Find the G. C. D. of 144, 264, and 540.

Solution. — 1. Factor each of the integers. (§ 24)

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$264 = 2 \times 2 \times 2 \times 3 \times 11$$
$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

2. 2 and 3 are the only common factors.
3. 2 appears only twice as a factor in 540.
3 appears only once as a factor in 264.
4. The G. C. D. $= 2 \times 2 \times 3 = 12$.

Rule. — To find the G. C. D. of two or more integers:

1. Find the prime factors of each of them.

2. Form the product of all the different prime factors, using each of them the least number of times that it occurs as a factor in any of the given numbers.

EXERCISE 24

Find the G. C. D. of:

1. 36 and 144.
2. 108 and 132.
3. 81 and 126.
4. 165 and 210.
5. 288 and 648.
6. 128 and 192.
7. 675 and 1125.
8. 72, 60, and 84.
9. 60, 105, and 150.
10. 36, 90, and 126.
11. 117, 81, and 126.
12. 104, 182, and 351.
13. 525, 3375, and 7425.
14. 160, 224, and 260.

15. A farmer has three pieces of timber whose lengths are 63, 84, and 105 feet, respectively. What is the length of the longest logs all of the same length that can be cut from them?

16. Two schools, containing 480 and 672 pupils, respectively, are divided into classes containing the same number of pupils. What is the greatest number of pupils that can be taken for a class?

SUPPLEMENTARY TOPICS

30. A. Proofs of the Rules of Divisibility. (Page 44)

Proof of I. — A number whose last digit is 0 consists of a number of tens. Hence any number ending in 0 can be divided by 10.

Thus, 560 consists of 56 tens, for 500 = 50 tens and 60 = 6 tens.

Proof of II. — If a number ends in 0, it consists of a number of tens. Each ten can be divided by 5. Hence the whole number can be divided by 5.

Thus, 360 = 36 tens, and therefore 72 fives.

If a number ends in 5, it consists of a number of tens and five units.

Thus, 365 consists of 36 tens and 5 units. The tens are divisible by 5 and also the 5 units are divisible by 5. Hence the whole number is divisible by 5.

Proof of III. — If a number ends in 0, it consists of a number of tens. Each ten is divisible by 2. Hence the number is divisible by 2.

If a number ends in 2, 4, 6, or 8, it consists of a number of tens plus 2, 4, 6, or 8 units.

The tens are divisible by 2, and also the units are divisible by 2. Hence the whole number is divisible by 2.

Proof of IV. — Consider the number 684.

1.
$$\begin{aligned}684 &= 6 \text{ hundreds} + 8 \text{ tens} + 4 \text{ units}\\ &= 6(99 + 1) + 8(9 + 1) + 4\\ &= 6 \times 99 + 6 + 8 \times 9 + 8 + 4\\ &= 6 \times 99 + 8 \times 9 + (6 + 8 + 4)\end{aligned}$$

2. But 6×99 and 8×9 are each divisible by 3, since 99 and 9 are divisible by 3.

3. Therefore 684 is divisible by 3 if $(6 + 8 + 4)$ is divisible by 3.

4. $$6 + 8 + 4 = 18 = 6 \times 3.$$

5. Therefore 684 *is* divisible by 3.

B. Greatest Common Divisor by a Division Process.

31. Finding the G. C. D. of Two or More Numbers that Cannot be Factored Easily.

Example 1. — Find the G. C. D. of 221 and 493.

$$\begin{array}{r|r|l} 221 & 493 & 2 \\ & \underline{442} & \\ & 51 & \end{array}$$

Solution. — 1. Dividing 493 by 221, the quotient is 2 and remainder is 51.

2. Any factor of 221 is also a factor of 442, since $442 = 2 \times 221$.

3. Hence any factor common to 493 and 221 must also be common to 493 and 442, and therefore to 493 and 51.

4. Also, any factor common to 221 and 51 must occur in the result when 51 is added to 2 × 221; *i.e.*, it must occur in 493.

5. Hence 221, 493, and 51 have the same common factors. In general, — *the G. C. D. of any two integers is the same as the G. C. D. of the less integer and the remainder obtained when the greater is divided by the less.*

6. Dividing 221 by 51, we have

7. ∴ the G. C. D. of 51 and 221 is the same as the G. C. D. of 17 and 51.

8. Dividing 51 by 17, the quotient is 3. Hence the G. C. D. of 51 and 17 is 17.

```
51)221(4
   204
    17
```

9. Therefore 17 is the G. C. D. of 51 and 221. (Step 7.)
Therefore 17 is the G. C. D. of 221 and 493. (Step 5.)

This solution illustrates the

Rule. — To find the G. C. D. of two integers:

1. Divide the greater by the less.

2. If there is a remainder, divide the previous divisor by it.

3. Continue in this manner, making each remainder a divisor of the preceding divisor until there is no longer a remainder.

4. The last divisor is the G. C. D. of the given integers.

Example 2. — Find the G. C. D. of 377 and 667.

Solution. — 1.

```
377)667(1
    377
    290)377(1
        290
         87)290(3
            261
             29)87(3
                87
```

2. Therefore the G. C. D. of 377 and 667 is 29.

NOTE. — If the G. C. D is 1, the integers are prime to each other. (§ 27)

EXERCISE 25

Find the G. C. D. of the following sets of integers:

1. 559 and 817.
2. 391 and 598.
3. 589 and 899.
4. 703 and 893.
5. 533 and 1271.
6. 731 and 1247.
7. 3658 and 4602.
8. 3703 and 6923.
9. 1591 and 2183.
10. 5605 and 6785.
11. 6059 and 7446.
12. 5312 and 10043.
13. 2291 and 3713.
14. 10057 and 11659.

Least Common Multiple of Integers

32. A **Multiple** of an integer is any integer which will contain the given integer as an exact divisor.

Thus, 12, 20, 32, etc., are all multiples of 4.

33. A **Common Multiple** of two or more integers is a multiple of each of them. It will contain each of them as an exact divisor.

Thus, 36 is a common multiple of 2, 3, and 9; also 18 and 54 are common multiples of 2, 3, and 9.

34. The **Least Common Multiple** of two or more integers (**L. C. M.**) is the smallest integer that will contain each of them exactly as a divisor.

Thus, the L. C. M. of 2, 3, and 9 is 18.

35. Finding the L. C. M. of Given Integers.

Example 1.—Find the L. C. M. of 40, 84, and 144.

Solution.—1. Factor each of the given integers. (§ 24)

$$40 = 2 \times 2 \times 2 \times 5$$
$$84 = 2 \times 2 \times 3 \times 7$$
$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

2. The L. C. M. must contain 144 and hence it must contain the factors 2, 2, 2, 2, 3, and 3.

3. In order to contain 84, the L. C. M. must contain the factors of 84; that is, the factors 2, 2, 3, and 7. It must therefore contain the factor 7 besides those named in Step 2.

4. In order to contain 40, the L. C. M. must contain the factor 5, as well as the factors 2, 2, 2.

5. Therefore the L. C. M. must have the factors 2, 2, 2, 2, 3, 3, 5, 7.

That is, the L. C. M. $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 5040$.

Rule. — To find the L. C. M. of two or more integers:

1. Find the prime factors of each of the integers.

2. Form the product of all the different prime factors that appear in Step 1, using each of them the greatest number of times that it occurs in any one of the given integers.

Example 2. — Find the L. C. M. of 24, 60, and 105.

Solution. — 1. $24 = 2 \times 2 \times 2 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

$105 = 3 \times 5 \times 7$

2. 2 appears as a factor three times in 24, and less frequently in the other integers.

3 appears as a factor only once in any of the integers.

5 appears as a factor once in 60.

7 appears as a factor once in 105.

3. $\therefore$ the L. C. M. $= 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$.

NOTE. — If the L. C. M. can be seen by inspection, then certainly get it that way, — do not take this longer way. Thus, the L. C. M. of 2, 3, and 4 is evidently 12.

EXERCISE 26

Find the L. C. M of:

1. 4, 6, and 9.
2. 3, 5, and 10.
3. 6, 7, and 8.
4. 8, 10, and 15.
5. 4, 14, and 21.
6. 4, 8, and 20.
7. 3, 6, and 15.
8. 8, 12, and 18.
9. 6, 9, and 12.
10. 18, 38, and 54.
11. 36, 104, and 24.
12. 33, 44, and 55.
13. 108 and 144.
14. 20, 75, and 180.
15. 32, 88, and 121.
16. 28, 49, and 147.
17. 15, 35, and 77.
18. 52, 81, and 117.
19. 98, 126, and 140.
20. 115, 138, and 230.

II. FRACTIONS

36. A **Common Fraction** consists of two integers, a **Numerator** and a **Denominator**. It is usually expressed by writing the numerator over the denominator, separated by a horizontal line. The numerator and the denominator together are called the terms of the fraction.

In elementary arithmetic, the denominator is regarded as indicating a number of parts into which a whole is divided and the numerator as indicating the number of the parts being considered.

Thus, $\frac{3}{4}$ is interpreted as three "one fourths" of a unit.

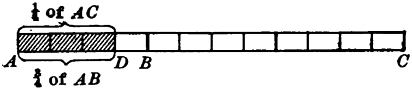
Mathematicians regard a fraction as indicating the division of the numerator by the denominator.

Thus, $\frac{3}{4}$ is interpreted by them as 3 divided by 4, or one fourth of three equal units.

Both meanings are useful in applications of fractions.

The two meanings are equivalent, in the case of integers.

In the following figure, rectangle AB represents a unit and AC represents 3 units. Study of the figure discloses that AD is three "one fourths" of the unit, AB, and is also one fourth of the three units, AC.



37. A **Proper Fraction** is a common fraction whose numerator is less than its denominator; as $\frac{5}{8}$.

An **Improper Fraction** is a common fraction whose numerator is greater than its denominator; as $\frac{7}{3}$.

A **Mixed Number** is the sum of an integer and a fraction; as $5 + \frac{3}{4}$, or, as it is usually written, $5\frac{3}{4}$.

EXERCISE 27

1. Illustrate geometrically the fraction $\frac{2}{3}$. What kind of fraction is this?

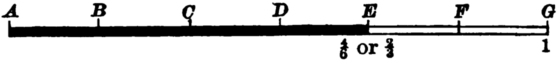
2. Illustrate geometrically the fraction $\frac{5}{2}$. What kind of fraction is this?

3. Illustrate geometrically the number $2\frac{1}{4}$. What kind of number is this?

4. Illustrate geometrically the fact that $\frac{1}{3}$ of 2 is the same as $\frac{2}{3}$ of 1.

38. Two Fundamental Laws of Fractions Verified.

In the figure below, let rectangle *AG* represent one unit. Let it be divided into six equal parts *AB*, *BC*, *CD*, *DE*, *EF*, and *FG*. Then *AC*, *CE*, and *EG* represent three equal parts of *AG*.



Hence *AE* represents $\frac{2}{3}$ of *AG* and also $\frac{4}{6}$ of *AG*.

Hence $\frac{2}{3} = \frac{4}{6}$.

Notice that $\frac{4}{6}$ can be obtained from $\frac{2}{3}$ by multiplying both numerator and denominator by 2.

Rule I. — The numerator and denominator of a fraction may each be multiplied by the same number without changing the value of the fraction.

Referring again to the fact that $\frac{2}{3} = \frac{4}{6}$, or that $\frac{4}{6} = \frac{2}{3}$, notice that $\frac{2}{3}$ can be obtained from $\frac{4}{6}$ by dividing both numerator and denominator by 2.

Rule II. — The numerator and denominator of a fraction may each be divided by the same number without changing the value of the fraction.

EXERCISE 28

1. *a.* Give a geometrical illustration of the fact that $\frac{1}{2}=\frac{2}{4}$. *b.* By what are each of the terms of $\frac{1}{2}$ multiplied to give $\frac{2}{4}$?

2. By what are the terms of $\frac{3}{4}$ multiplied to give $\frac{6}{8}$?

3. Multiply both numerator and denominator of $\frac{2}{3}$ by:
a. 2 *b.* 3 *c.* 5 *d.* 10 *e.* 15
How do all the resulting fractions compare?

4. Divide both numerator and denominator of $\frac{20}{50}$ by:
a. 10 *b.* 5 *c.* 2
How do all the resulting fractions compare?

5. Write five fractions each equal to the fraction $\frac{3}{4}$, and tell how each is obtained from $\frac{3}{4}$.

6. Write five fractions each equal to the fraction $\frac{2}{7}$, and tell how each is obtained from $\frac{2}{7}$.

7. Write three fractions each equal to the fraction $\frac{60}{210}$, and having smaller terms than it. How is each obtained?

8. Write two fractions each equal to the fraction $\frac{60}{185}$.

9. Change each of the following fractions to an equal fraction having a smaller denominator and tell how you do it:
a. $\frac{36}{48}$ *b.* $\frac{4}{12}$ *c.* $\frac{16}{64}$ *d.* $\frac{18}{90}$ *e.* $\frac{28}{70}$
f. $\frac{21}{105}$ *g.* $\frac{72}{108}$ *h.* $\frac{18}{72}$ *i.* $\frac{35}{75}$ *j.* $\frac{80}{120}$

10. Change each of the following fractions to an equal fraction having a larger denominator and tell how you do it:
a. $\frac{2}{3}$ *b.* $\frac{3}{4}$ *c.* $\frac{5}{6}$ *d.* $\frac{7}{8}$ *e.* $\frac{5}{3}$ *f.* $\frac{7}{6}$
g. $\frac{3}{8}$ *h.* $\frac{4}{5}$ *i.* $\frac{9}{10}$ *j.* $\frac{7}{12}$ *k.* $\frac{3}{7}$ *l.* $\frac{4}{9}$

39. A fraction is in its **Lowest Terms** when its numerator and denominator do not have any common factor.

40. Reducing a Fraction to its Lowest Terms.

Example 1. — Reduce $\frac{35}{63}$ to its lowest terms.

Solution. — Dividing both numerator and denominator by 7,

$$\frac{35}{63}=\frac{5}{9}.$$

Example 2. — Reduce $\frac{1125}{2700}$ to its lowest terms.

Solution. — 1. Dividing both numerator and denominator by 25,

$$\frac{1125}{2700}=\frac{45}{108}.$$

2. Dividing both terms by 9, $\frac{45}{108}=\frac{5}{12}.$

3. $\therefore \frac{1125}{2700}=\frac{5}{12}.$

The solution should be arranged thus:

$$\frac{1125}{2700}=\frac{45}{108}=\frac{5}{12}.$$

Rule. — To reduce a fraction to its lowest terms:

1. Divide both terms by any common factor of them.

2. Divide both terms of the resulting fraction by any of their common factors.

3. Continue in this manner until there results a fraction whose terms do not have a common factor.

Note 1. — This process is equivalent to dividing both terms by their G. C. D., so a fraction can be reduced to lowest terms at once by dividing both terms by their G. C. D.

Note 2. — Special care should be taken when the last remaining factor in the numerator is 1, as in the following example.

Example 3. — Reduce $\frac{30}{210}$ to its lowest terms.

Solution.

$$\frac{30}{210}=\frac{\overset{\overset{1}{\cancel{3}}}{\cancel{30}}}{\underset{\underset{7}{\cancel{21}}}{\cancel{210}}}=\frac{1}{7}.$$

Example 4. — Reduce to its lowest terms

$$\frac{21\times 12\times 15}{20\times 14\times 36}.$$

Solution.
$$\frac{\overset{3}{\cancel{21}} \times \overset{1}{\cancel{12}} \times \overset{\overset{1}{\cancel{3}}}{\cancel{15}}}{\underset{4}{\cancel{20}} \times \underset{2}{\cancel{14}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{36}}} = \frac{3 \times 1 \times 1}{4 \times 2 \times 1} = \frac{3}{8}.$$

In this solution, 21 and 14 are each divided by 7, and the quotient 3 is written above 21 and 2 below 14. 12 and 36 are each divided by 12, and the quotient 1 is written above 12 and 3 below 36. 15 and 20 are each divided by 5, and the quotient 3 is written above 15 and 4 below 20. The 3 above 15 and the 3 below 36 are each divided by 3, and the quotients 1 are written down.

The final result is the product of the remaining factors in the numerator over the product of the remaining factors in the denominator.

NOTE. — The process of dividing out common factors of a numerator and a denominator is commonly called *cancellation.* It is preferable to think of it as a process of division.

EXERCISE 29

Express in their lowest terms the fractions:

1. $\frac{20}{36}$. **6.** $\frac{30}{64}$. **11.** $\frac{38}{16}$. **16.** $\frac{539}{154}$.

2. $\frac{15}{24}$. **7.** $\frac{120}{45}$. **12.** $\frac{27}{18}$. **17.** $\frac{209}{330}$.

3. $\frac{12}{32}$. **8.** $\frac{46}{12}$. **13.** $\frac{90}{135}$. **18.** $\frac{288}{2304}$.

4. $\frac{15}{35}$. **9.** $\frac{76}{32}$. **14.** $\frac{22}{132}$. **19.** $\frac{308}{176}$.

5. $\frac{22}{44}$. **10.** $\frac{14}{32}$. **15.** $\frac{126}{18}$. **20.** $\frac{1155}{550}$.

21. $\dfrac{49 \times 88}{55 \times 112}$. **26.** $\dfrac{8 \times 12 \times 14 \times 15}{10 \times 16 \times 18 \times 21}$.

22. $\dfrac{46 \times 63}{7 \times 23 \times 30}$. **27.** $\dfrac{34 \times 38 \times 39}{57 \times 85 \times 91}$.

23. $\dfrac{20 \times 108}{18 \times 28 \times 36}$. **28.** $\dfrac{27 \times 77 \times 105}{135 \times 165}$.

24. $\dfrac{16 \times 95 \times 96}{114 \times 128}$. **29.** $\dfrac{21 \times 26 \times 52}{39 \times 56 \times 117}$.

25. $\dfrac{21 \times 39 \times 55}{20 \times 26 \times 33}$. **30.** $\dfrac{54 \times 84 \times 270}{50 \times 162 \times 196}$.

31. A set of gimlets consists of one of each of the following sizes: $\frac{2}{32}$, $\frac{3}{32}$, $\frac{4}{32}$, $\frac{5}{32}$, $\frac{6}{32}$, $\frac{7}{32}$, $\frac{8}{32}$, $\frac{9}{32}$, $\frac{10}{32}$, $\frac{11}{32}$, $\frac{12}{32}$. Express each of these sizes in its lowest terms.

32. A set of auger bits consists of one of each of the following sizes: $\frac{4}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{8}{16}$, $\frac{9}{16}$, $\frac{10}{16}$, $\frac{11}{16}$, $\frac{12}{16}$, $\frac{13}{16}$, $\frac{14}{16}$, $\frac{15}{16}$, $\frac{16}{16}$. Express each of these sizes in its lowest terms.

41. Changing an Improper Fraction to an Integer or a Mixed Number.

Example 1. — Express $\frac{54}{9}$ as an integer.

$$\text{Since } \frac{54}{9} = 54 \div 9, \quad \therefore \frac{54}{9} = 6.$$

Example 2. — Express $\frac{38}{7}$ as a mixed number,

$$\frac{38}{7} = \frac{35}{7} + \frac{3}{7} = 5 + \frac{3}{7} = 5\frac{3}{7}$$

$$\begin{array}{r|r} & 5 \\ \hline 7 & 38 \\ & \underline{35} \\ & 3 \end{array}$$

Rule. — To express an improper fraction as an integer or a mixed number:

1. Divide the numerator by the denominator.

2. The fraction equals the quotient, or, if there is a remainder, the quotient plus the fraction formed by writing the remainder over the divisor. Reduce the fractional part of the result to lowest terms if necessary.

Example 3. — How many bushels of oats are there in 1578 pounds of oats if one bushel of oats weighs 32 pounds?

Solution. — The number of bushels $= \frac{1578}{32}$

$= 49\frac{10}{32}$

$= 49\frac{5}{16}$.

$$\begin{array}{r|r} & 49 \\ \hline 32 & 1578 \\ & \underline{128} \\ & 298 \\ & \underline{288} \\ & 10 \end{array}$$

EXERCISE 30

Express each of the following fractions as an integer or a mixed number:

1. $\frac{6}{2}$ lb.
2. $\frac{16}{3}$ yd.
3. $\frac{65}{12}$ in.
4. $\frac{37}{8}$ pk.
5. $\frac{39}{4}$ gal.
6. $\frac{645}{320}$ mi.
7. $\frac{54}{16}$ lb.
8. $\frac{6400}{2000}$ T.
9. $\frac{100}{12}$ gross.
10. $\frac{108}{72}$ bu. corn.
11. $\frac{250}{60}$ bu. wheat.
12. $\frac{220}{12}$ oz.
13. $\frac{38}{6}$.
14. $\frac{27}{8}$.
15. $\frac{45}{11}$.
16. $\frac{56}{9}$.
17. $\frac{74}{8}$.
18. $\frac{92}{12}$.
19. $\frac{108}{8}$.
20. $\frac{125}{12}$.
21. $\frac{152}{13}$.
22. $\frac{168}{20}$.
23. $\frac{185}{14}$.
24. $\frac{196}{16}$.
25. $\frac{45}{18}$.
26. $\frac{279}{81}$.
27. $\frac{597}{40}$.
28. $\frac{1194}{22}$.
29. $\frac{2167}{58}$.
30. $\frac{2786}{61}$.
31. $\frac{2858}{29}$.
32. $\frac{3157}{45}$.
33. $\frac{6763}{74}$.
34. $\frac{6027}{93}$.
35. $\frac{2369}{85}$.
36. $\frac{2412}{151}$.

42. Expressing a Fraction as an Equal Fraction Having a Larger Denominator.

Example 1. — Express $\frac{3}{4}$ as an equal fraction with denominator 28.

Solution. — 1. $28 \div 4 = 7.$

2. Multiply both terms of the fraction by 7.

$$\text{Then } \frac{3}{4} = \frac{7 \times 3}{7 \times 4} = \frac{21}{28}.$$

Rule. — To express a fraction as an equal fraction having a larger denominator:

1. **Divide the new denominator by the given denominator.**
2. **Multiply both terms of the fraction by the quotient.**

Example 2. — Express 22 as a fraction with denominator 12.

Solution. — 1. $22 = \frac{22}{1}.$

2. $12 \div 1 = 12.$

$$\therefore \frac{22}{1} = \frac{12 \times 22}{12 \times 1} = \frac{264}{12}.$$

EXERCISE 31

1. Express $\frac{2}{3}$ as an equal fraction having the denominator:

a. 12 *b.* 9 *c.* 15 *d.* 24 *e.* 36 *f.* 60

2. Express $\frac{3}{4}$ as an equal fraction having the denominator:

a. 12 *b.* 20 *c.* 36 *d.* 48 *e.* 60 *f.* 56

3. Express $\frac{5}{8}$ as an equal fraction having the denominator:

a. 16 *b.* 24 *c.* 40 *d.* 56 *e.* 72 *f.* 96

4. Express $\frac{3}{5}$ as an equal fraction having the denominator:

a. 15 *b.* 25 *c.* 35 *d.* 45 *e.* 100 *f.* 120

5. Express $\frac{5}{6}$ as an equal fraction having the denominator:

a. 12 *b.* 24 *c.* 36 *d.* 48 *e.* 96 *f.* 72

6. Express 6: *a.* as 8ths *b.* as 5ths *c.* as 10ths
7. Express 8: *a.* as 9ths *b.* as 4ths *c.* as 7ths
8. Express 9: *a.* as 5ths *b.* as 6ths *c.* as 8ths
9. Express 11: *a.* as 3ds *b.* as 5ths *c.* as 10ths
10. Express 13: *a.* as 4ths *b.* as 6ths *c.* as 11ths

43. Changing Two or More Fractions to Equal Fractions having their Lowest Common Denominator.

To add two fractions like $\frac{1}{3}$ and $\frac{1}{2}$, it is necessary to change $\frac{1}{3}$ to $\frac{2}{6}$ and $\frac{1}{2}$ to $\frac{3}{6}$. Then

$$\frac{1}{3}+\frac{1}{2}=\frac{2}{6}+\frac{3}{6}=\frac{5}{6}.$$

$\frac{2}{6}$ and $\frac{3}{6}$ have a common denominator. Until fractions do have a common denominator, they cannot be added or subtracted.

The lowest common multiple of the denominators of the fractions is used as the lowest common denominator of the fractions.

Example 1. — Express $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{4}{15}$ as equal fractions having their lowest common denominator.

Solution. — 1. The L. C. M. of 6, 10, and 15 is 30. (§ 35)

2. For $\frac{5}{6}$: $30 \div 6 = 5.$ $\frac{5}{6}=\frac{5\times 5}{5\times 6}=\frac{25}{30}.$

For $\frac{7}{10}$: $30 \div 10 = 3.$ $\frac{7}{10}=\frac{3\times 7}{3\times 10}=\frac{21}{30}.$

For $\frac{4}{15}$: $30 \div 15 = 2.$ $\frac{4}{15}=\frac{2\times 4}{2\times 15}=\frac{8}{30}.$

Rule. — To express fractions as equal fractions having their lowest common denominator:

1. Find the L. C. M. of their given denominators. (§ 35)

2. For each fraction, divide the L. C. D. by the given denominator, and then multiply the numerator and the denominator of the fraction by the quotient.

EXERCISE 32

Change the following fractions to equal fractions having their lowest common denominator.

1. $\frac{2}{3}$, $\frac{1}{2}$.
2. $\frac{1}{3}$, $\frac{3}{4}$.
3. $\frac{2}{3}$, $\frac{3}{5}$.
4. $\frac{3}{4}$, $\frac{2}{5}$.
5. $\frac{5}{6}$, $\frac{1}{2}$.
6. $\frac{3}{5}$, $\frac{1}{4}$.
7. $\frac{3}{4}$, $\frac{1}{6}$.
8. $\frac{7}{8}$, $\frac{1}{3}$.
9. $\frac{3}{8}$, $\frac{2}{5}$.
10. $\frac{2}{7}$, $\frac{1}{2}$.
11. $\frac{5}{8}$, $\frac{2}{3}$.
12. $\frac{3}{7}$, $\frac{5}{6}$.
13. $\frac{4}{5}$, $\frac{3}{10}$.
14. $\frac{5}{6}$, $\frac{3}{8}$.
15. $\frac{3}{2}$, $\frac{4}{5}$.
16. $\frac{5}{2}$, $\frac{7}{8}$.
17. $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$.
18. $\frac{4}{5}$, $\frac{2}{9}$, and $\frac{4}{3}$.
19. $\frac{5}{6}$, $\frac{7}{18}$, and $\frac{8}{9}$.
20. $\frac{31}{24}$, $\frac{21}{32}$, and $\frac{25}{48}$.
21. $\frac{3}{10}$, $\frac{13}{14}$, and $\frac{22}{35}$.
22. $\frac{7}{12}$, $\frac{9}{20}$, and $\frac{19}{30}$.
23. $\frac{11}{12}$, $\frac{15}{28}$, and $\frac{25}{42}$.
24. $\frac{5}{24}$, $\frac{23}{30}$, and $\frac{17}{40}$.
25. $\frac{3}{16}$, $\frac{5}{32}$, and $\frac{7}{8}$.
26. $\frac{1}{12}$, $\frac{3}{4}$, and $\frac{7}{8}$.
27. $\frac{5}{64}$, $\frac{3}{8}$, and $\frac{7}{16}$.
28. $\frac{2}{32}$, $\frac{5}{6}$, and $\frac{7}{12}$.
29. $\frac{7}{15}$, $\frac{3}{5}$, and $\frac{9}{10}$.
30. $\frac{5}{18}$, $\frac{7}{6}$, and $\frac{13}{12}$.

44. Adding Fractions.

Example 1. — Find the sum of $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{5}{6}$.

Solution. — 1. Express each fraction as an equal fraction with denominator 12.

$$\tfrac{3}{4} = \tfrac{9}{12};\ \tfrac{2}{3} = \tfrac{8}{12};\ \tfrac{5}{6} = \tfrac{10}{12}. \qquad (\S\ 35)$$

2. $$\therefore \tfrac{3}{4} + \tfrac{2}{3} + \tfrac{5}{6} = \tfrac{9}{12} + \tfrac{8}{12} + \tfrac{10}{12}$$
$$= \tfrac{27}{12} = \tfrac{9}{4} = 2\tfrac{1}{4}.$$

Rule. — To add two or more fractions:

1. Express them, if necessary, as equal fractions having their lowest common denominator. (§ 43)

2. Add the numerators of the resulting fractions for the numerator of the sum, and write the result over their L. C. D.

3. Reduce the resulting fraction to its lowest terms.

Mixed numbers are added by adding separately the integral parts and the fractional parts.

Example 2. — Find the sum of $3\frac{2}{3}$, $4\frac{3}{4}$, and $5\frac{5}{6}$.

Solution. — 1. $3\tfrac{2}{3} + 4\tfrac{3}{4} + 5\tfrac{5}{6} = 3\tfrac{8}{12} + 4\tfrac{9}{12} + 5\tfrac{10}{12}$.

2. $= 12\tfrac{27}{12}$.

3. But $\tfrac{27}{12} = 2\tfrac{3}{12}$, or $2\tfrac{1}{4}$.

4. $\therefore 12\tfrac{27}{12} = 12 + 2\tfrac{1}{4}$, or $14\tfrac{1}{4}$.

EXERCISE 33

Find the sums:

1. $\frac{1}{2} + \frac{3}{4}$.

2. $\frac{1}{2} + \frac{1}{3}$.

3. $\frac{1}{4} + \frac{1}{3}$.

4. $\frac{1}{2} + \frac{3}{5}$.

5. $\frac{2}{3} + \frac{3}{4}$.

6. $\frac{3}{8} + \frac{1}{4}$.

7. $\frac{2}{3} + \frac{5}{6}$.

8. $\frac{3}{4} + \frac{7}{8}$.

9. $\frac{1}{3} + \frac{3}{8}$.

10. $\frac{3}{2} + \frac{1}{6}$.

11. $\frac{3}{4} + \frac{5}{12}$.

12. $\frac{5}{8} + \frac{9}{16}$.

13. $\frac{4}{3} + \frac{3}{5}$.

14. $\frac{5}{2} + \frac{3}{8}$.

15. $\frac{3}{4} + \frac{4}{5}$.

16. $\frac{3}{4} + \frac{5}{6}$.

17. $\frac{4}{5} + \frac{1}{6}$.

18. $\frac{2}{5} + \frac{7}{8}$.

19. $\frac{5}{8} + \frac{5}{6}$.

20. $\frac{1}{8} + \frac{3}{7}$.

21. $\frac{7}{8} + \frac{5}{32} + \frac{3}{16}$.

22. $\frac{9}{10} + \frac{3}{20} + \frac{2}{5}$.

23. $\frac{7}{8} + \frac{11}{18} + \frac{1}{2}$.

24. $\frac{2}{15} + \frac{5}{21} + \frac{3}{5}$.

25. $\frac{2}{3} + \frac{7}{12} + \frac{3}{16}$.

26. $\frac{1}{15} + \frac{7}{33} + \frac{14}{55}$.

27. $\frac{9}{10} + \frac{15}{22} + \frac{28}{35}$.

28. $\frac{2}{3} + \frac{4}{7} + \frac{7}{12}$.

29. $\frac{5}{12} + \frac{5}{18} + \frac{7}{36}$.

30. $\frac{9}{14} + \frac{7}{15} + \frac{10}{21}$.

31. $1\frac{2}{3} + 1\frac{5}{6} + 1\frac{8}{9}$.

32. $1\frac{1}{3} + 2\frac{3}{5} + 3\frac{9}{10}$.

33. $7\frac{2}{3} + 3\frac{5}{6} + 1\frac{3}{4}$.

34. $3\frac{1}{3} + 5\frac{3}{8} + \frac{7}{24}$.

35. $6\frac{1}{3} + 5\frac{1}{5} + 4\frac{5}{8}$.

36. $3\frac{1}{4} + 9\frac{1}{6} + 7\frac{3}{8}$.

37. $4 + 3\frac{5}{9} + 2\frac{5}{18}$.

38. $6\frac{2}{3} + 2 + 8\frac{5}{6}$.

39. $5\frac{11}{12} + 3\frac{4}{15} + 2\frac{1}{6}$.

40. $7\frac{1}{4} + 2\frac{5}{6} + 3\frac{7}{8}$.

NOTE. — If other examples are wanted, add the sets of fractions in the examples of Exercise 32, p. 59.

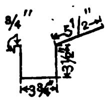
41. The manager of a restaurant ordered three whole hams. When delivered, they weighed $10\frac{3}{4}$, $11\frac{5}{8}$, and $12\frac{1}{4}$ pounds respectively. What was the total weight?

42. Five workmen, A, B, C, D, and E, had at the end of a week the following record of hours worked each day of the week preceding.

	MON.	TUES.	WED.	THURS.	FRI.	SAT.
A.	8	$8\frac{1}{2}$	$9\frac{1}{4}$	$8\frac{1}{3}$	$8\frac{3}{4}$	4
B.	8	8	$8\frac{1}{2}$	$9\frac{1}{4}$	8	$4\frac{1}{2}$
C.	$8\frac{1}{3}$	8	8	$8\frac{1}{2}$	9	$5\frac{1}{4}$
D.	$8\frac{3}{4}$	8	$8\frac{3}{4}$	$8\frac{1}{3}$	$8\frac{1}{2}$	$5\frac{2}{3}$
E.	8	$8\frac{1}{4}$	$8\frac{2}{3}$	$8\frac{1}{3}$	8	$4\frac{3}{4}$

How many hours did each man work during the week?

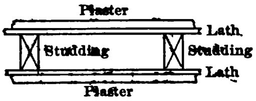
43. The figure at the right is a cross section of an iron gutter to be made by a tinsmith. It is made by bending a piece of material of the proper width. How wide must the material be?



44. A small drawing board is built up of three layers of wood. Two layers are $\frac{11}{32}''$ in thickness and one is $\frac{5}{32}''$. What is the total thickness of the board?

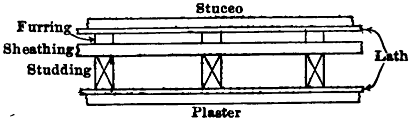
45. A veneered oak door has its stiles and rails built up as follows: there is a piece of quartered oak veneer $\frac{1}{16}''$ in thickness on both sides of a piece of plain oak $1\frac{5}{32}''$ in thickness. What is the total thickness of the rails and stiles?

46. What will be the total height of a set of sectional bookcases consisting of a base that is 10″ high, two sections $12\frac{1}{4}''$ high, one section $10\frac{1}{2}''$ high, and a top 3″ high?



47. The adjoining figure represents a section through a partition wall in a dwelling.

If the studding is $3\frac{3}{4}''$ wide, the lath $\frac{3}{8}''$ thick, and the plaster $\frac{1}{2}''$ thick, how thick is the partition?



48. The adjoining figure represents a section through an outside wall of a stuccoed house.

If the studding is $3\frac{3}{4}''$ wide, the lath $\frac{3}{8}''$ thick, the inside plaster $\frac{1}{2}''$ thick, the sheathing $\frac{7}{8}''$ thick, the furring strips $\frac{7}{8}''$ thick, and the stucco $\frac{3}{4}''$ thick, what is the total thickness of the wall?

45. Expressing a Mixed Number as an Improper Fraction.

Example 1. — Express $2\frac{3}{4}$ as an improper fraction.

Solution. — 1. $2\frac{3}{4}$ means $2 + \frac{3}{4}$.

2. But $2 = \frac{8}{4}$, and therefore $2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$.

Rule. — To express a mixed number as an improper fraction:

1. Multiply the integral part by the denominator of the fractional part, and to the product add the numerator of the fractional part.

2. Write the result of Step 1 over the denominator of the fractional part.

Example 2. — Express $9\frac{7}{8}$ as an improper fraction.

Solution. — 1. $8 \times 9 = 72;\ 72 + 7 = 79.$

2. $\therefore 9\frac{7}{8} = \frac{79}{8}.$

EXERCISE 34

Express each of the following as an improper fraction.

1. $3\frac{2}{5}$.
2. $5\frac{3}{4}$.
3. $4\frac{5}{8}$.
4. $6\frac{2}{3}$.
5. $4\frac{3}{7}$.
6. $5\frac{5}{6}$.
7. $7\frac{3}{5}$.
8. $9\frac{3}{8}$.
9. $10\frac{1}{3}$.
10. $12\frac{2}{3}$.
11. $8\frac{10}{11}$.
12. $10\frac{7}{9}$.
13. $6\frac{3}{4}$.
14. $5\frac{4}{7}$.
15. $3\frac{7}{8}$.
16. $12\frac{5}{6}$.
17. $18\frac{2}{3}$.
18. $15\frac{3}{7}$.
19. $21\frac{3}{4}$.
20. $4\frac{6}{7}$.
21. $4\frac{3}{11}$.
22. $8\frac{5}{6}$.
23. $4\frac{1}{12}$.
24. $3\frac{3}{16}$.
25. $11\frac{7}{10}$.
26. $14\frac{2}{3}$.
27. $9\frac{1}{4}$.
28. $8\frac{9}{11}$.
29. $10\frac{5}{6}$.
30. $13\frac{3}{10}$.
31. $9\frac{5}{8}$.
32. $12\frac{5}{8}$.
33. $17\frac{4}{5}$.
34. $54\frac{3}{100}$.
35. $40\frac{5}{16}$.
36. $5\frac{31}{64}$.
37. $27\frac{1}{2}$.
38. $66\frac{2}{3}$.
39. $71\frac{3}{7}$.
40. $96\frac{4}{5}$.

46. Subtracting Fractions.

Example 1. — Subtract $\frac{9}{14}$ from $\frac{5}{6}$.

Solution. — $\frac{5}{6} - \frac{9}{14} = \frac{35}{42} - \frac{27}{42} = \frac{8}{42} = \frac{4}{21}$.

Rule. — To subtract one fraction from another:

1. Reduce the fractions if necessary to their L. C. D.

2. Subtract the numerator of the subtrahend fraction from that of the minuend, and write the result over the L. C. D.

3. Reduce the final result to lowest terms.

Example 2. — Subtract $2\frac{3}{5}$ from $5\frac{5}{6}$.

Solution. — $5\frac{5}{6} - 2\frac{3}{5} = 5\frac{25}{30} - 2\frac{18}{30} = 3\frac{7}{30}$.

Rule. — To subtract mixed numbers:

1. Express the fractional parts of the mixed numbers in terms of their L. C. D.

2. If the fractional part of the subtrahend is less than that of the minuend, subtract the integral parts and the fractional parts separately. (See Example 2.)

3. **If the fractional part of the subtrahend is greater than that of the minuend, add 1 from the integral part of the minuend to the fractional part of it, and then subtract as in Step 2 of this rule. (See Example 3.)**

Example 3. — Subtract $3\frac{5}{9}$ from $5\frac{1}{6}$.

Solution. — 1. $5\frac{1}{6} - 3\frac{5}{9} = 5\frac{3}{18} - 3\frac{10}{18}$

2. $= 4\frac{21}{18} - 3\frac{10}{18}$ $(5\frac{3}{18} = 4\frac{21}{18}.)$

3. $= 1\frac{11}{18}$

EXERCISE 35

Find the value of:

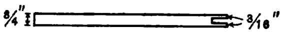
1. $\frac{1}{2} - \frac{1}{3}$.
2. $\frac{2}{3} - \frac{1}{2}$.
3. $\frac{3}{4} - \frac{1}{3}$.
4. $\frac{3}{5} - \frac{1}{4}$.
5. $\frac{5}{6} - \frac{2}{3}$.
6. $\frac{7}{8} - \frac{2}{5}$.
7. $\frac{5}{6} - \frac{3}{4}$.
8. $\frac{7}{8} - \frac{5}{6}$.
9. $\frac{9}{10} - \frac{3}{4}$.
10. $\frac{7}{8} - \frac{2}{3}$.
11. $\frac{7}{12} - \frac{1}{4}$.
12. $\frac{9}{16} - \frac{3}{8}$.
13. $\frac{13}{16} - \frac{3}{4}$.
14. $\frac{9}{10} - \frac{7}{8}$.
15. $\frac{9}{16} - \frac{2}{5}$.
16. $\frac{17}{32} - \frac{3}{8}$.
17. $\frac{33}{64} - \frac{5}{16}$.
18. $\frac{5}{12} - \frac{3}{8}$.
19. $\frac{7}{12} - \frac{5}{16}$.
20. $\frac{25}{32} - \frac{3}{4}$.
21. $5 - \frac{3}{8}$.
22. $7 - \frac{4}{5}$.
23. $10 - \frac{5}{8}$.
24. $6 - \frac{2}{3}$.
25. $8 - \frac{5}{9}$.
26. $12 - \frac{3}{7}$.
27. $5\frac{3}{4} - \frac{1}{3}$.
28. $6\frac{5}{9} - \frac{2}{3}$.
29. $8\frac{9}{16} - 3\frac{1}{4}$.
30. $15\frac{3}{16} - 2\frac{1}{8}$.
31. $20\frac{7}{32} - 5\frac{3}{4}$.
32. $33\frac{5}{6} - 12\frac{2}{3}$.
33. $99\frac{3}{8} - 2\frac{3}{4}$.
34. $36\frac{1}{2} - 7\frac{5}{8}$.
35. $18\frac{3}{5} - 3\frac{7}{10}$.
36. $22\frac{3}{7} - 5\frac{2}{5}$.
37. $131\frac{7}{8} - 128\frac{5}{8}$.
38. $107\frac{1}{4} - 105\frac{7}{8}$.
39. $99\frac{3}{4} - 98\frac{5}{6}$.
40. $114\frac{1}{8} - 109\frac{3}{4}$.

41. In the following table are given the names of certain corporations, the highest price paid in 1916 for a share of their stocks, and the final price paid for a share at the close of 1917.

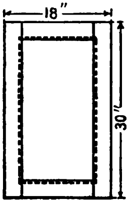
How many dollars per share was the decrease in value for each?

Name of Corporation	1916 High Price	1917 Closing Price
American Sugar Refining Co.	$ $125\frac{5}{8}$	$ $97\frac{1}{2}$
American Telephone and Telegraph	$134\frac{1}{2}$	$104\frac{3}{8}$
American Woolen Co.	$55\frac{5}{8}$	$42\frac{1}{2}$
Baldwin Locomotive Co.	$118\frac{5}{8}$	$57\frac{3}{4}$
Cuban Cane Sugar	$76\frac{3}{8}$	$26\frac{5}{8}$
Ohio Cities Gas	$124\frac{1}{4}$	$35\frac{7}{8}$
United Cigar Stores Co.	$105\frac{3}{4}$	$89\frac{7}{8}$
F. W. Woolworth	$141\frac{3}{4}$	$107\frac{1}{8}$

42. In one end of a piece of wood which is $\frac{3}{4}''$ thick, a slot is to be cut so that $\frac{3}{16}''$ of wood shall remain on each side of the slot. How wide will the slot be?



43. The adjoining figure represents a panel door. The outside pieces are each $2\frac{3}{8}''$ wide. Assume that the panel fits into a groove $\frac{3}{8}''$ deep as shown by the dotted lines. How large must the panel be made?



Multiplication of Fractions

47. Multiplying a Fraction by an Integer.

Example 1. — $2 \times \frac{3}{8} = \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$.

Notice that the final result can be obtained at once by writing the numerator over the quotient of 8 divided by 2.

Similarly, $3 \times \frac{4}{9} = \frac{4}{3}$, and $5 \times \frac{7}{20} = \frac{7}{4}$.

Example 2. — $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}$.

Notice that the final result can be obtained at once by multiplying the numerator 2 by 3, and writing the result over the denominator 5.

Similarly, $2 \times \frac{3}{7} = \frac{6}{7}$, and $5 \times \frac{3}{4} = \frac{15}{4}$.

Rule. — To multiply a fraction by an integer:

1. When the denominator can be divided by the integer,

write the given numerator over the quotient of the denominator divided by the given integer.

2. Otherwise, multiply the numerator of the given fraction by the integer, and write the result over the given denominator. Reduce the result to lowest terms.

Examples of the second kind are solved rapidly as follows:

Example 3.—$24 \times \frac{7}{8} = \overset{3}{\cancel{24}} \times \frac{7}{\cancel{8}} = 3 \times 7 = 21.$

48. Multiplying a Mixed Number by an Integer.

Example 1.—Multiply $3\frac{2}{3}$ by 5.

Solution.— $5 \times 3\frac{2}{3} = 15 + \frac{10}{3} = 15 + 3\frac{1}{3} = 18\frac{1}{3}.$

Rule.—To multiply a mixed number by an integer:

Multiply the integral part and the fractional part separately and add the results.

EXERCISE 36

1. Multiply each of the following fractions:

$\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{3}{8}, \frac{4}{5}, \frac{7}{10}, \frac{4}{9}, \frac{7}{8}, \frac{11}{12}.$

a. by 2 *b.* by 3 *c.* by 4 *d.* by 5 *e.* by 8 *f.* by 20

2. Multiply $\frac{3}{20}$ by: *a.* 2 *b.* 5 *c.* 7 *d.* 10 *e.* 13

3. Multiply $\frac{7}{36}$ by: *a.* 4 *b.* 6 *c.* 11 *d.* 13 *e.* 12

4. Multiply $\frac{9}{28}$ by: *a.* 7 *b.* 5 *c.* 14 *d.* 11 *e.* 28

Find each of the following products:

5. $12 \times \frac{3}{32}$.
6. $45 \times \frac{2}{3}$.
7. $62 \times \frac{3}{31}$.
8. $95 \times \frac{4}{5}$.
9. $60 \times \frac{3}{7}$.
10. $72 \times \frac{7}{80}$.
11. $256 \times \frac{3}{8}$.
12. $1432 \times \frac{15}{32}$.
13. $\frac{2}{3}$ of 90.
14. $\frac{7}{8}$ of 234.
15. $\frac{3}{16}$ of 84.
16. $\frac{5}{12}$ of 642.
17. $\frac{3}{4}$ of 965.
18. $\frac{5}{6}$ of 1578.
19. $\frac{7}{5}$ of 365.
20. $\frac{9}{10}$ of 273.
21. $9 \times 3\frac{2}{3}$.
22. $10 \times 5\frac{3}{4}$.

23. $12 \times 6\frac{5}{8}$.

24. $14 \times 7\frac{5}{6}$.

25. $21 \times 3\frac{2}{7}$.

26. $65 \times 5\frac{1}{3}$.

27. $92 \times 8\frac{3}{4}$.

28. $24 \times 12\frac{1}{[illegible]}$.

29. $45 \times 16\frac{2}{3}$.

30. $136 \times 8\frac{1}{8}$.

31. $224 \times 14\frac{2}{7}$.

32. $638 \times 11\frac{1}{9}$.

33. $83 \times 21\frac{1}{2}$.

34. $75 \times 33\frac{1}{3}$.

35. $98 \times 66\frac{1}{3}$.

36. $17 \times 55\frac{2}{5}$.

37. $212 \times 4\frac{5}{8}$.

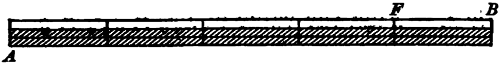
38. $64 \times 3\frac{7}{16}$.

39. $48 \times 11\frac{2}{3}$.

40. $96 \times 2\frac{3}{32}$.

49. Multiplying a Fraction by a Fraction.

In the following figure is a geometrical illustration of $\frac{2}{3}$ of $\frac{4}{5}$.



Rectangle *AF* is clearly $\frac{4}{5}$ of rectangle *AB*. The shaded part of *AF* is $\frac{2}{3}$ of *AF*.

Therefore this shaded part of *AF* is $\frac{2}{3}$ of $\frac{4}{5}$ of *AB*.

It is clear from the figure that this shaded part of *AF* is also $\frac{8}{15}$ of *AB*.

Therefore $\frac{2}{3}$ of $\frac{4}{5}$ of *AB* is $\frac{8}{15}$ of *AB*.

Notice that $\frac{8}{15}$ can be obtained by writing 2×4 over 3×5.

Rule. — To multiply one fraction by another:

1. Multiply the numerators of the fractions for the numerator of the product, the denominators for the denominator of the product.

2. Reduce the resulting fraction to its lowest terms.

Thus, $\frac{7}{8} \times \frac{3}{5} = \frac{21}{40}$, and $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$.

Since a fraction is reduced to lowest terms by dividing both numerator and denominator by any common factors of them, it is preferable to remove such common factors, as in the following:

Example 1. — Find the value of $\frac{6}{7} \times \frac{5}{9} \times \frac{11}{15}$.

Solution. — $$\frac{\overset{2}{\cancel{6}}}{7} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}} \times \frac{11}{\underset{3}{\cancel{15}}} = \frac{22}{63}.$$

In this solution, 6 and 9 are each divided by 3, and 5 and 15 are each divided by 5, the quotients being written either above or below the number divided. The result is obtained by multiplying the resulting factors of the numerator for the numerator and those of the denominator for the denominator of the product.

In multiplication examples involving mixed numbers, it is usually preferable to change the mixed numbers to improper fractions.

Example 2. — Find the value of $1\frac{7}{20} \times 2\frac{2}{15} \times \frac{1}{9}$.

Solution. — $$1\tfrac{7}{20} \times 2\tfrac{2}{15} \times \tfrac{1}{9} = \frac{\overset{1}{\cancel{\overset{3}{\cancel{27}}}}}{\underset{5}{\cancel{20}}} \times \frac{\overset{8}{\cancel{32}}}{\underset{5}{\cancel{15}}} \times \frac{1}{\underset{1}{\cancel{9}}} = \frac{8}{25}.$$

EXERCISE 37

Find the following products:

1. $\frac{2}{7} \times \frac{21}{8}$.

2. $\frac{3}{8} \times \frac{7}{5}$.

3. $\frac{9}{16} \times \frac{11}{36}$.

4. $\frac{5}{11} \times \frac{7}{4}$.

5. $\frac{15}{32} \times \frac{30}{75}$.

6. $\frac{18}{42} \times \frac{24}{27}$.

7. $\frac{76}{25} \times \frac{35}{19}$.

8. $\frac{18}{25} \times \frac{20}{27} \times \frac{45}{64}$.

9. $\frac{10}{27} \times \frac{12}{25} \times \frac{55}{8}$.

10. $\frac{7}{8} \times \frac{6}{49} \times \frac{10}{63}$.

11. $\frac{21}{25} \times \frac{55}{98} \times \frac{35}{6}$.

12. $\frac{13}{12} \times \frac{20}{39} \times \frac{24}{45}$.

13. $\frac{22}{7} \times \frac{13}{2} \times 15$.

14. $\frac{14}{27} \times \frac{9}{70} \times \frac{54}{13}$.

15. $3\frac{5}{6} \times 5\frac{1}{7}$.

16. $16\frac{2}{3} \times 3\frac{6}{25}$.

17. $37\frac{1}{2} \times 4\frac{2}{15}$.

18. $33\frac{1}{3} \times \frac{9}{100}$.

19. $87\frac{1}{2} \times 2\frac{4}{5}$.

20. $1\frac{4}{21} \times 2\frac{8}{45} \times 1\frac{1}{35}$.

21. $1\frac{12}{25} \times 2\frac{16}{37} \times \frac{5}{18}$.

Find the cost of:

22. 12 lb. of sugar at 9¾¢ per lb

23. 14 T. of coal at $11⅗ per T.

24. 2⅜ lb. round steak at 23¢ per pound.

25. 4⅞ lb. chicken at 24¢ per pound.

26. $37\frac{1}{2}$ lb. chicken at $18\frac{1}{2}$¢ per pound.

27. 13 gal. gasoline at $23\frac{2}{10}$¢ per gallon.

28. 85 bu. oats at $89\frac{3}{4}$¢ per bushel.

29. 275 bu. corn at 1.26\frac{3}{8}$ per bushel.

30. 1450 bu. #3 white oats at $91\frac{1}{2}$¢ per bushel.

31. 750 bu. flaxseed at 1.71\frac{3}{4}$ per bushel.

32. 2500 bu. wheat at 1.45\frac{3}{8}$ per bushel.

Note. — In many of the following examples, the prices are wholesale prices before the war.

33. 12,500 lb. peanuts at $3\frac{3}{4}$¢ per pound.

34. 3250 lb. prunes at $7\frac{1}{4}$¢ per pound.

35. 875 lb. lard at $9\frac{3}{4}$¢ per pound.

36. 9876 qt. milk at $4\frac{33}{100}$¢ per quart.

37. 10,965 lb. sugar at $4\frac{11}{100}$¢ per pound.

38. 1375 yd. calico at $4\frac{3}{4}$¢ per yard.

39. 1985 yd. gingham at $5\frac{7}{8}$¢ per yd.

40. 1575 yd. overcoating at 1.91\frac{1}{4}$ per yard.

41. 1250 yd. overcoating at 1.41\frac{3}{4}$ per yard.

42. 2750 yd. sheeting at $5\frac{3}{4}$¢ per yard.

43. 965 lb. oxide of zinc at $5\frac{3}{8}$¢ per pound.

44. A meat market owner purchased a whole beef weighing 748 lb. at $21\frac{1}{4}$¢ per pound. Also $77\frac{1}{2}$ lb. of heavy pork loins at $27\frac{1}{4}$¢ per pound, 68 lb. medium loins at $28\frac{1}{2}$¢ per pound, and $82\frac{3}{4}$ lb. light loins at $29\frac{1}{4}$¢ per pound.

What was the total cost?

45. Find the cost of 12 T. of bran at 27\frac{3}{4}$ per ton.

46. A recipe for golden corn cake calls for $\frac{3}{4}$ cup (c.) of corn meal, $1\frac{1}{2}$ c. of sugar, 4 teaspoonfuls (t.) of baking-powder, $\frac{1}{2}$ t. of salt, 1 c. of milk, 1 egg, and 1 tablespoonfull (T.) of melted butter.

What quantities should be used for one and one-half portions of this recipe?

47. The following recipe is given for coffee sauce to be served with ice cream:

$1\frac{1}{2}$ c. of milk, $\frac{1}{2}$ c. of ground coffee, $\frac{1}{3}$ c. of sugar, $\frac{3}{4}$ T. of arrowroot, and salt.

How much of each should be used to make one half of the recipe?

48. A recipe for orange snow pudding is:

1 T. of gelatine, $\frac{1}{4}$ c. of cold water, $\frac{3}{4}$ c. of boiling water, $\frac{1}{2}$ c. of sugar, $\frac{3}{4}$ c. of orange juice, 2 T. of lemon juice, and the white of 1 egg.

Determine the quantities to use for one third portion of this pudding; also for one and one half portions.

49. In feeding dairy cows, it is suggested that $\frac{2}{7}$ pound of concentrated feed (bran, middlings, etc.) be fed for every pound of milk produced daily.

How many pounds of feed should each of the following cows receive?

Cow # 1 produces 28 lb. per day; cow # 2, 30 lb. per day; cow # 3, 25 lb. per day; cow # 4, 33 lb. per day.

50. A recipe for plum preserves calls for three fourths as much sugar as fruit, and one cup of water to each pound of sugar. How much water and sugar are required for 14 lb. of fruit? for 9 lb.? for 17 lb.?

51. In a certain institution, members of the faculty receive as salary for service during the summer session $\frac{2}{15}$ of their annual salary. Determine the amounts to be paid to persons whose salaries are:

a. $1200 *b.* $1750 *c.* $2250

52. What is the cost of $4\frac{3}{4}$ T. of coal at $11.70 per ton?

53. What is the cost of $1\frac{1}{2}$ yd. of cloth at $16\frac{2}{3}$¢ per yard?

54. What is the cost of 276 lb. of sugar at $8\frac{3}{4}$¢ per pound?

55. What is the cost of $17\frac{3}{5}$ A. of land at $235 per acre?

56. Find the cost of $18\frac{3}{5}$ ft. of steel rod at $.35 per foot, and $24\frac{1}{6}$ ft. of rod at $.46 per foot.

57. A machinist is working at the rate of 42¢ per hour, with "time and one half" for overtime. (The full day is 8 hours.) What should be the amount of his wages at the end of a week in which he worked 8 hr. on Monday, 9 on Tuesday, 10 on Wednesday, $9\frac{1}{2}$ on Thursday, $8\frac{1}{2}$ on Friday, and 8 on Saturday?

58. A member of a club received the following statement of his indebtedness to the club for use of the billiard room:

May 8, $1\frac{1}{4}$ hr.; May 12, $1\frac{2}{3}$ hr.; May 18, $2\frac{1}{6}$ hr.; May 24, $1\frac{1}{2}$ hr.; May 30, $1\frac{3}{4}$ hr. What was the amount of his bill at 30¢ per hour?

59. Find the cost of 237 bales of silk averaging $125\frac{1}{2}$ yd. at $2.43 per yard.

Division of Fractions

50. Dividing a Fraction by an Integer. — To divide a fraction by an integer like 2 means to find the fraction which must be multiplied by 2 to produce the given fraction as product.

Thus, $\frac{6}{7} \div 2 = \frac{3}{7}$, for $2 \times \frac{3}{7} = \frac{6}{7}$.

Similarly, $\frac{12}{17} \div 3 = \frac{4}{17}$, for $3 \times \frac{4}{17} = \frac{12}{17}$.

Also, $\frac{3}{4} \div 2 = \frac{3}{8}$, for $2 \times \frac{3}{8} = \frac{3}{4}$.

Similarly, $\frac{2}{5} \div 3 = \frac{2}{15}$, for $3 \times \frac{2}{15} = \frac{2}{5}$.

Rule. — To divide a fraction by an integer:

1. If possible, divide the numerator by the integer and write the result over the denominator.

2. Otherwise, multiply the denominator by the integer and write it below the given numerator.

Thus, $\frac{40}{3} \div 5 = \frac{8}{3}$, and $\frac{4}{3} \div 5 = \frac{4}{15}$.

51. Dividing an Integer or a Fraction by a Fraction.

Example 1. — Divide 3 by $\frac{2}{5}$.

Solution. — 1. $3 = \frac{15}{5}$.

2. $\therefore 3 \div \frac{2}{5} =$ 15 fifths ÷ 2 fifths $= 15 \div 2 = \frac{15}{2}$.

Notice now that $\frac{15}{2} = 3 \times \frac{5}{2}$, so that $3 \div \frac{2}{5} = 3 \times \frac{5}{2} = \frac{15}{2}$. $\frac{5}{2}$ is said to be the fraction $\frac{2}{5}$ *inverted*.

Example 2. — $\frac{3}{4}$ by $\frac{2}{3}$.

Solution. — 1. $\frac{3}{4} = \frac{9}{12}$, and $\frac{2}{3} = \frac{8}{12}$.

2. $\therefore \frac{3}{4} \div \frac{2}{3} = \frac{9}{12} \div \frac{8}{12} = 9 \div 8 = \frac{9}{8}$.

Notice again that $\frac{9}{8} = \frac{3}{4} \times \frac{3}{2}$, so that $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2}$. The quotient is found by multiplying $\frac{3}{4}$ by the fraction $\frac{2}{3}$ inverted.

Rule. — To divide a number by a fraction:

1. **Multiply the dividend by the inverted divisor.**
2. **Reduce the result of Step 1 to lowest terms.**

Example 3. — Divide $\frac{25}{12}$ by $\frac{35}{18}$.

Solution. — $$\frac{25}{12} \div \frac{35}{18} = \frac{25}{12} \times \frac{18}{35} = \frac{\overset{5}{\cancel{25}}}{\underset{2}{\cancel{12}}} \times \frac{\overset{3}{\cancel{18}}}{\underset{7}{\cancel{35}}} = \frac{15}{14}.$$

If mixed numbers appear, change them to improper fractions.

Example 4. — Divide $13\frac{1}{2}$ by $2\frac{1}{4}$.

Solution. — $$13\tfrac{1}{2} \div 2\tfrac{1}{4} = \tfrac{27}{2} \div \tfrac{9}{4}.$$
$$= \frac{27}{2} \times \frac{4}{9} = \frac{\overset{3}{\cancel{27}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{9}}} = 6.$$

EXERCISE 38

Find the value of:

1. $\frac{5}{8} \div \frac{3}{4}$.	4. $\frac{7}{8} \div \frac{9}{16}$.	7. $\frac{2}{3} \div \frac{5}{12}$.
2. $\frac{3}{5} \div \frac{2}{10}$.	5. $\frac{3}{4} \div \frac{3}{8}$.	8. $\frac{13}{16} \div \frac{7}{8}$.
3. $\frac{5}{6} \div \frac{2}{3}$.	6. $\frac{3}{8} \div \frac{6}{5}$.	9. $\frac{15}{16} \div \frac{5}{8}$.

10. $\frac{21}{32} \div \frac{7}{8}$.

11. $\frac{7}{12} \div \frac{3}{32}$.

12. $16 \div \frac{4}{5}$.

13. $22 \div \frac{8}{9}$.

14. $35 \div \frac{5}{7}$.

15. $24 \div \frac{8}{9}$.

16. $32 \div \frac{3}{4}$.

17. $12 \div \frac{5}{6}$.

18. $10 \div \frac{7}{8}$.

19. $15 \div \frac{2}{3}$.

20. $\frac{5}{8} \div 4$.

21. $\frac{7}{12} \div 3$.

22. $\frac{9}{16} \div 5$.

23. $\frac{5}{6} \div 3$.

24. $\frac{3}{4} \div 4$.

25. $\frac{7}{8} \div 4$.

26. $5\frac{1}{2} \div 4$.

27. $6\frac{2}{3} \div 5$.

28. $2\frac{1}{16} \div 3$.

29. $3\frac{9}{22} \div 100$.

30. $37\frac{1}{2} \div 15$.

31. $45 \div 3\frac{4}{7}$.

32. $16\frac{2}{3} \div 10$.

33. $87\frac{1}{2} \div 45$.

34. $66\frac{2}{3} \div 25$.

35. $2\frac{1}{16} \div 12$.

36. $1\frac{19}{35} \div \frac{90}{7}$.

37. $3\frac{1}{5} \div 2\frac{14}{25}$.

38. $\frac{11}{9} \div 2\frac{3}{7}$.

39. $\frac{63}{52} \div \frac{9}{13}$.

40. $28 \div \frac{42}{11}$.

41. $\frac{110}{21} \div 88$.

42. $3\frac{11}{15} \div 96$.

43. $18\frac{1}{12} \div 4\frac{10}{13}$.

44. $\frac{36}{85} \div \frac{78}{17}$.

45. $\frac{480}{19} \div 51\frac{1}{5}$.

46. $1\frac{13}{98} \div \frac{74}{161}$.

47. $(\frac{2}{3} + \frac{3}{4}) \div (\frac{5}{6} - \frac{4}{5})$.

48. $(\frac{7}{6} - \frac{5}{8}) \div (\frac{11}{9} + \frac{7}{12})$.

49. $(\frac{1}{5} \times \frac{7}{12}) \times (\frac{2}{3} \div \frac{4}{9})$.

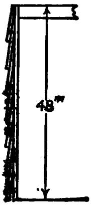
50. $\frac{40}{21} \div (\frac{4}{3} - \frac{7}{9})$.

51. Toweling comes in 10-yd. bolts. How many towels 30″ long (finished length) could be made if each had a $\frac{3}{4}$″ hem at each end?

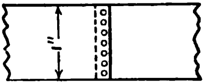
52. For a certain garment $2\frac{3}{8}$ yd. of material is required. How many such garments can be cut from a 12-yd. bolt of the material?

53. A finished oak board $7\frac{3}{4}$″ wide is to be rip sawed into four strips of equal width. If the saw cut is $\frac{1}{16}$″ in width, how wide will the strips be?

54. The sides of a low shed are 48″ high. They are to be covered with a finished siding $7\frac{1}{2}$″ in width. These are overlapped as shown in the figure. How much must the lower six be overlapped so that seven pieces will just cover the side of the shed?



55. Two pieces of sheet iron 1 ft. wide are to be riveted together. Rivets are to be placed in holes with centers $\frac{3}{4}''$ from the outside edges. Between these two rivets, five other rivets are to be placed. What will be the distance between the centers of two successive rivets?



56. How many yards of cloth at $66\frac{2}{3}$¢ per yd. can be bought for $ 8.00?

57. If $12\frac{2}{3}$ cords of wood cost $116.66 how much does one cord cost?

Complex Fractions

52. A **Complex Fraction** is a fraction in which either the numerator or the denominator, or both, involve fractions.

Thus $\frac{\frac{5}{6}}{\frac{10}{9}}$ and $\frac{11}{\frac{2}{3}}$ are complex fractions.

Since a fraction is merely an indicated division, complex fractions are simplified by performing the division according to the rule in § 51.

Example 1. — Simplify $\frac{\frac{5}{6}}{\frac{10}{9}}$.

Solution. — $\frac{\frac{5}{6}}{\frac{10}{9}} = \frac{5}{6} \div \frac{10}{9} = \frac{\cancel{5}}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} = \frac{3}{4}.$

Example 2. — Simplify $\frac{2\frac{1}{3}}{\frac{28}{33}}$.

Solution. — $\frac{2\frac{1}{3}}{\frac{28}{33}} = \frac{7}{3} \div \frac{28}{33} = \frac{\cancel{7}}{\cancel{3}} \times \frac{\overset{11}{\cancel{33}}}{\underset{4}{\cancel{28}}} = \frac{11}{4}.$

EXERCISE 39

1. $\frac{\frac{14}{25}}{\frac{21}{20}}$.
3. $\frac{\frac{9}{10}}{\frac{3}{25}}$.
5. $\frac{2\frac{1}{2}}{\frac{2}{3}}$.
7. $\frac{4\frac{3}{5}}{\frac{8}{15}}$.
2. $\frac{\frac{7}{16}}{\frac{14}{22}}$.
4. $\frac{\frac{24}{33}}{\frac{8}{11}}$.
6. $\frac{3\frac{1}{4}}{\frac{5}{8}}$.
8. $\frac{6\frac{3}{7}}{\frac{15}{28}}$.

9. $\dfrac{3\frac{1}{2}}{1\frac{1}{4}}$.

10. $\dfrac{5\frac{3}{8}}{2\frac{1}{2}}$.

11. $\dfrac{7\frac{2}{3}}{2\frac{1}{6}}$.

12. $\dfrac{10\frac{1}{5}}{2\frac{1}{8}}$.

13. $\dfrac{2\frac{2}{5}}{4\frac{1}{4}}$.

14. $\dfrac{\frac{39}{8}}{52}$.

15. $\dfrac{56}{3\frac{8}{9}}$.

16. $\dfrac{1\frac{2}{33}}{2\frac{5}{22}}$.

17. $\dfrac{51}{\frac{34}{11}}$.

18. $\dfrac{39}{\frac{26}{5}}$.

19. $\dfrac{11\frac{4}{7}}{6\frac{3}{18}}$.

20. $\dfrac{\frac{55}{57}}{\frac{75}{76}}$.

21. $\dfrac{1\frac{13}{15}}{2\frac{16}{24}}$.

22. $\dfrac{\frac{32}{63}}{\frac{80}{147}}$.

23. $\dfrac{\frac{27}{35}}{\frac{3}{7}}$.

24. $\dfrac{\frac{66}{185}}{2\frac{17}{74}}$.

25. $\dfrac{\frac{5}{2}+\frac{11}{8}}{\frac{31}{4}+\frac{26}{5}}$.

26. $\dfrac{5\frac{3}{4}-1\frac{1}{6}}{4\frac{2}{3}-3\frac{5}{8}}$.

27. $\dfrac{2\frac{1}{15}+1\frac{5}{6}}{8-5\frac{2}{5}}$.

28. $\dfrac{3\frac{1}{2}+2\frac{2}{3}}{4\frac{1}{6}-1\frac{1}{4}}$.

Fractional Parts

53. Finding What Part One Number is of Another.

Example. — What part of 35 yr. is 10 yr.?

Solution. — 1. 1 yr. is $\frac{1}{35}$ of 35 yr.

2. Then 10 yr. is 10 × $\frac{1}{35}$ or $\frac{10}{35}$ of 35 yr.

3. ∴ the result is $\frac{10}{35}$ or $\frac{2}{7}$.

In this example, 10 yr. is regarded as a *part* of 35 yr., which is considered the *whole*.

Rule. — To find what part one number is of another:

1. Form the fraction whose numerator is the number regarded as the part and whose denominator is considered the whole.

2. Reduce the fraction to lowest terms.

EXERCISE 40

1. What part of 60 is:

a. 10 *b.* 15 *c.* 20 *d.* 25 *e.* 35 *f.* 45

2. What part of 108 is:

a. 6 *b.* 18 *c.* 26 *d.* 42 *e.* 63 *f.* 84

3. 15 is what part of:

a. 45 *b.* 100 *c.* 80 *d.* 135 *e.* 210 *f.* 350

4. Determine what part 28 is of:

a. 56 *b.* 72 *c.* 90 *d.* 133 *e.* 150 *f.* 140

5. What part of $11\frac{2}{3}$ is 5? is 14? is 20? is 10?

6. What part of 24 is $2\frac{4}{7}$? is $3\frac{1}{8}$? is $4\frac{4}{5}$? is $5\frac{1}{2}$?

7. What part of:

a. 36 is 27? *b.* 49 is 70? *c.* 2340 is 455?

8. What part of:

a. $\frac{27}{20}$ is $\frac{12}{25}$? *b.* $\frac{12}{35}$ is $\frac{16}{21}$? *c.* $\frac{34}{57}$ is $\frac{51}{38}$?

9. $4\frac{1}{2}$ is what part of $3\frac{3}{4}$?

10. What part of $11\frac{2}{3}$ is $6\frac{1}{4}$?

11. What part of 40 is $6\frac{5}{11}$?

12. What part of $4\frac{8}{11}$ is $7\frac{3}{7}$?

13. What part of $2\frac{8}{45}$ is $2\frac{1}{10}$?

14. 2765 is what part of 5690?

15. $3\frac{3}{8}$ is what part of $20\frac{1}{4}$?

16. What part of a long ton (2240 lb.) is a short ton (2000 lb.)?

17. Out of 260 gauze bandages made by a Red Cross group, 125 were made by one person. What part of the total was made by this person?

18. An auto tire, guaranteed to run 3500 miles, showed serious defects after being run 1050 miles. What part of the guaranteed milage was gotten?

19. In most states a bushel of corn on the cob weighs 70 lb. and a bushel of shelled corn 56 lb. What part of a bushel of corn on the cob is a bushel of shelled corn?

20. A cubic foot of pure ice weighs 56 lb. and of fresh water $62\frac{1}{2}$ lb. What part of the weight of a cubic foot of fresh water is that of a cubic foot of pure ice?

21. A girl, employed at the rate of $5.00 per week, works $4\frac{1}{2}$ days. How much should she receive?

22. Out of 250 eggs placed in an incubator, 165 chicks were hatched. What part of the eggs placed in the incubator were lost?

If the eggs cost $ 12.50, what was the value of the eggs lost?

23. A recipe for wild grape jelly calls for 1 pk. of wild grapes, 1 qt. of vinegar, 6 lb. of sugar, and $\frac{1}{4}$ c. each of cloves and cinnamon. How much of each of these ingredients should be used with 4 qt. of grapes? with 10 qt.?

24. A recipe for raspberry and currant preserves calls for 6 lb. of currants, 8 qt. of raspberries, and 6 lb. of sugar. How much sugar and raspberries should be used with 3 lb. of currants? with 5 lb.? with $3\frac{1}{2}$ lb.?

25. A recipe for spiced currants calls for 7 lb. of currants, 5 lb. of sugar, 3 T. of cinnamon, 3 T. of cloves, and 1 pt. of vinegar.

How much of each of these ingredients should be used with 5 lb. of currants? with 1 lb.? with 13 lb.?

54. Finding a Number When Some Fractional Part of It is Given.

Example 1. — An auto takes 3 hours to go $\frac{2}{5}$ of a certain trip. How long will it take to go the whole distance at the same rate?

Solution — 1. 3 hr. is $\frac{2}{5}$ of the total time.

2. $\therefore \frac{1}{5}$ of the total time is $\frac{1}{2}$ of 3 hr. or $1\frac{1}{2}$ hr.

3. $\therefore \frac{5}{5}$ or the whole time is $5 \times 1\frac{1}{2}$ hr. or $7\frac{1}{2}$ hr.

Notice that $7\frac{1}{2}$ is $3 \div \frac{2}{5}$ or $3 \times \frac{5}{2}$.

Example 2. — 36 is $\frac{4}{7}$ of what number?

Solution. — 1. $\frac{1}{7}$ of the number is $\frac{1}{4}$ of 36 or 9.

2. $\therefore$ the number is 7×9 or 63.

Notice again that $63 = 36 \div \frac{4}{7}$ or $36 \times \frac{7}{4}$.

Rule. — To find a number when a fractional part is given: Divide the number by the fractional part.

EXERCISE 41

1. 24 is $\frac{3}{4}$ of what number? Explain the solution.

2. 35 is $\frac{5}{6}$ of what number? Explain the solution.

3. 48 is *a.* $\frac{2}{3}$ of what number? *b.* $\frac{3}{4}$ of what number?
c. $\frac{8}{9}$ of what number? *d.* $\frac{6}{11}$ of what number?

4. Of what number is 32 two thirds?

5. $500 is $\frac{2}{15}$ of what amount?

6. Of what number is 68 four fifths?

7. It took 72 days to complete $\frac{3}{5}$ of the construction of a certain boat. How much longer will it take to complete the construction?

8. A man received a bonus of $ 276 at the end of a year of work. If the bonus was $\frac{2}{9}$ of his year's salary, what was his regular salary?

9. A poultryman finds that he raises about $\frac{4}{7}$ as many chicks as he has eggs in his incubators. If he wants to have at least 500 chicks, how many eggs must he place in his incubators?

10. A carpenter finds that a given number of square feet of flooring will cover only about $\frac{4}{5}$ as many square feet of floor. How many square feet of flooring must he order so that he will have enough to cover 850 sq. ft. of floor surface?

Aliquot Parts

55. An **Aliquot Part** of a number is a part of the number contained in it an integral number of times.

Thus, 2 is an aliquot part of 10; also $2\frac{1}{2}$, $3\frac{1}{3}$, etc.

56. The aliquot parts of 100 are particularly important.

Complete the following:

Table of Aliquot Parts of 100

$\mathbf{\frac{1}{2}}$ =
$\mathbf{\frac{1}{3}}$ =
$\frac{2}{3}$ =
$\mathbf{\frac{1}{4}}$ =
$\frac{3}{4}$ =
$\mathbf{\frac{1}{5}}$ =
$\frac{2}{5}$ =
$\frac{3}{5}$ =
$\frac{4}{5}$ =
$\mathbf{\frac{1}{6}}$ =
$\frac{5}{6}$ =
$\mathbf{\frac{1}{7}}$ =
$\mathbf{\frac{1}{8}}$ =
$\frac{3}{8}$ =
$\frac{5}{8}$ =
$\frac{7}{8}$ =
$\mathbf{\frac{1}{9}}$ =
$\mathbf{\frac{1}{10}}$ =
$\frac{3}{10}$ =
$\frac{7}{10}$ =
$\frac{9}{10}$ =
$\mathbf{\frac{1}{11}}$ =
$\mathbf{\frac{1}{12}}$ =
$\mathbf{\frac{1}{15}}$ =
$\mathbf{\frac{1}{16}}$ =
$\mathbf{\frac{1}{20}}$ =
$\mathbf{\frac{1}{25}}$ =
$\frac{1}{30}$ =
$\frac{1}{40}$ =
$\frac{1}{50}$ =

NOTE. — Pupils should memorize as many of these as possible; they should by all means memorize those printed in heavy black type.

57. By means of the aliquot parts printed in heavy type, others can be determined.

Thus, since $\frac{1}{25}$ of 100 = 4, $\frac{7}{25}$ = 28.

Again, since $\frac{1}{15}$ of 100 is $6\frac{2}{3}$, $\frac{4}{15}$ is $26\frac{2}{3}$.

EXERCISE 42

Give the following parts of 100:

1. $\frac{2}{7}$.
2. $\frac{5}{7}$.
3. $\frac{2}{9}$.
4. $\frac{5}{9}$.
5. $\frac{7}{9}$.
6. $\frac{3}{10}$.
7. $\frac{7}{11}$.
8. $\frac{5}{12}$.
9. $\frac{2}{15}$.
10. $\frac{4}{11}$.
11. $\frac{3}{16}$.
12. $\frac{7}{20}$.
13. $\frac{4}{25}$.
14. $\frac{11}{40}$.
15. $\frac{3}{50}$.
16. $\frac{5}{16}$.
17. $\frac{7}{25}$.
18. $\frac{9}{40}$.
19. $\frac{11}{50}$.
20. $\frac{7}{16}$.
21. $\frac{7}{12}$.
22. $\frac{4}{9}$.
23. $\frac{5}{11}$.
24. $\frac{2}{12}$.
25. $\frac{6}{11}$.
26. $\frac{4}{15}$.
27. $\frac{8}{15}$.
28. $\frac{11}{12}$.
29. $\frac{7}{15}$.
30. $\frac{8}{11}$.
31. $\frac{9}{16}$.
32. $\frac{13}{25}$.
33. $\frac{6}{16}$.
34. $\frac{9}{11}$.
35. $\frac{11}{15}$.
36. $\frac{10}{11}$.
37. $\frac{17}{25}$.
38. $\frac{11}{16}$.
39. $\frac{13}{40}$.
40. $\frac{15}{16}$.
41. $\frac{11}{30}$.
42. $\frac{17}{50}$.

What part of $1.00 is:

43. 40 ¢?
44. 70 ¢?
45. 4 ¢?
46. 75 ¢?
47. 60 ¢?
48. $12\frac{1}{2}$ ¢?
49. $16\frac{2}{3}$ ¢?
50. $33\frac{1}{3}$ ¢?
51. $8\frac{1}{3}$ ¢?
52. $6\frac{1}{4}$ ¢?
53. $37\frac{1}{2}$ ¢?
54. $6\frac{2}{3}$ ¢?
55. $62\frac{1}{2}$ ¢?
56. $18\frac{3}{4}$ ¢?
57. $87\frac{1}{2}$ ¢?
58. $3\frac{1}{3}$ ¢?
59. $2\frac{1}{2}$ ¢?
60. $7\frac{1}{2}$ ¢?

58. The aliquot parts of \$1.00 are used constantly in finding the cost of articles.

Example 1. — What is the cost of 75 lb. of sugar at $8\frac{1}{3}$¢ per pound?

Solution. — 1. $8\frac{1}{3}$¢ = \$$\frac{1}{12}$.

2. ∴ 75 lb. cost 75 × \$$\frac{1}{12}$, or \$$6\frac{1}{4}$ or \$6.25.

Example 2. — What is the cost of $12\frac{1}{2}$ yd. of cloth at \$1.75 per yard?

Solution. — 1. Since $12\frac{1}{2} \times 175 = 175 \times 12\frac{1}{2}$,
the cost of $12\frac{1}{2}$ yd. at \$1.75 per yard is the same as
the cost of 175 yd. at $12\frac{1}{2}$¢ per yard, or at \$$\frac{1}{8}$ per yard.

2. ∴ the cost is 175 × \$$\frac{1}{8}$, or \$$21\frac{7}{8}$ or \$$21.87\frac{1}{2}$.

EXERCISE 43

Find the cost:

1. At $33\frac{1}{3}$¢ per yard of:

a. 15 yd. *b.* 24 yd. *c.* 12 yd. *d.* 18 yd. *e.* 20 yd.
f. 10 yd. *g.* 8 yd. *h.* 11 yd. *i.* 5 yd. *j.* 16 yd.

2. At $12\frac{1}{2}$¢ per yard of the quantities in Example 1.

3. At $6\frac{1}{4}$¢ per yard of the quantities in Example 1.

4. At $8\frac{1}{3}$¢ per yard of the quantities in Example 1.

5. At $16\frac{2}{3}$¢ per yard of the quantities in Example 1.

6. At $37\frac{1}{2}$¢ per yard of the quantities in Example 1.

7. At 75¢ per yard of the quantities in Example 1.

8. At 60¢ per yard of the quantities in Example 1.

9. At $87\frac{1}{2}$¢ per yard of the quantities in Example 1.

10. At \$$1.33\frac{1}{3}$ per yard of the quantities in Example 1.

11. At \$1.40 per yard of the quantities in Example 1.

12. At \$$1.37\frac{1}{2}$ per yard of the quantities in Example 1.

Find the cost of:

13. 30 articles at:

a. $33\frac{1}{3}$¢ each *b.* 40¢ each *c.* $12\frac{1}{2}$¢ each
d. $\$1.66\frac{2}{3}$ each *e.* $16\frac{2}{3}$¢ each *f.* $37\frac{1}{2}$¢ each

14. Of 240 articles at:

a. $66\frac{2}{3}$¢ each *b.* 75¢ each *c.* $6\frac{2}{3}$¢ each
d. $1.25 each *e.* $8\frac{1}{3}$¢ each *f.* $62\frac{1}{2}$¢ each

15. Of 320 articles at the prices quoted in Example 13.

16. Of 1200 articles at the prices quoted in Example 14.

17. Of 25 articles at:

a. 84¢ each *b.* 73¢ each *c.* 12¢ each *d.* 19¢ each

18. Of 50 lb. of articles of food at:

a. 9¢ per pound *b.* 17¢ per pound *c.* 29¢ per pound
d. 34¢ per pound *e.* 47¢ per pound *f.* 69¢ per pound

19. Of $12\frac{1}{2}$ yd. of cloth at:

a. 18¢ per yard *b.* 24¢ per yard *c.* 27¢ per yard
d. 45¢ per yard *e.* 69¢ per yard *f.* 98¢ per yard

20. Of $8\frac{1}{3}$ doz. articles at:

a. 36¢ per dozen *b.* 75¢ per dozen
c. 84¢ per dozen *d.* $1.00 per dozen
e. $1.20 per dozen *f.* $1.50 per dozen

Find the total cost of the following purchases:

21. 15 lb. sugar at $8\frac{1}{3}$¢ per pound.
$12\frac{1}{2}$ lb. prunes at 14¢ per pound.
12 cans of corn at $16\frac{2}{3}$¢ per can.
25 lb. oatmeal at 7¢ per pound.

22. 35 lb. of oats at $2\frac{1}{2}$¢ per pound.
25 lb. barley at $3\frac{1}{3}$¢ per pound.

23. 50 lb. of coffee at 29¢ per pound.
25 lb. of coffee at 37¢ per pound.
15 lb. tea at $62\frac{1}{2}$¢ per pound.
75 lb. tea at $37\frac{1}{2}$¢ per pound.

24. 17 tons coal at $12.50 per ton. ($12.50 = $\frac{1}{8}$ of $100.)
8 tons coal at $ $6\frac{2}{3}$ per ton.

25. 150 bu. barley at $ $1.12\frac{1}{2}$ per bushel.
250 bu. oats at 73¢ per bushel. (250 = $\frac{1}{4}$ of 1000.)
300 bu. wheat at $ $2.16\frac{2}{3}$ per bushel.

Miscellaneous Fractional Examples

EXERCISE 44

Some of the following examples are of the nature of puzzles rather than practical. They are included because examples like them sometimes appear on examination papers, and because, being unusual, they will challenge the ability of the pupil.

1. Find the sum of $\frac{11}{12}$ of $5.67 and $\frac{3}{4}$ of $8.75.

2. If $13\frac{5}{6}$ yards of cloth can be bought for $33.50, what will $15\frac{5}{8}$ yards cost at the same rate?

3. A, B, and C together received $17.40 for doing a piece of work. If A did $\frac{1}{8}$ of the work, B $\frac{1}{4}$, and C the remainder, what should each receive?

4. Which is the greater, $\frac{7}{8}$ of $16.12 or $\frac{5}{6}$ of $17.67, and how much?

5. If $15\frac{5}{6}$ yards of a certain cloth can be bought for $12.73, how many yards of the same cloth can be bought for $18.76?

6. Three men, A, B, and C, can do a piece of work in 18, 24, and 36 hours respectively. How long will it take them to do the work if they work together? If they receive $11.25 for doing the work, how much should each receive.

SUGGESTION. A does $\frac{1}{18}$ in one hour, B $\frac{1}{24}$, and C $\frac{1}{36}$.

7. A man left $\frac{1}{2}$ of his estate to his wife, $\frac{5}{8}$ of the remainder to his son, and the rest to his daughter. The wife received $546.75 more than the daughter. What did each receive?

8. A man spent $\frac{2}{5}$ of his money for provisions, $\frac{7}{8}$ of the remainder for clothing, and had $12.87 left. How much had he at first?

9. If $5\frac{3}{8}$ pounds of coffee can be bought for 2.36\frac{1}{2}$, how many pounds can be bought for $6.38?

10. A, B, and C can do a piece of work in 6, 14, and 21 days, respectively. B and C worked alone for 5 days, when they were joined by A, after which the work was completed by them working together. If they received $216 for doing the work, how much should each receive?

11. A man left $\frac{1}{3}$ of his property to his wife, $\frac{1}{4}$ to his elder son, $\frac{1}{5}$ to his younger son, $\frac{1}{6}$ to his daughter, and the balance to a charitable institution. If the charitable institution received $3697.50, how much did each of the family receive?

12. A man invested $\frac{5}{8}$ of his wealth in real estate, $\frac{3}{10}$ of the remainder in railway shares, and the balance in city bonds. The amount invested in real estate exceeded by $8700 the amount invested in city bonds. Find the amount of each investment.

13. A man divided $220 between his two sons in such a way that the younger received $\frac{2}{3}$ as much as the elder. How much did each receive?

14. An acre of ground contains 43,560 square feet.

a. What part of an acre is a lot containing 5400 square feet?

b. What part of an acre is a lot containing 7200 square feet?

c. If one of the smaller lots sold for $ 900, what would be the value of an acre at the same rate?

d. If one of the larger lots sold for $ 1050, what would be the value of an acre at the same rate?

15. A tank has two pipes. One of them can fill it in $8\frac{1}{2}$ minutes, and the other can empty it in $12\frac{3}{4}$ minutes. How many minutes will it take to fill the tank if both are opened at the same time?

III. DECIMAL FRACTIONS

59. Powers of Ten. — The numbers 100, 1000, etc., can be obtained by multiplying together a certain number of tens.

Thus, $100 = 10 \times 10$, and $1000 = 10 \times 10 \times 10$.

Such numbers are called **Powers of Ten.** Notice that the number of *zeros* in the number is the same as the number of *tens* that must be multiplied together to give that number.

Thus, 10,000 is the product of four tens.

60. Decimal Fractions are fractions having as denominators powers of ten, when written as follows:

$\frac{3}{10}$ is written .3; $\frac{8}{100}$ is written .08; etc.

The period placed before the number is called the **Decimal Point.** The figures to the right of the decimal point are said to be in the *first decimal place, the second, the third,* and so on.

61. A figure written in the

a. first decimal place denotes tenths;
b. second decimal place denotes hundredths;
c. third decimal place denotes thousandths;
d. fourth decimal place denotes ten thousandths;
e. fifth decimal place denotes hundred thousandths;
f. sixth decimal place denotes millionths; etc.

Thus, .037 means $\frac{0}{10} + \frac{3}{100} + \frac{7}{1000}$.

But $\frac{3}{100} + \frac{7}{1000} = \frac{30}{1000} + \frac{7}{1000} = \frac{37}{1000}$,

and $\therefore .037 = \frac{37}{1000}$.

Similarly, .245 means $\frac{2}{10} + \frac{4}{100} + \frac{5}{1000}$.

But $\frac{2}{10} + \frac{4}{100} + \frac{5}{1000} = \frac{200}{1000} + \frac{40}{1000} + \frac{5}{1000} = \frac{245}{1000}$,
and $\therefore .245 = \frac{245}{1000}$.

Rule. — To read a fraction written in decimal form:

1. Read the number on the right of the decimal point as if it were an integer, disregarding any zeros immediately to the right of the decimal point.

2. Then repeat the name of the fractional part indicated by the right-most digit.

3. If there are digits to the left of the decimal point, read the number indicated by them, then the word "and," and finally the decimal fraction.

Thus, .261 is read *two hundred sixty-one thousandths;*
.0261 is read *two hundred sixty-one ten thousandths;*
.000261 is read *two hundred sixty-one millionths;* etc.
Also, 2.305 is read *two and three hundred five thousandths.*

EXERCISE 45

1. What fractional part is indicated by a number in the third decimal place? in the fifth? in the second? in the seventh? in the eighth?

Read the following numbers:

2. .53.
3. .183.
4. .995.
5. 4.27.
6. 12.052.
7. 90.065.
8. .0039.
9. 8.208.
10. 24.0071.
11. .84073.
12. 3.689313.
13. 5.90854.
14. 2574.009.
15. 67.0203.
16. 736.89245.

17. *a.* Compare the fractions .20 and .2.
b. Compare the fractions .32 and .320.
c. Compare the fractions .5, .50, and .500.
d. What effect upon the value of a decimal fraction does it have to annex zeros to the right of it?

18. What is the common value of .4, .40, .400, etc.?

19. *a.* Determine the value of .2 and .02. (2 is called the significant figure.)

b. Determine the values of .3, and .003.

c. Determine the values of .5 and .0005.

d. Does it change the value of a decimal fraction to place one or more zeros between the decimal point and the first significant figure?

20. What is the effect upon a decimal fraction of placing between the decimal point and the first significant figure:

a. one zero; *b.* two zeros; *c.* three zeros?

Write the following numbers:

21. Fifty-two and forty-three hundredths.

22. One hundred fifty-eight thousandths.

23. Nine and thirteen ten thousandths.

24. Fifty-nine thousand, three hundred ninety-eight millionths.

25. Twenty-six and eight hundred five thousand, three hundred three millionths.

26. Read the following sentences based upon statistics in a recent Bulletin of the U. S. Bureau of Labor.

The average wholesale price during the year 1915:

a. Of lath was $ 3.8386 per thousand.

b. Of single-strength window glass in small sizes was $ 2.4225 per 50 square feet.

c. Of tin plate was $ 3.2417 per 100 lb. box.

d. Of barbed wire fencing was $ 2.5348 per 100 lb.

27. If a bar of steel at a temperature of 32° F. is heated $1\frac{4}{5}^\circ$, its length will increase .000011 of itself, a bar of iron .000012 of itself, a bar of copper .000017 of itself, and a bar of lead .000027 of itself. (From a textbook on Physics.)

28. The diameter of No. 1 wire is given as follows on different measuring instruments:

Brown and Sharpe instrument	.2893 in.
Washburn instrument	.2830 in.
U. S. Standard	.2812 in.

(From a textbook for machinists.)

62. Changing a Decimal Fraction to a Common Fraction.

Example 1. — $11.28 = 11 + \frac{28}{100} = \frac{1128}{100}$.

Example 2. — $.0523 = \frac{523}{10000}$.

Example 3. — $2.035 = 2 + \frac{35}{1000} = \frac{2035}{1000}$.

Rule. — To change a decimal fraction to a common fraction:

Write a fraction having for its numerator, the given digits with the decimal point omitted; and for its denominator, 1 followed by as many zeros as there are places to the right of the decimal point.

Thus, $2.065 = \frac{2065}{1000}$; $32.05 = \frac{3205}{100}$; and $.0347 = \frac{347}{10000}$.

EXERCISE 46

Express the following as common fractions in their lowest terms:

1. 2.8.
2. .005.
3. .684.
4. 1.85.
5. 8.512.
6. 75.44.
7. 30.75.
8. .1975.
9. 68.461.
10. .025.
11. .0376.
12. .0096.
13. 3.0875.
14. .08309.
15. .00125.
16. .0625.
17. 8.75.
18. 25.25.
19. .0375.
20. 4.235.
21. 10.25.
22. 83.125.
23. 6.25.
24. 13.042.
25. 5.375.

63. Adding and Subtracting Decimals.

When numbers involving decimals are written so that the decimal points are in a vertical line, the digits representing the same fractional parts fall in vertical columns. Consequently, the numbers can be added at once.

Example 1. — Add 7.89, 31.4, and .086.

Solution. — The explanation of this solution is left to the class.

$$\begin{array}{l} 7.89\\ 31.4\\ .086\\ \hline 39.376 \end{array}$$

If desired, zeros may be added to the right of some of the numbers in order to make all of them have the same number of decimal places, as arranged at the right.

$$\begin{array}{r} 7.890\\ 31.400\\ .086\\ \hline 39.376 \end{array}$$

Example 2. — Subtract 98.725 from 162.07.

Solution. — The solution may be arranged as at the right. Notice that a zero is annexed to the right of the minuend in order to make it and the subtrahend each have three decimal places.

$$\begin{array}{r} 162.070\\ 98.725\\ \hline 63.345 \end{array}$$

EXERCISE 47

Add the following:

1. 25.5, 2.0076, 1.7852, and 3.084.
2. 2.601, .9763, 35.083, and .00745.
3. 165, .94468, .0051, and 39.236.
4. 7.3075, 8.62, .03874, and 2.319.
5. 5.0805, .00007, 2.637, and .014976.
6. .39665, 93.3896, .0562, and 805.74.
7. 9.9, .00616, 75.1208, and 16.004.
8. 5.06212, 35.49, 87.65, 4290.24, and 297.638.
9. 2.0342, 69.21, 8.463, .12058, and 3.028.
10. 14.069, 20.743, .085, 9.263, and .00584.

Subtract:

11. .4167 from 5.2765.
12. .0825 from .32943.
13. .317 from 2.
14. .00964 from .0205.
15. 2.0843 from 11.365.
16. .5431 from 2.9063.
17. .06397 from 10.
18. .08948 from 3.52866.
19. 47.6007 from 830.453.
20. .00099 from .06009.

Find the sums:

21.	**22.**	**23.**	**24.**	**25.**
2516.34	809.142	1702.3	4617.335	33.862
375.081	76.053	56.7798	11.008	518.0325
9.004	4820.279	231.0205	700.8318	7.664
61.2865	36.0086	7.88	4.215	43.2218
649.75	7.251	339.021	63.08	.9086
31.1092	237.85	65.7081	3010.508	8201.22
3750.546	49.9957	.9959	715.244	66.192
879.0732	3654.0075	8.114	29.9592	735.11
.5531	68.132	563.7732	2.007	5.368
25.864	911.0768	8104.22	687.14	71.6842

64. Multiplying Decimals.

Example 1. — $.3 \times .2 = \frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = .06$.

Example 2. — $.4 \times .12 = \frac{4}{10} \times \frac{12}{100} = \frac{48}{1000} = .048$.

Example 3. — $.25 \times .83 = \frac{25}{100} \times \frac{83}{100} = \frac{2075}{10000} = .2075$.

Notice that the number of decimal places in the product in each of these examples equals the sum of the number in the multiplier and in the multiplicand. Hence,

Rule. — To multiply a decimal by a decimal:

1. Multiply the numbers, disregarding any decimal points.

2. Point off from the right as many decimal places as there are in the multiplier and the multiplicand together, prefixing zeros to the product if necessary.

Example 4. — Multiply .764 by .0108.

Solution. — In this example, the sum of the number of decimal places in the multiplier and in the multiplicand is seven. After multiplying, there are only five figures in the product. Two zeros are therefore prefixed to the product 82512.

$$\begin{array}{r} .764 \\ \underline{.0108} \\ 6112 \\ \underline{7640} \\ .0082512 \end{array}$$

EXERCISE 48

Multiply:

1. 8.27 by 27.3.
2. .096 by 5.61.
3. .0708 by .365.
4. .6528 by 9.7.
5. 5.648 by .082.
6. 2.835 by 34.1.
7. 45.76 by .0207.
8. .5114 by .3043.
9. .0682 by 16.
10. 4.486 by 5.38.
11. 18.052 by .75.
12. 21.96 by 4.78.
13. .0782 by 63.05.
14. .9256 by .0821.
15. 35.702 by .012.
16. 42.004 by 3.404.
17. .2705 by .234.
18. 9.631 by 3.1416.
19. 2.8 by .7854.
20. 3.8 by .00011.
21. 1.85 by 26.3 by 4.32.
22. 5.06 by 3.14 by .243.
23. 65.842 by .85 by .125.
24. .875 by 2500 by .05.

25. At 39° F. temperature (approximately), a cubic foot of copper weighs 8.45 times as much as water; of cork, .24 times as much as water; of gold, 19.26 times as much; of ice, .92 times as much; of lead, 11.35 times as much; of steel, 7.83 times as much; of air, .001293 times as much.

Determine the weight of a cubic foot of each of these materials if a cubic foot of water weighs 62.5 pounds.

26. If an object is allowed to fall freely from any height for any number of seconds, the distance through which it will fall may be found by multiplying 16.08 feet by the square of the number of seconds. (The square of a number is that number multiplied by itself.)

Determine how far an object will fall in:

a. 3 seconds; *b.* 4 seconds; *c.* 5 seconds.

Determine also how far it falls during the 4th second, and during the 5th second.

27. A farmer sold 15 loads of wheat, averaging 83.25 bushels to a load, at $1.95 per bushel. How much money was due him?

28. Find the cost of 12 bales of silk averaging 73.625 yards to a bale, at $1.93 per yard.

29. The width of the slot, for the screw driver, in the head of a flat-head machine screw, is found by multiplying the diameter of the body of the screw by .173 and then adding .015.

Thus, if the diameter is .086 inch, the width of the slot is $(.173 \times .086 + .015)$ inch.

a. Compute the result for this illustration.

b. Compute the width of the slot when the diameter is .268 inch.

65. Multiplying by 10, 100, 1000, etc.

Example 1.—
$$10 \times .3 = 10 \times \tfrac{3}{10} = 3.$$
$$10 \times .03 = 10 \times \tfrac{3}{100} = \tfrac{3}{10} = .3.$$
$$10 \times .003 = 10 \times \tfrac{3}{1000} = \tfrac{3}{100} = .03.$$

Example 2.—
$$100 \times .3 = 100 \times \tfrac{3}{10} = 30.$$
$$100 \times .03 = 100 \times \tfrac{3}{100} = 3.$$
$$100 \times .003 = 100 \times \tfrac{3}{1000} = \tfrac{3}{10} = .3.$$

Multiplying a decimal by 10 has the effect of moving the decimal point of the multiplicand one place to the right; multiplying by 100, of moving the decimal point two places to the right. Hence,

Rule.—To multiply a number by 10, 100, 1000, etc., move the decimal point of the multiplicand as many places to the right as there are zeros in the multiplier, annexing zeros to the multiplicand if necessary.

Thus, $100 \times 2.653 = 265.3$;
$1000 \times 3.84 = 3840.$

66. Multiplying by .1, .01, .001, etc.

Example 1.— .1 × .3 = .03.
.1 × .03 = .003.
.1 × .003 = .0003.

Example 2.— .01 × .3 = .003.
.01 × .03 = .0003.
.01 × .003 = .00003.

Multiplying a decimal by .1 has the effect of moving the decimal point of the multiplicand one place to the left; multiplying by .01, of moving the decimal place two places to the left. Hence,

Rule.—To multiply a number by .1, .01, .001, etc., move the decimal point of the multiplicand as many places to the left as there are decimal places in the multiplier, prefixing zeros to the multiplicand if necessary.

Thus, .001 × 5 = .005; and .0001 × 26.3 = .00263.

EXERCISE 49

1. Multiply each of the following numbers by 10.

a. 27.6 *b.* 3.85 *c.* 2.608 *d.* .5703 *e.* 642
f. 30.451 *g.* 51.008 *h.* 431.8 *i.* 500.09 *j.* .089

2. Multiply each of the numbers in Example 1 by 100.

3. Multiply each of the numbers in Example 1 by 1000.

4. Multiply each of the numbers in Example 1 by .1.

5. Multiply each of the numbers in Example 1 by .01.

6. Multiply each of the numbers in Example 1 by .001.

Multiply:

7. 85.2 by 10.
8. 377 by .01.
9. .0065 by 1000.
10. .0208 by 100.
11. 12.61 by .01.
12. 5839 by .00001.
13. 1.417 by 10,000.
14. 368.8 by .001.
15. 937,628 by .00001.
16. .06023 by 100,000.

What is the cost of:

17. 10 dozen eggs at $ .55 per dozen?

18. 100 feet of oak lumber at 12.5¢ per foot?

19. 1000 bushels of oats at 93.75¢ per bushel?

20. 100 feet of garden hose at 13.25¢ per foot?

21. 1000 tons of chestnut coal at $ 5.3256 per ton?

NOTE. — In this and the following examples, the prices are average wholesale prices of 1915.

22. 1000 tons of anthracite egg coal at $ 5.0446?

23. 1000 pounds of Ohio wool at $ .5714 per pound?

24. 1000 pounds of worsted yarn at $ .7875 per pound?

25. 100 gallons of denatured alchohol at $ .3717 per gallon?

26. 100 pounds of candles at $ .0725 per pound?

27. 1000 yards of all wool French serge at $ .3325 per yard?

28. 10,000 yards of all wool storm serge dress goods at $ .5574 per yard?

29. 100 dozen men's cotton shirts at $ 4.1875 per dozen?

30. 100 yards of Middlesex suiting, 15 ounce variety, 55–56 inches wide, at $ 1.5638 per yard?

31. 1000 yards bleached muslin shirting at $ .0782 per yard? 1000 yards bleached muslin shirting at $ .1001 per yard?

32. 10,000 pounds of linen shoe thread at $ 1.0763 per pound? 1000 spools of cotton thread at $ .0359 per spool?

33. 100 barrels of salt pork at $ 18.3173 per barrel?

34. 10,000 quarts of milk at $ .0376 per quart?

35. 100,000 pounds of poultry at $ .1373 per pound?

36. 10,000 pounds of fresh beef at $.1255 per pound?

37. 100,000 dozen eggs at $ .2657 per dozen?

38. 10,000 gallons of denatured alcohol at $ .3717 per gallon?

Division of Decimals

67. Dividing a Decimal by an Integer.

Example 1. — $.8 \div 2 = \frac{8}{10} \div 2 = \frac{4}{10} = .4.$

Example 2. — $.045 \div 3 = \frac{45}{1000} \div 3 = \frac{15}{1000} = .015.$

Example 3. — $12.18 \div 6 = \frac{1218}{100} \div 6 = \frac{203}{100} = 2.03.$

Notice that the number of decimal places in the quotient in each example is the same as the number in the dividend. Hence,

Rule. — To divide a decimal by an integer:

1. Carry out the division, disregarding the decimal point.

2. Point off from the right in the quotient as many decimal places as there are in the dividend.

The following example illustrates the most convenient arrangement of the solution.

Example 4. — Divide 4742.66 by 754.

Solution. — Notice that each figure of the quotient is directly above the last figure of the partial product obtained by multiplying the divisor by that figure of the quotient. Notice also that the decimal point of the quotient is directly above that of the dividend.

$$
\begin{array}{rr}
 & 6.29 \\
754 & \overline{)4742.66} \\
 & \underline{4524} \\
 & 218\,6 \\
 & \underline{150\,8} \\
 & 67\,86 \\
 & \underline{67\,86}
\end{array}
$$

Example 5. — Divide 237.5 by 68, carrying out the division to three decimal places.

Solution. — Notice that two zeros are annexed to the dividend so that there will be three decimal places in it.

The final result may be written 3.492^{+}

$$
\begin{array}{rr}
 & 3.492 \\
68 & \overline{)237.500} \\
 & \underline{204} \\
 & 33\,5 \\
 & \underline{27\,2} \\
 & 6\,30 \\
 & \underline{6\,12} \\
 & 180 \\
 & \underline{136} \\
 & 44
\end{array}
$$

68. Dividing a Number by a Decimal Divisor.

Notice the following divisions.

$$\begin{array}{r}4\\ 2\overline{)8}\end{array};\ \text{also}\ \begin{array}{r}4\\ 20\overline{)80}\end{array};\ \text{also}\ \begin{array}{r}4\\ 200\overline{)800}\end{array}.$$

Observe that the dividend and the divisor in the second division have each been multiplied by 10, and in the third division have each been multiplied by 100. All three quotients, however, are the same. Hence,

Rule I. — The dividend and the divisor may each be multiplied by 10, 100, etc., without changing the value of the quotient.

This fact aids in solving

Example 1. — Divide 6.804 by 2.1.

Solution. — The divisor can be made an integer by multiplying it by 10. Hence multiply both dividend and divisor by 10.

$$\left.\begin{array}{c}\text{Then the solution of}\\ 2.1\overline{)6.804}\end{array}\right\}\ \text{is the same as}\ \left\{\begin{array}{r}\text{the solution of}\\ 3.24\\ 21\overline{)68.04}\\ \underline{63}\\ 5\,0\\ \underline{4\,2}\\ 84\\ \underline{84}\end{array}\right.$$

The second dividend and divisor are obtained by moving the decimal points of the original ones one place to the right.

Rule II. — To divide a number by a decimal divisor:

1. **Move the decimal point of the divisor to the right until the divisor becomes an integer; then move the decimal point of the dividend an equal number of places to the right, annexing zeros as may be necessary.** (See Rule above.)

2. **Then carry out the division as directed by the Rule in § 67.**

Example 2. — Divide 2.75918 by .7261.

Solution. — Notice that the original decimal points have been crossed out, and new ones inserted four places to the right.

$$
\begin{array}{rr}
 & 3.8 \\
\underline{\cancel{.}7261.} & \overline{)2\cancel{.}7591.8} \\
 & \underline{2\;1783} \\
 & 5808\;8 \\
 & \underline{5808\;8}
\end{array}
$$

Example 3. — Divide .06084 by 2.34.

Solution. — In this example, there must be three decimal places in the quotient, of which 2 and 6 occupy the second and third places. A zero must be prefixed to 26 before locating the decimal point, in order to have as many places in the quotient as in the dividend.

$$
\begin{array}{rr}
 & .026 \\
\underline{2\cancel{.}34.} & \overline{)\cancel{.}06.084} \\
 & \underline{4\;68} \\
 & 1\;404 \\
 & \underline{1\;404}
\end{array}
$$

EXERCISE 50

Divide. If the division is not exact, obtain a quotient having three decimal places.

1. 15 by .3.
2. 28 by .04.
3. 3.6 by .6.
4. .42 by .7.
5. 5.4 by .03.
6. .66 by 3.
7. 8.4 by 1.2.
8. 9.6 by .32.
9. 1.24 by .004.
10. 25.6 by .008.
11. 100 by .05.
12. 376.8 by .006.
13. 46.35 by .15.
14. .48 by .0012.
15. 284.24 by .038.
16. 4.6292 by 2.84.
17. .49245 by .549.
18. 29,379.7 by .47.
19. 19.7635 by 8.41.
20. .31236 by .00685.
21. 462.90881 by .9107.
22. 691.8015 by 71.74.
23. 609.4429 by .01234.
24. 332.45 by .0488.

In the following problems, carry out the result as indicated.

25. During the year 1916, 298,826 immigrants came to this country. What was the average number per month? (One decimal place.)

26. How many miles will a train run in one hour at the rate of one mile:

a. in 1′ 20″ (1 minute and 20 seconds); *b.* in 2′ 3″; *c.* in 1′ 43″; *d.* in 1′ 10″?

27. If it takes a train 4 hours and 20 minutes to run 140 miles, what is the rate per hour? (One decimal place.)

28. A short ton of coal occupies about 38.4 cubic feet of space. About how many tons of coal can be placed in a bin containing 700 cubic feet? (Two decimal places.)

29. On an auto trip of 157 miles, 14 gallons of gasoline were consumed. How many miles did the machine run on one gallon of gasoline?

30. An autoist finds that his four tires are practically worn out at the end of 4250 miles of service. If the tires cost him $33.75 each, what was the expense for tires in cents per mile? (Cents and one decimal.)

31. In 1910, the population of the country was 91,972,266 and the number of acres of land was 1,903,289,600. What was the average number of acres per inhabitant? (One decimal.)

32. In 1910, the total land area of the country was 2,973,890 square miles. What was the population per square mile? (One decimal.)

33. In 1915, there were produced in the United States, 281,104,104 barrels of crude petroleum, valued at $179,462,890. What was the value per barrel? (One decimal place beyond cents.)

34. In 1914, 19,153,786 children were enrolled in the schools of the country. The total cost of instruction was $555,077,146. What was the cost per pupil? (One decimal place beyond cents.)

35. In a recent year, there were in the United States 24,825 telegraph offices.

a. From these offices, 75,135,405 messages were sent. What was the average number per office? (One decimal place.)

b. The net profits of the operating companies were $ 7,274,900. What was the net profit per office? (One decimal place beyond cents.)

36. The exports of certain articles of food from the United States during the fiscal year ending June 30 in the last year before the Great European War and in the year 1916, the second year of that war, were:

ARTICLE	THE YEAR ENDING JUNE 30, 1914	THE YEAR ENDING JUNE 30, 1916
Fresh beef	$6,394,404	$231,215,075
Bacon	193,964,252	579,808,786
Fresh pork	2,668,020	63,005,524
Butter	3,693,597	13,503,279
Cheese	2,427,577	44,394,251

How many times as great was the value of the exports in 1916 as they were in 1914? (One decimal place.)

69. Dividing a Decimal by 10, 100, 1000, etc.

By § 67, 84.6 ÷ 10 = 8.46.

Similarly, 92.4 ÷ 100 = .924.

Rule. — To divide a number by 10, 100, 1000, etc.

Move the decimal point of the dividend as many places to the left as there are zeros in the divisor.

An important application of this rule is found in

Example 1. — Divide 4716.28 by 62,800.

$$\begin{array}{r|l} & .07\;51 \\ \hline 628.00_{\wedge} & 47.16_{\wedge}28 \\ & \underline{43\;96} \\ & 3\;20\;2 \\ & \underline{3\;14\;0} \\ & 6\;28 \\ & \underline{6\;28} \end{array}$$

Solution. — In order to remove the two zeros from the divisor, move the decimal point two places to the *left*, by dividing by 100. Then also move the decimal point of the dividend two places to the *left*.

This does not change the quotient.

EXERCISE 51

1. Divide each of the following numbers by 10.

a. 37.8 *b.* 92.83 *c.* 5.637 *d.* 426.54 *e.* .695
f. 10.342 *g.* .008 *h.* .125 *i.* 60.086 *j.* 402.54

2. Divide each of the numbers in Example 1 by 100.

3. Divide each of the numbers in Example 1 by 1000.

4. Divide each of the numbers in Example 1 by 200.

5. Divide each of the numbers in Example 1 by 500.

Divide:

6. 630 by 500.
7. 2697.5 by 8300.
8. 683.37 by 270.
9. .463 by 1000.
10. 7.815 by 2000.
11. 85,536 by 648,000.
12. 5156.8 by 4000.
13. 27,865 by 3000.
14. 82,648 by 2400.
15. 2,756,830 by 4300.
16. 1,276,534 by 478,000.
17. 2,376,583.45 by 27,500.

18. How many tons of coal are there in 7680 pounds? What is the value at $ 8.75 per ton?

19. How many thousand feet of lumber are there in a pile of wood containing 15,350 feet? What is the value at $ 36 per thousand?

20. How many hundreds of pounds of glucose are there in three shipments consisting of 2346, 5485, and 4275 pounds respectively? What is the value at the price $ 2.2942 per hundred?

21. A "square" of roofing material is 100 square feet. How many squares of roofing material are required to cover a roof containing 2760 square feet?

22. How many tons of cabbage are there in three wagon loads consisting of 1675 pounds, 1845 pounds, and 1780 pounds? What is the value at the average price of $ 13.05 per ton?

23. How many hundreds of pounds of tobacco are there in five loads weighing 2765, 2430, 3125, 2875, 2945 pounds respectively? What is the value of the whole lot at $13.78 per 100 pounds?

24. Find the value of 1578 pounds of beans at $5.81 per hundred.

25. Find the value of three lots of sheep weighing respectively 1345 pounds, 2253 pounds, and 1682 pounds at $5.92 per hundred.

26. Find the value of five lots of steers weighing respectively 5765, 8342, 9724, 7586, and 8794 pounds at $11.35 per hundred pounds.

27. Find the value at $15.72 per hundred of five hogs weighing respectively 258, 274, 264, 278, and 285 pounds.

28. Find the value of 12,500 ft. of lumber at $34 per thousand.

29. Find the value of 6500 shingles at $5.75 per thousand.

30. Find the cost of 775 lb. of oil meal at $3.10 per hundred.

31. Find the cost of 1500 lb. of bran at $2.15 per hundred.

32. Find the cost of 650 lb. of middlings at $2.25 per hundred.

33. Find the cost of 4500 pads of paper at $37.50 per thousand.

34. Find the cost of 1155 lb. of timothy seed at $8.35 per hundred.

35. Find the cost of 845 lb. of clover seed at $36.85 per hundred.

36. Find the cost at $2.15 per hundred of:

a. 940 lb.; *b.* 1125 lb.; *c.* 650 lb.

37. Find the cost at $27.50 per thousand of:
a. 4500 ft. of lumber; *b.* 3750 ft.; *c.* 8475 ft.

38. Find the cost at $8.25 per hundred of:
a. 275 lb. of timothy seed; *b.* 169 lb.; *c.* 735 lb.

39. Find the cost at $37.50 per hundred of:
a. 624 lb. of clover seed; *b.* 369 lb.; *c.* 472 lb.

40. Find the value at $19.86 per hundred of:
a. 12,400 lb. of pork; *b.* of 8250 lb.; *c.* of 7785 lb.

70. Changing a Common Fraction to a Decimal Fraction.

Example 1. — Change $\frac{57}{125}$ to decimal form.

Solution. — 1. $\frac{57}{125} = 57 \div 125 = .456.$

$$\begin{array}{r@{}l} & .456 \\ 125 & \overline{)57.000} \\ & \underline{50\,0} \\ & 7\,00 \\ & \underline{6\,25} \\ & 750 \\ & \underline{750} \end{array}$$

In some cases, there is not an exact decimal equivalent for a given common fraction. In such cases, a decimal is often derived which represents the given fraction for a certain number of decimal places.

Example 2. — Express $\frac{7}{12}$ as a decimal fraction correct to three decimal places.

Solution. —

$$\begin{array}{r@{}l} & .583 \\ 12 & \overline{)7.000} \\ & \underline{6\,0} \\ & 1\,00 \\ & \underline{96} \\ & 40 \\ & \underline{36} \\ & 4 \end{array}$$

$\frac{7}{12} = .583^{+}$.

Sometimes the result is written $.583\frac{4}{12}$ or $.583\frac{1}{3}$; but this is seldom a useful form of the result.

A more common practice is to increase the last figure of the decimal by 1 if the remainder is equal to or more than one half the divisor.

Thus, $.725\frac{6}{7}$ may be written .726,
and $.725\frac{2}{7}$ may be written .725.

EXERCISE 52

Change the following fractions to decimals, giving the first three decimal places:

1. $\frac{7}{9}$.
2. $\frac{1}{5}$.
3. $\frac{5}{16}$.
4. $\frac{1}{3}$.
5. $\frac{5}{21}$.
6. $\frac{1}{9}$.
7. $\frac{2}{7}$.
8. $\frac{3}{13}$.
9. $\frac{23}{45}$.
10. $\frac{11}{23}$.
11. $\frac{2}{9}$.
12. $\frac{15}{32}$.
13. $\frac{14}{27}$.
14. $\frac{3}{11}$.
15. $\frac{15}{42}$.
16. $\frac{17}{9}$.
17. $\frac{17}{26}$.
18. $\frac{9}{32}$.
19. $\frac{11}{12}$.
20. $\frac{23}{16}$.
21. $\frac{183}{40}$.
22. $\frac{256}{73}$.
23. $\frac{341}{5000}$.
24. $\frac{625}{89}$.
25. $\frac{433}{1280}$.
26. $\frac{21}{312}$.
27. $\frac{19}{189}$.
28. $\frac{1562}{73}$.
29. $\frac{253}{8100}$.
30. $\frac{31}{480}$.
31. $\frac{857}{649}$.
32. $\frac{23}{27}$.
33. $\frac{5754}{236}$.
34. $\frac{8275}{94882}$.
35. $\frac{24575}{123582}$.

36. Change $\frac{3}{8}$ to decimal form by use of "aliquot parts" of 100.

Solution. — $\frac{3}{8} = \frac{3}{8}$ of $\frac{100}{100} = \frac{37\frac{1}{2}}{100} = .37^1 = .375.$

Similarly express in decimal form:

37. $\frac{1}{4}$.
38. $\frac{1}{2}$.
39. $\frac{1}{3}$.
40. $\frac{2}{3}$.
41. $\frac{1}{5}$.
42. $\frac{3}{5}$.
43. $\frac{1}{6}$.
44. $\frac{1}{8}$.
45. $\frac{3}{8}$.
46. $\frac{1}{12}$.
47. $\frac{5}{12}$.
48. $\frac{7}{8}$.
49. $\frac{5}{6}$.
50. $\frac{7}{12}$.
51. $\frac{1}{16}$.
52. $\frac{1}{15}$.
53. $\frac{5}{8}$.
54. $\frac{4}{5}$.
55. $\frac{7}{25}$.
56. $\frac{9}{40}$.

Miscellaneous Examples

EXERCISE 53

1. Find the weight of a cubic inch of material if a cubic foot weighs the amount indicated below, by dividing by 1728, to three decimal places:

a. cast iron, 450 lb. per cubic foot.

b. steel, 489 lb. per cubic foot.

c. lead, 711 lb. per cubic foot.

2. The number of cubic inches in a steel shaft 2 in. in diameter and 12 ft. long is

$$.7854 \times 12 \times 12 \times 2 \times 2.$$

Determine the number of cubic inches, and then from the result of part (*b*) of Example 1, find the weight of the shaft.

3. *a.* Find the total of the following grains sold to a mill by a farmer:

1468 bu. of oats at 73¢ per bushel.
545 bu. of wheat at $2.13 per bushel.

b. The farmer bought:

850 lb. of bran at $1.75 per hundred pounds.
225 lb. of oil meal at $3.15 per hundred.
475 lb. of middlings at $2.10 per hundred.

How much did the mill man still owe the farmer?

4. In 1914, 580,058 teachers were employed in the schools of the United States. Their salaries amounted to $323,610,915. What was the average salary?

5. In 1916, there were in the United States about 21,166,000 horses, valued at $101.60 each on the average. What was the total value of the horses?

6. In 1915, there were in the country on the Indian Reservations of the United States 324,878 Indians. The

reservations embraced 33,324,261 acres of ground. What was the average acreage for each Indian?

7. In 1913, the year before the Great War, 688 vessels were built in Great Britain with a total tonnage of 1,932,153; in Germany, 162 vessels with a tonnage of 465,226; in the United States, 182 vessels with a tonnage of 228,232. Arrange these data in a table. Determine the average tonnage per vessel for each country, carrying out the division to only the first decimal place.

8. *a.* A gallon contains 231 cubic inches; a cubic foot, 1728 cubic inches. How many gallons of water can be held in a vessel having a capacity of 1 cubic foot?

b. A cubic foot of water weighs about 62.5 pounds. What is the weight of a gallon of water, correct to two decimal places?

9. The specific gravity of a substance is the weight of any "volume" of that substance divided by the weight of an equal volume of water.

Weigh a quantity of milk in a vessel, — say a full quart measure of milk. Then in the same vessel weigh the same amount of water. Find the specific gravity of milk by dividing the weight of the milk by the weight of the water.

10. Determine in the same manner the specific gravity of some sand.

11. Find the total cost of:

78 lb. of goods at $.12½ per pound.
54 lb. of goods at $.40 per pound.
112 lb. of goods at $.37½ per pound.
27 lb. of goods at $.83⅓ per pound.

12. The following computations become necessary in a certain textbook which treats of the effects, both upon the cannon itself and upon a projectile placed in its chamber, of burning in the cannon a certain amount of powder.

Find the result of:

a. $1400.4 \div 3.280869$ to two decimal places.

b. $\frac{2116.3 \times 12.3909}{273}$ to three decimal places.

c. $1 + \frac{96.056}{.16892 \times 1400.4}$ to three decimal places.

d. $\frac{5 \times 86.878 \times 333}{144 \times 64.6967}$ to three decimal places.

e. $\frac{876 \times 294\frac{1}{9}}{2 \times 3 \times 144} - 14.697$ to three decimal places.

f. $\frac{19.6 \times 2116.3}{29.9215}$ to one decimal place.

g. $\frac{5}{9}(60 - 32) + 273$ to two decimal places.

h. $\frac{.000006941 \times 180 \times 1694 \times 1694}{70 \times 61.08}$ to four decimal places.

13. To find the number of threads per inch on a standard machine screw, divide 6.5 by the diameter of the screw plus .02.

Thus, if the diameter is .0828″, the number of threads is

$$\frac{6.5}{.0828 + .02}.$$

a. Find the result in this case.

b. Find the result also if the diameter is .1210″.

14. The length of a belt running around a 14″ pulley and a 20″ pulley placed 12 ft. apart is found by solving the following indicated computation:

$$3.1416(10 + 7) + (2 \times 144) + \frac{(10 - 7)^2}{144}.$$

Find the result.

15. The horse power developed by a certain steam engine is found by completing the following indicated computation:

$$\frac{40 \times 4 \times 63.62 \times 120}{3 \times 33000}.$$

Find the result correct to one decimal place.

16. The number of pounds of pull necessary to raise 120 pounds of weight a distance of 30 inches by means of a certain combination of pulleys is found by completing the following indicated computation:

$$\frac{120 \times (4.5 - 3)^2}{2 \times 4.5}.$$

Find the result.

17. The annual statement of a certain life insurance company for the year 1917 was summarized in the following form:

Ledger Assets Jan. 1, 1917		$4,879,871.50
Income During 1917		
Premium Income	$1,564,539.38	
Interest and Other Income . . .	318,477.79	?
Total of Assets and Income . .		?
Disbursements		
Paid Policy Holders	$ 332,812.45	
All Other Disbursements . . .	678,339.75	?
Balance		?
Ledger Assets Jan. 1, 1918		
Real Estate	$ 222,766.84	
Mortgage Loans	4,353,177.40	
Policy Loans	723,224.20	
Bonds	217,239.12	
Cash in Banks	108,276.16	
Agents' Balances, Notes . . .	127,052.75	?

Find the totals where indicated by the question marks, and also the balance where indicated.

IV. DENOMINATE NUMBERS

71. Review the tables of measures, if necessary, given at the back of the book. **Units of Measure**, by which corresponding quantities are measured, are given in these tables. The **Measure** of a concrete quantity is a *number* of units of measure of the same kind.

Thus, a line may be eight feet in length, — the measure being eight and the unit one foot. Again, a lot may contain 2345 square feet, — the measure being 2345 and the unit one square foot.

72. Expressions like 8 ft., 2345 sq. ft., 3 gal. + 1 qt., etc. are called **Denominate Numbers.**

73. Expressing Denominate Numbers in Terms of Smaller Units.

Example 1. — Express 3 A. 50 sq. rd. in terms of square rods.

Solution. — 1. 3 A. = 3 × 160 sq. rd. = 480 sq. rd.

2. ∴ 3 A. 50 sq. rd. = 480 sq. rd. + 50 sq. rd. = 530 sq. rd.

Example 2. — Express $\frac{4}{25}$ T. in pounds.

Solution. — $\frac{4}{25}$ T. = $\frac{4}{25}$ × 2000 lb. = 320 lb.

Example 3. — Express $\frac{8}{9}$ rd. in terms of smaller units.

Solution. — 1. $\frac{8}{9}$ rd. = $\frac{8}{9}$ × $5\frac{1}{2}$ yd.

= $\frac{8}{9}$ × $\frac{11}{2}$ yd. = $\frac{44}{9}$ yd. = $4\frac{8}{9}$ yd.

2. $\frac{8}{9}$ yd. = $\frac{8}{9}$ × 3 ft. = $2\frac{2}{3}$ ft.

3. $\frac{2}{3}$ ft. = $\frac{2}{3}$ × 12 in. = 8 in.

4. ∴ $\frac{8}{9}$ rd. = 4 yd. 2 ft. 8 in.

EXERCISE 54

Reduce:

1. 5 mi. to rods; to feet.
2. 12 gal. to quarts; to pints.
3. 15 sq. yd. to square feet; to square inches.
4. 1 wk. to hours; to minutes.
5. 3 T. to hundredweight; to pounds.
6. 8 bu. to quarts; to pints.
7. 15° to minutes; to seconds.
8. 3 cu. yd. to cubic feet; to cubic inches.
9. 4 sq. rd. to square yards; to square feet.
10. 24 rd. to yards; to feet.
11. $\frac{2}{3}$ cu. yd. to cubic feet; to cubic inches.
12. $\frac{7}{8}$ mi. to rods; to feet.
13. 35 rd. to yards; to feet.
14. $\frac{27}{60}$° to minutes; to seconds.
15. $\frac{11}{16}$ bu. to pecks; to quarts; to pints.
16. $\frac{1}{10}$ A. to square rods; to square feet.
17. $\frac{3}{5}$ T. to hundredweight; to pounds.
18. 9.5 bu. to pecks; to pints.
19. .56 wk. to hours; to minutes.
20. 8.6 sq. rd. to square feet.
21. 3.6 gal. to quarts; to pints.
22. 12 gal. 3 qt. to pints.
23. 1 T. 15 cwt. 81 lb. to pounds.
24. 3 A. 5 sq. rd. to square yards.
25. 365 da. 5 hr. to hours.
26. 2 mi. 253 rd. to feet.
27. 8 bu. 6 pk. to quarts.

28. 3 yd. 2 ft. 5 in. to inches.

29. 4 gal. $3\frac{1}{2}$ qt. to pints.

30. 5 sq. yd. 7 sq. ft. to square feet.

31. 15 cu. yd. 8 cu. ft. to cubic feet.

Express the following in terms of smaller units as in Example 3, page 108.

32. $\frac{7}{8}$ gal.
33. $\frac{36}{64}$ bu.
34. 2.51 hr.
35 $\frac{56}{33}$ rd.
36. 2.8125 lb.
37. .75 wk.
38. $\frac{4}{5}$ rd.
39. .625 bu.
40. $\frac{2}{3}$ T.
41. $\frac{1}{3}$ yr.
42. $\frac{1}{6}$ mi.
43. $\frac{1}{4}$ rd.

74. Expressing Denominate Numbers in Terms of Larger Units.

Example 1. — Express 5870 ft. in rods.

Solution. — 1. Since $16\frac{1}{2}$ ft. = 1 rd.,
2. 5870 ft. = $5870 \div 16\frac{1}{2}$
3. $= 5870 \times \frac{2}{33}$
4. $= 355\frac{25}{33}$ rd.
5. $= 355.7^{+}$ rd.

The approximate decimal form of the result is probably the most useful form.

Example 2. — Express 1523 ft. in terms of larger units with integral multipliers.

Solution. — 1. Since 3 ft. = 1 yd.,
1523 ft. = $1523 \div 3$ = 507 yd. 2 ft.
2. Since $5\frac{1}{2}$ yd. = 1 rd.
507 yd. = $507 \div 5\frac{1}{2}$
= 92 rd. 1 yd.
3. ∴ 1523 ft. = 92 rd. 1 yd. 2 ft.

$\frac{1523}{3} = 507\frac{2}{3}$ yd.
= 507 yd. 2 ft.

$507 \div 5\frac{1}{2}$
$= 507 \times {}^{2}$
$= \frac{1014}{11} = 92\frac{2}{11}$ rd.
= 92 rd. 1 yd.

NOTE. — This process has long been called *Reduction Ascending.*

EXERCISE 55

Express in common and also in decimal fractional form:

1. 3 pk. as part of a bushel.
2. 7 pt. as part of a gallon.
3. 35 min. as part of an hour.
4. 250 rd. as part of a mile.
5. 90 sq. in. as part of a square foot.
6. 15 cu. ft. as part of a cubic yard.
7. 100 sq. rd. as part of an acre.
8. 1000 yd. as part of a mile.
9. 30,000 sq. ft. as part of an acre.
10. 20 in. as part of a yard.
11. 3500 ft. as part of a mile.
12. 1350 lb. as part of a ton.
13. 75 rd. as part of a mile.
14. 250 da. as part of a year.
15. 30 wk. as part of a year.
16. 14 in. as part of a yard.
17. 175 rd. as part of a mile.
18. 120 sq. rd. as part of an acre.
19. 30 in. as part of a yard.
20. 1400 lb. as part of a ton.

Express in fractional form and also in decimal form correct to two decimal places as in Example 1.

21. 1259 rd. as miles.
22. 7843 lb. as tons.
23. 15,786 ft. as miles.
24. 278 qt. as gallons.
25. 73,840 sq. ft. as acres.
26. 10,000 ft. as miles.

27. 3500 yd. as miles.

28. 23,450 lb. of wheat as bushels of wheat.

(See Tables of Measures for the number of pounds in a bushel.)

29. 31,275 lb. of oats as bushels of oats.

30. 16,870 lb. barley as bushels of barley.

31. 480 qt. as bushels.

32. 9365 cu. ft. as cubic yards.

33. 15,000 ft. as miles.

34. 7500 yards as miles.

35. 10,000 doz. as gross.

36. 6500 da. as years.

37. 30,000 lb. as tons.

38. 150,000 sq. ft. as acres.

39. 15 cwt. 6 lb. as tons.

40. 2 pk. 2 qt. as part of a bushel.

41. 16 hr. 25 min. as hours.

42. 2 qt. 1 pt. as part of a gallon.

43. 8 ft. 11 in. as feet.

44. 4 yd. 1 ft. $2\frac{2}{5}$ in. as part of a yard.

45. 71 lb. 12 oz. as part of a hundredweight.

46. 3 pk. 1 qt. as part of a bushel.

47. 3 qt. 1 pt. as part of a gallon.

48. 4 da. 20 hr. as part of a week.

Express in terms of larger units as in Example 2.

49. 975 pt. dry measure.

50. 2357 oz. avoirdupois.

51. 2609 pt. liquid measure.

52. 45,432 in.

53. 113,292 sq. ft.

54. 80,000 seconds of time.

55. 98,582 sq. ft.

56. 3758 lb.

57. 5432 yd.

58. 2654 ft.

59. 5000 yd.

60. 8500 ft.

61. 27,000 lb.

62. 15,000 yd.

75. Adding and Subtracting Denominate Numbers.

Example 1.—Add 17 lb. 10 oz., 12 lb. 9 oz., and 8 lb. 7 oz.

Solution.—1.

$$\begin{array}{r} 17\text{ lb.} + 10\text{ oz.}\\ 12\text{ lb.} + 9\text{ oz.}\\ 8\text{ lb.} + 7\text{ oz.}\\ \hline 37\text{ lb.} + 26\text{ oz.}\end{array}$$

2. But 26 oz. = 1 lb. + 10 oz.

3. ∴ 37 lb. + 26 oz. = 38 lb. + 10 oz.

Example 2.—Subtract 3 mi. 201 rd. from 6 mi. 76 rd.

Solution.—1.

$$\begin{array}{rcr} 6\text{ mi.} + 76\text{ rd.} & = & 5\text{ mi.} + 396\text{ rd.}\\ \underline{3\text{ mi.} + 201\text{ rd.}} & = & \underline{3\text{ mi.} + 201\text{ rd.}}\\ & & 2\text{ mi.} + 195\text{ rd.}\end{array}$$

In this solution, 1 mi. of the 6 mi. is changed to 320 rd. and added to the 76 rd., making 396 rd.

EXERCISE 56

Find the sum of:

1. 12 gal. 3 qt. 1 pt., 7 gal. 1 qt., and 24 gal. 2 qt. 1 pt.

2. 4 cwt. 67 lb., 11 cwt. 49 lb., and 2 cwt. 83 lb.

3. 34 bu. 3 pk. 7 qt., 46 bu. 1 pk. 5 qt. 1 pt., 55 bu. 2 pk. 6 qt., and 27 bu. 3 pk. 1 pt.

4. 71 da. 22 hr. 18 min., 36 da. 16 hr. 48 min., 60 da. 11 hr. 32 min., and 49 da. 9 hr. 23 min.

5. 45° 38′ 40″, 123° 17′ 33″, 78° 44′ 55″, and 65° 46′ 12″.

6. 7 mi. 163 rd., 2 mi. 313 rd. 5 yd., 9 mi. 36 rd. 4 yd., 6 mi. 3 yd., and 244 rd. 2 yd.

7. 13 T. 11 cwt., 8 T. 12 cwt. 64 lb., 5 T. 8 cwt. 74 lb., 12 T. 36 lb., and 14 cwt. 63 lb.

8. 165 cu. ft. 828 cu. in., 23 cu. ft. 562 cu. in., and 37 cu. ft. 983 cu. in.

9. 3 sq. mi. 288 A., 543 A. 19 sq. rd., 74 A. 53 sq. rd., and 610 A. 149 sq. rd.

10. 12 mi. 98 rd. 5 yd., 8 mi. 212 rd. 4 yd., 156 rd. 5 yd., 18 mi. 313 rd. 4 yd., 36 mi. 286 rd., and 3 mi. 125 rd. 5 yd.

Subtract:

11. 2 bu. 3 pk. 7 qt. from 13 bu. 2 pk. 5 qt.

12. 49° 52′ 41″ from 87° 39′ 25″.

13. 16° 37′ from 97° 42′ 15″.

14. 38 gal. 3 qt. 1 pt. from 48 gal. 3 qt.

15. 42 lb. 7 oz. from 52 lb. 3 oz.

16. 6 cu. yd. 26 cu. ft. 752 cu. in. from 18 cu. yd. 9 cu. ft.

17. 7 mi. 16 rd. from 15 mi. 38 rd.

18. 14 T. 16 cwt. from 23 T. 11 cwt.

19. 9 A. 147 sq. rd. from 15 A.

20. 1 da. 20 hr. from 5 da. 15 hr. 48 min.

21. 4 A. 152 sq. rd. from 78 A. 153 sq. rd.

22. The sides of a field are 8 rd. 1 yd. 2 ft., 9 rd. 1 yd. 2 ft., 12 rd. 1 yd., and 10 rd. 1 yd. 1 ft. What is the length of a fence around it?

23. To trim a girl's garment there are needed 2 pieces of lace 13 inches long, one piece $1\frac{1}{3}$ yd. long, and one piece $3\frac{3}{8}$ yd. long. How much lace is required for the garment?

24. A man agreed to build 115 rd. of fence. On the first day, he built 27 rd. 3 yd.; and the second, 32 rd. 5 yd.; and on the third day, 26 rd. 1 yd. How much remained to be built?

25. From a barrel containing 50 gal. of lubricating oil, a dealer sold the following quantities: 5 gal. 3 qt., 3 gal., 1 qt., 5 pt., 5 gal., $2\frac{1}{2}$ gal., 7 qt., 2 gal. 1 pt., 3 gal. 2 qt., 3 pt.

How much should be left in the barrel?

76. Finding the Time between Two Dates.

Example 1.— Find the exact number of days from the 13th of June to the 10th of October of the same year.

Solution. — 1.	There remain in June	17 da.
	There are in July	31 da.
	There are in August	31 da.
	There are in September	30 da.
	There are in October	10 da.
2. ∴ the total number of days =		119 da.

NOTE. — The first of the given dates is not included, but the last is included.

Example 2. — Express in years, months, and days the time between Apr. 12, 1915 and Nov. 20, 1918.

Solution. — 1. From Apr. 12, 1915 to Apr. 12, 1918 is 3 yr.
2. From Apr. 12 to Nov. 12, 1918 is 7 mo.
3. From Nov. 12 to Nov. 20, 1918 is 8 da.
4. ∴ the total time is 3 yr. 7 mo. 8 da.

This method of solution is useful when the exact time in years, months, and days is wanted.

NOTE. — When reckoning from the last day of a month having 31 days, the number of months from then until the last day of the month preceding the final date is taken as the number of months, and then the number of days is counted.

Thus, the time from Mar. 31 to Oct. 17 is found as follows:
From Mar. 31 to Sept 30 is 6 months.
From Sept. 30 to Oct. 17 is 17 days.
Hence, the whole time is 6 months and 17 days.

Example 3. — How long is it from Feb. 13, 1915 to May 6, 1918?

		YR.	MO.	DA.		YR.	MO.	DA.
Solution. — May 6,	1918 =	1918	5	6	=	1918	4	36
Feb. 13,	1915 =	1915	2	13	=	1915	2	13
						3	2	23

Since 13 da. cannot be subtracted from 6 da., 1 mo. of the 5 mo. in the minuend is changed to 30 days and added to the 6 da., making 36 da.

EXERCISE 57

Find the exact number of days from:

1. March 23, 1918 to Dec. 11, 1918.
2. April 12, 1918 to Jan. 18, 1919.
3. Sept. 4, 1918 to Dec. 29, 1918.
4. July 7, 1918 to May 25, 1919.
5. Nov. 22, 1919 to Aug. 8, 1920.
6. Jan. 17, 1919 to Sept. 26, 1919.
7. Feb. 3, 1919 to Oct. 16, 1919.
8. Sept. 22, 1918 to March 12, 1919.
9. May 3, 1920 to Nov. 24, 1920.
10. April 26, 1918 to Feb. 19, 1919.

Find, as in Example 2, p. 115, the time in years, months, and days from:

11. March 13, 1915 to Jan. 28, 1920.
12. Feb. 17, 1914 to June 8, 1919.
13. April 26, 1918 to March 23, 1920.
14. Aug. 21, 1917 to May 15, 1921.
15. Dec. 13, 1917 to Dec. 4, 1919.
16. Oct. 28, 1916 to March 18, 1922.
17. Jan. 12, 1912 to April 4, 1918.
18. Feb. 17, 1909 to June 6, 1919.
19. Nov. 6, 1914 to Feb. 15, 1918.
20. Oct. 23, 1915 to May 1, 1919.

21–30. Find, as in Example 3, p. 115, the time between the dates given in Examples 11 to 20, and compare your results with those found in Examples 11 to 20.

77. Multiplying Denominate Numbers.

Example.—Multiply 2 mi. 135 rd. 5 yd. by 4.

Solution. — 1.

$$\begin{array}{rrr} 2\text{ mi.} & 135\text{ rd.} & 5\text{ yd.} \\ & & 4 \\ \hline 8\text{ mi.} & 540\text{ rd.} & 20\text{ yd.} \end{array}$$

2. But 20 yd. = 3 rd. + $3\frac{1}{2}$ yd.

∴ the distance = 8 mi. 543 rd. $3\frac{1}{2}$ yd.

3. 543 rd. = 1 mi. 223 rd.

∴ the distance = 9 mi. 223 rd. $3\frac{1}{2}$ yd.

EXERCISE 58

Multiply:

1. 13 bu. 3 pk. by 3.

2. 8 ft. 7 in. by 5.

3. 4 gal. 3 qt. by 3.

4. 7 sq. ft. 32 sq. in. by 8.

5. 5 rd. 4 yd. by 4.

6. 7 bu. 3 pk. 5 qt. by 2.

7. 26° 38′ by 3.

8. 15 gal. 3 qt. 1 pt. by 6.

9. 18 mi. 275 rd. 5 yd. by 4.

10. 5 da. 19 hr. 37 min. by 10.

11. 16 T. 8 cwt. 85 lb. by 5.

12. 37 cu. ft. 560 cu. in. by 11.

13. 12 lb. 7 oz. by 25.

14. 16 rd. 1 yd. 2 ft. by 9.

15. 5 A. 87 sq. rd. by 10.

16. How many feet of wood are required for 26 pieces each 3′ 4″ in length?

17. How many feet of iron rod are needed to make 185 marking stakes, each 1′ 8″ in length?

18. Find the value of 35 A. 68 sq. rd. of land at $225 per acre.

19. What is the value of 18,240 square feet of land at $750 per acre?

20. How much concrete is required to make 150 concrete stepping stones each containing 875 cu. in. of concrete?

21. How many yards of carpet must be bought to give 5 strips, each 4 yd. 2 ft. 8 in. in length?

78. Dividing Denominate Numbers by an Integer or a Fraction.

Example.—Divide 47 A. + 51 sq. rd. by 3.

Solution.—1. 47 A. ÷ 3 = 15 A. + remainder of 2 A., or 320 sq. rd.

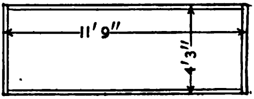
2. 320 sq. rd. + 51 sq. rd. = 371 sq. rd.
371 sq. rd. ÷ 3 = $123\frac{2}{3}$ sq. rd.

3. ∴ (47 A. + 51 sq. rd.) ÷ 3 = 15 A. + $123\frac{2}{3}$ sq. rd

EXERCISE 59

Divide:

1. 35 bu. by 3.
2. 53 gal. 3 qt. by 5.
3. 11 ft. 10 in. by 4.
4. 22° 30′ by 4.
5. 135 A. 80 sq. rd. by 2.
6. 41 lb. 4 oz. by 5.
7. 17 T. 5 cwt. by 3.
8. 25 sq. yd. 3 sq. ft. by 6.
9. 19 rd. $2\frac{1}{2}$ yd. by 2.
10. 22 A. 152 sq. rd. by 6.
11. 234 T. 17 cwt. 10 lb. by 10.
12. 35 bu. 3 pk. 5 qt. by 3.
13. 10′ 9″ by 3.
14. 16 rd. $9\frac{1}{2}$ yd. by 15.
15. 3 wk. 5 da. 4 hr. by 4.
16. Multiply 13 lb. 10 oz. by $3\frac{1}{3}$.
17. Divide 2 T. 2 cwt. by $\frac{3}{7}$.
18. Multiply 7 mi. 210 rd. 5 yd. by 4.
19. Multiply 12 cu. ft. 25 cu. in. by $\frac{4}{7}$.
20. Multiply 38° 58′ 16″ by $\frac{3}{2}$.
21. A carpenter is asked to build four cold-frame sashes to cover the cold frame pictured. What will be the outside dimensions of each sash?



22. The distance between the first floor and the second floor of a house is 10′ 6″. If the stairway is to contain 17 steps (including the landing on the second floor), how much rise is there in each step?

79. Dividing a Denominate Number by a Denominate Number.

Example 1.—Divide 12 yd. 2 ft. by 1 yd. 1 ft. 9 in.

Solution.—1. 12 yd. 2 ft. = 38 ft. = 456 in.
2. 1 yd. 1 ft. 9 in. = 57 in.
3. 456 in. ÷ 57 in. = 8.

Example 2.—What part of 7 gal. 2 qt. is 3 gal. 3 qt.?

Solution.—1. By § 53, the part is used as the dividend and the whole as the divisor.
2. 7 gal. 2 qt. = 30 qt.
3. and 3 gal. 3 qt. = 15 qt.

$$\therefore \text{the fractional part} = \frac{15 \text{ qt.}}{30 \text{ qt.}} = \frac{1}{2}.$$

Hence 3 gal. 3 qt. is $\frac{1}{2}$ of 7 gal. 2 qt.

EXERCISE 60

Divide:

1. 15 bu. by 3 pk.
2. 54 gal. by 4 gal. 2 qt.
3. 18 T. by 2 T. 5 cwt.
4. 171 gal. 3 qt. 1 pt. by 3 gal. 1 pt.
5. 6 T. 10 cwt. 84 lb. by 1 T. 12 cwt. 71 lb.
6. 502 da. 21 hr. 45 min. by 4 da. 3 hr. 45 min.
7. 10 mi. 165 rd. $2\frac{1}{2}$ yd. by 224 rd. 2 yd.

What part of

8. 5 da. is 1 da. 13 hr. 30 min.?

9. A steel rod 15 ft. long is put in an automatic machine to be worked up into pieces $2\frac{1}{2}$ in. in length. If $\frac{1}{8}$ in. is allowed for cutting on each piece, how many pieces will be obtained from the rod?

10. How many bricks each containing 64 cu. in. of material can be made from 50 cu. ft. of the raw material?

11. About how many concrete blocks each containing 900 cu. in. of concrete can be made from 1 cu. yd. of concrete?

12. How many pieces of tape 30″ long can be cut from a 10-yd. bolt of tape?

13. If an estate of 460 acres is divided equally among 6 children, how many acres and square rods will each child receive?

14. The capacity of a tank is 31 cu. ft. 528 cu. in. How many barrels will it contain if the capacity of a barrel is 2 cu. ft. 1152 cu. in.?

15. What is the rate of a train in feet per minute which is traveling 38 miles per hour?

16. What is the value of a carload of wheat weighing 8 T. 17 cwt. at $ 1.85 per bushel if a bushel of wheat weighs 60 lb.?

17. A man burned 9 T. 13 cwt. of coal in his furnace between Nov. 8 and Feb. 17 inclusive. How much did he burn per day?

18. Sound travels about 1085 ft. per second. How long will it take the sound of a gun to reach a point $2\frac{1}{2}$ miles away?

19. It takes a train 9 sec. to go 528 ft. (*a*) How long will it take it to go one mile? (*b*) How many miles will it travel in one hour?

20. If the moon revolves around the earth in 27 da. 7 hr. 42 min., over what part of its orbit does it travel in 1 da.? Express the result decimally to two places.

21. If a steel rail weighs 63 lb. per yard, how many tons of rail will be required to lay a mile of single-track railroad?

22. If a cubic foot of water weighs $62\frac{1}{2}$ lb. and lead is 11.44 times as heavy as water, how many cubic inches of lead are there in a piece weighing 43 lb. 8 oz.?

23. If a cubic foot of water weighs 1000 oz. and iron is 7.6 times as heavy as water, what is the weight of 405 cu. in. of iron?

24. If a railroad company is asked to put in a switch track one quarter mile in length, how many 60-ft. rails are needed?

25. Find the cost at $ 23.75 per ton of a load of hay weighing 4435 lb., if the weight of the empty wagon is 1670 lb.

26. A farmer delivered at the mill loads weighing net: 2875 lb., 2962 lb., 2885 lb., 3018 lb., 2992 lb., 3084 lb., 3115 lb., and 2925 lb.

Find the amount due him if oats were selling at 67 ¢ per bushel. (See Tables for number of pounds in a bushel.)

27. A millman shipped a car containing 54,912 lb. of wheat. What was due him if wheat was selling at $ 2.15 per bushel?

28. He also shipped 25,228 lb. of rye, which was sold at $ 2.83 per bushel. What was due him?

29. Find the total value of the following five loads of wheat at $ 2.15 per bushel:

3175 lb.; 3064 lb.; 3220 lb.; 2985 lb.; 3043 lb.

30. A boy bought at a mill the following order of grains for chicken feed:

125 lb. of bran at $ 1.85 per hundredweight.
125 lb. of middlings at $ 2.10 per hundredweight.
75 lb. of corn meal at $ 3.65 per hundredweight.
25 lb. of oil meal at $ 3.25 per hundredweight.
100 lb. shelled corn at $ 1.83 per bushel.
100 lb. oats at $ .78 per bushel.

Find the total of his bill.

EXERCISE 61

Miscellaneous Examples

1. An aviator flew from San Antonio, Texas, to Rantoul Field in Illinois during the summer of 1918 at an average rate of 139 mi. per hour. What was the average time for a mile, expressed in seconds?

2. How far from the earth, in miles, is an aviator who is up 11,000 ft.?

3. How far will a projectile go in a minute, if it is moving 2040 ft. per second?

4. What is the total cost at $12.35 per ton of the following loads of coal:

4720 lb.; 4564 lb.; 4488 lb.; 4675 lb.; 4810 lb.?

5. Change 2.32 A. to square rods.

6. *a.* Change 18,000 ft. to miles.

b. Express 18,000 ft. in terms of higher units.

7. A barrel of gasoline, containing 54½ gallons, is consumed in the gas machine of a country home in about 7 weeks. *a.* What is the amount consumed per day? *b.* What is the expense per day if the gasoline costs $.43½ per gallon?

8. A tobacco grower delivered at the warehouse three loads of tobacco weighing respectively 3278 lb., 3795 lb., and 4015 lb. What was the value if tobacco was selling for $3.25 per hundredweight?

9. Measure the diameter of a silver dollar. If 1,000,000 silver dollars were placed in a line, how far would they reach?

10. It was estimated that 11,590,000,000 gal. of milk were produced in the United States in 1915. How much was this per capita if the population was approximately 100,000,000?

SUPPLEMENTARY TOPICS

Whether or not the following topics should be taught, depends upon local conditions.

A. *Metric Measures.* B. *Foreign Money.*
C. *Apothecaries' Weights.*

A. The Metric System of Measures

80. The **Metric System of Measures** is employed exclusively now in many foreign countries. It is also used in the sciences, and in certain of the Departments of the United States Government.

Many of our manufacturers must make articles for foreigners according to directions given in metric units.

The system was first adopted in France in 1799.

MEASURES OF LENGTH

81. The fundamental unit of length is the **Meter**. This was intended to be *one ten-millionth* of a quadrant of the meridian of the earth running through Paris. Compared with the English units of length, one meter equals about 39.37 inches.

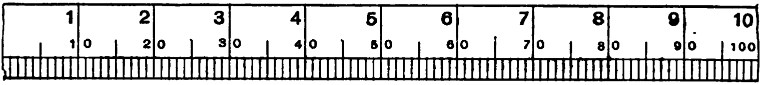
Table

10 millimeters (*mm.*)	= 1 centimeter	(*cm.*)
10 centimeters	= 1 decimeter	(*dm.*)
10 decimeters	= 1 **meter**	(*m.*)
10 meters	= 1 dekameter	(*Dm.*)
10 dekameters	= 1 hektometer	(*Hm.*)
10 hektometers	= 1 **kilometer**	(*Km.*)
10 kilometers	= 1 miriameter	(*Mm.*)

EQUIVALENTS

1 meter = 39.37 in.
1 yard = .914 m.
1 kilometer = about $\frac{5}{8}$ mile.
1 mile = 1.6093 Km.

From the table it is clear that there are in a meter 10 decimeters, 100 centimeters, and 1000 millimeters.



The prefixes of the word "meter" always have the same meaning:

thus, *milli* means *one thousandth* of the unit;
centi means *one hundredth* of the unit;
deci means *one tenth* of the unit;
Deka means *ten* times the unit;
Hecta means *one hundred* times the unit;
Kilo means *one thousand* times the unit;
Miria means *ten thousand* times the unit.

Measures of Surface

Table

100 square millimeters (*sq. mm.*) = 1 square centimeter (*sq. cm.*)
100 square centimeters = 1 square decimeter (*sq. dm.*)
100 square decimeters = 1 square meter (*sq. m.*)
= 1 centar (*Ca.*)
100 square meters, or centars = 1 square dekameter (*sq. Dm.*)
= 1 ar (*a.*)
100 square dekameters, or ars = 1 square hektometer (*sq. Hm.*)
= 1 hektar (*Ha.*)
100 square hektometers = 1 square kilometer (*sq. Km.*)

The square meter is used in measuring ordinary surfaces; the square kilometer, in measuring the areas of countries; the ar and hektar, in measuring land.

Equivalents

1 sq. cm. = .155 sq. in.	1 sq. in. = 6.452 sq. cm.
1 sq. m. = 1.196 sq. yd.	1 sq. yd. = .8361 sq. m.
1 sq. Km. = .3861 sq. mi.	1 sq. rd. = .2529 a.
1 a. = 3.954 sq. rd.	1 sq. mi. = 2.59 sq. Km.
1 Ha. = 2.471 A.	1 A. = .4047 Ha.

MEASURES OF CONTENTS

Table

1000 cubic millimeters (*cu. mm.*) = 1 cubic centimeter (*cu. cm.*)
1000 cubic centimeters = 1 cubic decimeter (*cu. dm.*)
1000 cubic decimeters = 1 cubic meter (*cu. m.*)

EQUIVALENTS

1 cu. m. = 1.308 cu. yd.   1 cu. yd. = .7645 cu. m.

MEASURES OF CAPACITY

Table

10 milliliters (*ml.*) = 1 centiliter (*cl.*)
10 centiliters = 1 deciliter (*dl.*)
10 deciliters = 1 liter (*l.*)
10 liters = 1 dekaliter (*Dl.*)
10 dekaliters = 1 hektoliter (*Hl.*)
10 hektoliters = 1 kiloliter (*Kl.*)

EQUIVALENTS

1 liter = 1.0567 liq. qt.   1 liquid qt. = .9463 l.
1 liter = .9081 dry qt.   1 dry qt. = 1.101 l.
1 hektoliter = 3.531 cu. ft.   1 gallon = 3.785 l.
1 hektoliter = 26.417 gal.   1 bushel = .3524 Hl.

NOTE. — The liter is used in measuring liquids and small fruit; the hektoliter, in measuring grain, vegetables, and liquids in casks.

MEASURES OF WEIGHT

The principal unit of weight is the **Gram**. It is the weight of a cubic centimeter of distilled water at 39.2° Fahrenheit.

Table

10 milligrams = 1 centigram (*cg.*)
10 centigrams = 1 decigram (*dg.*)
10 decigrams = 1 **gram** (*g.*)
10 grams = 1 dekagram (*Dg.*)
10 dekagrams = 1 hektogram (*Hg.*)
10 hektograms = 1 **kilogram** (*Kg.*)
1000 kilograms = 1 metric ton (*T.*)

EQUIVALENTS

1 kilogram = 2.2046 lb. avoirdupois 1 lb. avoirdupois = .4536 Kg.

NOTE.—The gram is used in weighing letters, precious metals, and jewels, and in mixing medical prescriptions; the kilogram, in weighing ordinary articles; the metric ton, in weighing heavy articles.

EXERCISE 62

1. By holding the edge of a ruler against the scale on p. 124, determine the approximate number of millimeters in 3 in.; also in 1.25 in.

2. Similarly determine the approximate English length equivalent to 45 mm.; also to 93 mm.

3. Measure the diameter of a *penny* in millimeters.

4. Measure the diameter of a five-cent piece in millimeters.

5. Measure the length and the width of your school room in the metric system.

(If your class does not possess a *meter stick*, the teacher in charge of Physics classes in your school certainly will have one that you can use.)

6. *Class Exercise.* Represent upon the blackboard a square meter. Inside it, draw a square representing a square yard.

82. Reducing Denominate Numbers in the Metric System.

In the tables of measures of length, weight, and capacity, any unit is ten times the next smaller unit of the table. Hence to change any number of these units to the next smaller units, merely move the decimal point one place to the right; to change to the next larger unit, move the decimal point one place to the left.

Thus, 25 m. = 250 dm. = 2500 cm. = 25000 mm.;
and 25 m. = 2.5 Dm. = .25 Hm. = .025 Km.

Expressions like 2 m. 5 dm. 3 mm. are therefore unnecessary in the metric system, for

2 m. 5 dm. 3 mm. = 2000 mm. 500 mm. 3 mm.
= 2503 mm.

In changing units of surface or square measure, the decimal point must be moved two places to the right or left for every change to a smaller or a larger unit respectively, for in the table of surface measure, each unit is one hundred times the next smaller unit.

Similarly, in changing cubic units, the decimal point must be moved three places.

EXERCISE 63

Express:

1. 4.72 m. as centimeters; as millimeters; as kilometers.
2. .073 Kl. as liters; as centiliters.
3. 8.325 Kg. as grams; as decigrams; as milligrams.
4. 73.85 Km. as meters; as dekameters.
5. .0548 sq. Hm. as square meters.
6. 3 m. 5 dm. 4 cm. as centimeters; as meters.
7. 7 Kg. 6 Dg. 5 g. as grams.
8. 5 l. 4 dl. 6 ml. as liters.
9. 78.56 m. as centimeters; as hectometers.
10. 735 cu. m. as cubic decimeters.

11. Add 78.26 mg., 8.53 dg., 3 g. (Change all of them to milligrams.)

12. Add 47.6 m., 83.05 Dm., .45 Km.

13. Add 314 sq. m., 5.62 sq. m., and .043 sq. Dm.

14. Subtract 89.7 Dg. from 2.816 Kg.

15. Subtract 2.482 mm. from .5016 m.

16. Subtract 60.4 dl. from 4.307 Hl.

17. Multiply 5.03 m. by 2.5 and express the result in hectometers.

18. Multiply .8259 sq. dm. by 35 and express the result in square meters.

19. Multiply 47.7 mg. by .6 and express the result in centigrams.

20. Divide 93.98 ml. by 25.4 dl.

21. Divide .0735 Kg. by 2.3 and express the result to three decimal places in grams.

22. Divide 4.686 Hm. by 825 m.

23. Divide .0602784 cu. cm. by .644 and express the result in cubic millimeters.

24. Multiply 56.82 dm. by 4.3.

25. Divide 4.7835 Km. by 635 m.

83. Only very simple problems involving conversion of metric units to English units or vice versa are likely to occur.

Example 1.—Express 15.3 Km. in miles.

Solution.—1. Since 1 Km. = .6214 mi.
2. 15.3 Km. = 15.3 × .6214 mi.
= 9.5^{+} mi.

Example 2.—Express 1 ton in kilograms.

Solution.—1. Since 1 lb. = .4536 kg.
2. ∴ 2000 lb. = 2000 × .4536 kg. = 907.2 kg.

EXERCISE 64

Express:

1. 1 yd. in decimeters.
2. 1 pk. in dekaliters.
3. 1 sq. ft. in square decimeters.
4. 1 cu. m. in cubic feet.
5. 1 centar in square feet.
6. 1 pt. (liquid) in deciliters.
7. 31 Hm. in rods.
8. 288 sq. ft. in square meters.
9. 5 yd. 2 ft. in meters.
10. 20 cu. ft. in cubic meters.
11. 25 lb. 14 oz. in kilograms.
12. 12 bu. 3 pk. in dekaliters.
13. 3 mi. 200 rd. in kilometers.
14. 3500 kilograms in pounds.
15. 2750 kilometers in miles.
16. 15 hectars in acres.
17. 40 acres in hectars.
18. 60 lb. (1 bu. wheat) in kilograms.
19. 31½ gal. (1 bbl.) in kiloliters.
20. 15 Km. in miles.

21. A merchant bought 300 liters of wine at 83¢ per liter and sold it at $5.00 per gallon. What were his profits?

22. A merchant bought silk at $1.95 per meter and sold it at $3.00 per yard. How much did he gain on 100 meters?

23. A merchant wants about 500 yards of a certain grade of silk. How many meters should he order?

24. A wine merchant received 1000 liters of wine. How many quart bottles will he be able to fill?

25. Foreign letters from the United States weighing 15 g. or less may be mailed for 5¢ postage. What is the equivalent weight in English units?

26. How many kiloliters of gasoline are there in a barrel containing 54.5 gallons?

27. A French grain dealer ordered through a broker 10,000 kilograms of wheat. How many bushels should the broker purchase?

28. A broker must fill an order for 25,000 kilograms of oats. How many bushels must he order?

29. What is the equivalent in French units of 100,000 gallons of gasoline?

30. What is the equivalent in French units of 50,000 pounds of beef?

31. On Sept. 2, the English troops continued "their attacks on a front of 45 Km." (From official war report.) What is the English equivalent of this distance?

B. Foreign Money

84.

English Money

4 farthings (far.) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£)
1 *s.* = $\$.243^{+}$; 1 £ = \$4.8665

French Money

100 centimes = 1 franc (fr.) = \$.193.

German Money

100 Pfennigs = 1 Mark (M) = \$.238.

Mexican Money

100 centavos = 1 peso = \$.498.

EXERCISE 65

1. Change 1 £ to pence.

2. Change 100 *d.* to pounds.

3. Express 75 *s.* in higher denominations.

4. Change 10,000 centimes to francs.

5. How much United States money should a soldier receive for 75 fr.?

6. How many francs should a traveler receive for $ 150, if no charge is made for the exchange?

7. How much United States money will a traveler receive in exchange for 75 £ ?

8. A United States soldier bought in France articles costing 3 fr. 50 cen.; 2 fr. 40 cen.; 5 fr. 85 cen. and 75 cen. How much did he spend?

9. Express $\frac{3}{4}$ £ in shillings; in pence.

10. Express .45 £ in shillings; in pence.

11. Express .78 fr. in centimes.

12. Express $\frac{3}{5}$ fr. in centimes.

13. How much change should be received from a 5-pound note offered in payment of purchases amounting to 3 £ 17 *s.* 10 *d.*?

14. An American traveler going to England exchanges $ 1000 in United States money for English money. How much should he receive?

15. What will be the cost of 15 doz. towels at 18 *s.* 6 *d.* per dozen?

16. *a.* The European Edition of the New York Herald in the summer of 1918 sold for 20 centimes per copy. How much is the equivalent in American money?

b. The annual subscription rate outside of Paris was 75 fr. per year. How much is this in American money?

C. Apothecaries' Measures

85. Apothecaries' measures, liquid and dry, are given in the collection of tables at the back of the book.

Reduction, both ascending and descending, and other operations with these measures are performed as with other measures.

EXERCISE 66

1. How many scruples are there in 1 pound?
2. Change 8℥ 5ʒ to drams.
3. Change 2℔ 3℥ 7ʒ to drams.
4. Change 75 gr. to drams.
5. Change 250 gr. to ounces.
6. Change 96℈ to ounces; to drams.
7. Change 1 pt. to fluid drams.
8. Change 1½ gal. to fluid ounces.
9. Change 26fʒ to fluid ounces.
10. Change 4240 minims to pints.
11. Express in higher units 8027 gr.
12. Express in higher units 35,798 minims.
13. Express in higher units 951,000 ʒ.
14. Add 8℥ 5ʒ, 10℥ 7ʒ 3℈, 6ʒ 2℈, and 6℥ 4ʒ 9℈.
15. From 3℔ 11℥, take 15℥.
16. Add 3O 15f℥ and 1O 8f℥.
17. From 1 gal. take 5O 12f℥.
18. How many bottles of hand lotion each containing 3f℥ can be filled from 3 quarts of the lotion?
19. From 14O 4f℥ take 9O 13f℥.
20. How much quinine is required to fill 15 dozen 3-grain capsules?

21. How much of each of the following drugs must be taken to make enough for 24 powders, each of which is to contain the following amounts:

Extract nux vomica, 6 gr.; caffeine citrate, 24 gr.; sodium salicylate, ½ ʒ; sodium phosphate, 1 ʒ?

V. INVOLUTION AND EVOLUTION

86. If a number is used as a factor two or more times, the result is called a **Power** of the number.

Thus, 3 × 3 or 9 is the second power of 3;
and 5 × 5 × 5 or 125 is the third power of 5.

5 × 5 × 5 is written 5^3. The small number written at the right and a little above the 5 is called an **Exponent**; the number 5 is called the **Base.**

An integral **Exponent** indicates the number of times that the base is used as a factor.

Thus, 2^5 means 2 × 2 × 2 × 2 × 2 or 32.

87. **Involution** is the process of raising a number to a required integral power.

The second power of a number is called the **Square** of the number.

Thus, 7 squared or 7^2 is 49.

The third power of a number is called the **Cube** of the number.

Thus, 4 cubed or 4^3 is 4 × 4 × 4 or 64.

EXERCISE 67

Find the value of:

1. 5^2.
2. 3^3.
3. 6^2.
4. 2^4.
5. 8^2.
6. 10^3.
7. 3^4.
8. 5^4.
9. 6^3.
10. 20^2.
11. 10^4.
12. 7^3.
13. 9^2.
14. 25^2.
15. 30^3.
16. 11^2.
17. 4^4.
18. 3^5.
19. 2^6.
20. 6^4.

21. $(\frac{1}{2})^2$.
22. $(\frac{2}{3})^3$.
23. $(\frac{1}{3})^2$.
24. $(\frac{2}{5})^3$.
25. $(\frac{3}{4})^2$.
26. $(\frac{5}{6})^2$.
27. $(\frac{1}{3})^4$.
28. $(\frac{4}{11})^2$.
29. 12^3.
30. 85^2.
31. $(2.5)^2$.
32. $(1.1)^2$.
33. $(3.5)^2$.
34. $(4.3)^2$.
35. $(1.5)^2$.
36. $(1.3)^2$.
37. $(.25)^2$.
38. $(.04)^2$.
39. $(.21)^2$.
40. $(2.03)^2$.
41. $(.003)^2$.
42. $(3.12)^2$.
43. $(20.13)^2$.
44. $(.215)^2$.

45. Memorize the square of each of the integers 1, 2, 3, ... etc., up to 20 inclusive.

46. Determine and memorize the cube of each of the integers 1, 2, 3, ... etc., up to 10 inclusive.

88. If one number is the square of a second, then the second is called the **Square Root** of the first.

Thus, since $25 = 5^2$, then 5 is the square root of 25.

If one number is the cube of a second, then the second is called the **Cube Root** of the first.

Thus, since $8 = 2^3$, then 2 is the cube root of 8.

89. **Evolution** is the process of finding a required root of a number.

90. The **Radical Sign**, $\sqrt{\ }$, when written over a number, indicates some root of the number. The particular root wanted is indicated by writing an **Index** to the left of and a little above the radical sign.

$\sqrt[3]{8}$ means the cube root of 8, or 2.

$\sqrt[4]{81}$ means the fourth root of 81. This is 3.

When no index is written, then the square root is indicated.

Thus, $\sqrt{25}$ means the square root of 25, or 5.

91. **Finding Roots by Inspection.** — Certain square and cube roots can be given at sight if pupils have done as directed in Examples 45 and 46 of Exercise 67.

EXERCISE 68

Give:

1. $\sqrt[2]{81}$.

2. $\sqrt[3]{64}$.

3. $\sqrt{144}$.

4. $\sqrt[3]{27}$

5. $\sqrt{121}$.

6. $\sqrt[3]{1000}$.

7. $\sqrt{196}$.

8. $\sqrt[3]{125}$.

9. $\sqrt{256}$.

10. $\sqrt[3]{512}$.

11. $\sqrt{400}$.

12. $\sqrt[3]{343}$.

13. $\sqrt{324}$.

14. $\sqrt[3]{729}$.

15. $\sqrt{361}$.

16. $\sqrt[3]{216}$.

17. Find $\sqrt{\frac{16}{25}}$.

Solution. — $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$, for $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$.

18. Find $\sqrt[3]{\frac{8}{125}}$.

Solution. — $\sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}} = \frac{2}{5}$, for $\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$.

Find similarly:

19. $\sqrt{\frac{4}{9}}$.

20. $\sqrt{\frac{25}{49}}$.

21. $\sqrt{\frac{9}{100}}$.

22. $\sqrt{\frac{16}{81}}$.

23. $\sqrt{\frac{64}{49}}$.

24. $\sqrt{\frac{225}{9}}$.

25. $\sqrt{\frac{169}{100}}$.

26. $\sqrt{\frac{196}{144}}$.

27. $\sqrt{\frac{49}{100}}$.

28. $\sqrt[3]{\frac{8}{27}}$.

29. $\sqrt[3]{\frac{27}{64}}$.

30. $\sqrt[3]{\frac{1}{8}}$.

31. $\sqrt[3]{\frac{1}{1000}}$.

32. $\sqrt[3]{\frac{1}{729}}$.

33. $\sqrt[3]{\frac{1}{64}}$.

34. $\sqrt[3]{\frac{27}{125}}$.

35. $\sqrt{\frac{81}{169}}$.

36. $\sqrt[3]{\frac{27}{125}}$.

37. $\sqrt{\frac{1}{256}}$.

38. $\sqrt[3]{\frac{125}{8}}$.

39. $\sqrt[3]{\frac{1}{27}}$.

40. $\sqrt[3]{\frac{64}{125}}$.

41. $\sqrt{\frac{324}{25}}$.

42. $\sqrt[3]{\frac{216}{343}}$.

43. $\sqrt[3]{\frac{27}{1000}}$.

44. $\sqrt[3]{\frac{125}{512}}$.

45. $\sqrt{\frac{361}{400}}$.

46. $\sqrt{\frac{121}{256}}$.

47. $\sqrt[3]{\frac{64}{27}}$.

48. $\sqrt{\frac{169}{225}}$.

92. Computing the actual or approximate **Square Root** is based upon the following method of squaring a number.

To find 74^2.

Notice that $74 = 70 + 4$. To multiply $70 + 4$ by $70 + 4$, one may first multiply by 70 and then by 4 as below.

$$\begin{aligned} 70 \times (70 + 4) &= 70^2 + 70 \times 4 \\ 4 \times (70 + 4) &= \underline{\quad\quad\quad 70 \times 4 + 4^2} \\ \text{Hence } (70 + 4)^2 &= 70^2 + 2 \times 70 \times 4 + 4^2. \end{aligned}$$

That is, *to square a number of two figures, first square the tens,* add twice the product of the tens and the units and finally add the square of the units.

Thus, $23^2 = 20^2 + 2 \times 20 \times 3 + 3^2 = 400 + 120 + 9 = 529$.

93. Notice next that $1^2 = 1$, $10^2 = 100$, $100^2 = 10{,}000$; etc.

From these results it is clear that

a. The square of a number of one figure has either one or two figures in it;

b. The square of a number of two figures has either three or four figures in it;

c. The square of a number of three figures has either five or six figures in it.

Therefore, working backward, if a power be divided into groups of two figures each, beginning with the units' figure, for each such group there will be one figure in the square root. The groups are called **Periods.**

Thus, 6889 becomes 68 89. The square root must have in it two figures. It cannot have three figures by (*c*) above, and it must have more than one by (*a*) above.

Similarly 45,632 becomes 4 56 32. Its square root must have in it three figures, a hundreds', a tens', and a units' figure.

If there are figures to the right of the decimal point, divide them also into periods of two figures each, starting from the decimal point.

Thus, 4357.265 becomes 43 57.26 50. The last period on the decimal side is completed by annexing a zero to the given number. The square root of this number must have two figures to the left of the decimal point and two to the right of it.

94. The Process of Computing the Square Root.

Example 1. — Find the square root of 6889.

Solution. — 1. 6889 divided into periods becomes 68 89. There are in the square root a tens' and a units' figure.

2. The tens' figure must be 8, for $80^2 = 6400$ and $90^2 = 8100$.

That is, the square root must be 80 plus a number x less than 10.

$$\begin{array}{lll} \therefore 6889 \text{ must} & = 80^2 + 2 \times 80 \times x + x^2. & \\ & = 80^3 + x(2 \times 80 + x) & 68\ 89 \\ \text{From 6889, subtract } 80^2 \text{ or 6400.} & & \underline{64\ 00} \\ \text{Then } 2 \times 80 \times x + x^2 \text{ must be} & & 4\ 89 \end{array}$$

To find x, form the *trial divisor* 2×80 or 160. This is contained in 489 three times with a remainder. This suggests that $x = 3$.

To the trial divisor 160 add 3, making 163. This is the complete divisor. 3×163 exactly equals 489.

The square root then is $80 + 3$ or 83. This can be proved by squaring 83.

This solution is arranged in the following form.

$$\begin{array}{lr|l} & & 8\cancel{0} + 3 \\ & & 68\ 89 \\ & & \underline{64\ \cancel{0}\cancel{0}} \\ \text{Trial divisor } 2 \times 8\cancel{0}; & 160 & 4\ 89 \\ 489 \div 160 = 3^+. \quad \text{Add 3.} & \underline{3} & \\ \text{Complete divisor} & 163 & \underline{4\ 89} \end{array}$$

Hence the square root is 83.

NOTE. — The zeros crossed out are usually omitted in the solution.

Rule. — To find the square root of a number:

1. Separate the number into periods. (§ 93)

2. Find the greatest square number in the leftmost period; write its square root in the root; subtract the square itself from the leftmost period, and to the result annex the next period.

3. Form the trial divisor by doubling the root already found and annexing zero to the result.

4. Divide the remainder in Step 2, by the trial divisor. Annex the quotient to the root already found, and add the quotient to the trial divisor, forming the complete divisor.

5. Multiply the complete divisor by the root figure last obtained and subtract the product from the remainder of Step 2.

6. If other periods remain, repeat Steps 3, 4, and 5 until there is no longer a remainder, or until the desired number of decimal places has been obtained for the root.

Note. 1. — It sometimes happens that, on multiplying a complete divisor, the product is greater than the remainder. In such cases, the figure of the root last obtained is too great, and the next smaller integer must be substituted for it.

Note 2. — If the remainder, in Step 4, is less than the trial divisor, the next figure of the root is zero. Annex zero to the root, and bring down the next period. Then start again with Step 3.

Example 2. — Find the square root of 4944.9024.

Solution. — The first trial divisor is 140. Since this is more than 44, the first remainder, annex zero to the root, obtaining 70. Then bring down the next period, 90.

The second trial divisor is then 1400. Dividing 4490 by it, gives 3 as the next figure of the root.

```
           70.32
       |49 44.90 24
       |49
  1400|44 90
     3|
  1403|42 09
 14060| 2 81 24
     2|
 14062| 2 81 24
```

Example 3. — Find the square root of 12, correct to three decimal places.

Solution. — The solution adjoining is according to the rule.

The square root to three places is 3.464.

```
       3.464
     |12 00 00 00
     | 9
  60|3 00
   4|
  64|2 56
 680| 44 00
   6|
 686| 41 16
6920|  2 84 00
   4|
6924|  2 76 96
          7 04
```

EXERCISE 69

Find the square root of:

1. 1849.
2. 3844.
3. 3136.
4. 5476.
5. 8281.
6. 10,609.
7. 55,225.
8. 264,196.

9. 104,329.
10. 14.3641.
11. 2735.29.
12. 33.1776.
13. .521284.
14. 6811.2009.
15. 44,605.44.
16. 24.7009.
17. 3344.3089.
18. 5838.4881.
19. 224,486.44.
20. .256036.

Find the approximate square root of the following, correct to three decimal places:

21. 7.
22. 23.
23. 35.
24. 47.
25. 17.3.
26. 2.85.
27. 37.2.
28. 145.63.
29. 72.05.
30. 643.27.
31. 58.908.
32. 375.625.
33. 125.48.
34. 3410.73.
35. 6714.95.
36. 18,956.241.

95. Tables are employed to avoid the computation **of Powers** and of **Roots** of numbers. On p. 141 is a table of the squares, cubes, square roots, and cube roots of all numbers from 1 to 100 inclusive. In the column headed by the letter N is the number. Opposite the number, in the appropriate column is found its square, its cube, its square root, or its cube root.

Thus, in the second column of numbers is found the number 64. In the first column to the right is the square of 64, namely 4096. In the second column to the right, is the cube of 64, namely 262,144. In the next column to the right, is the square root, namely 8. And in the last column, is the cube root of 64, namely 4.

EXERCISE 70

Determine from the table, or with the help of the table:

1. 39^2.
2. 76^3.
3. 58^2.
4. 47^2.
5. 29^3.
6. $\sqrt{38}$.
7. $\sqrt[3]{82}$.
8. $\sqrt{91}$.
9. $\sqrt[3]{56}$.
10. $\sqrt{70}$.
11. 86^2.
12. $\sqrt[3]{99}$.
13. 23^3.
14. $\sqrt[3]{59}$.
15. $\sqrt{65}$.
16. 46^2.
17. $\sqrt[3]{71}$.
18. $\sqrt{37}$.
19. $\sqrt[3]{28}$.
20. $\sqrt[3]{78}$.
21. $(\frac{13}{28})^2$.
22. $(\frac{12}{37})^3$.
23. $(\frac{47}{15})^2$.
24. $(\frac{28}{61})^3$.
25. $\sqrt[3]{\frac{97}{27}}$.
26. $\sqrt{\frac{89}{64}}$.
27. $\sqrt{\frac{95}{49}}$.
28. $\sqrt[3]{\frac{57}{8}}$.
29. $\sqrt{\frac{77}{25}}$.
30. $\sqrt[3]{\frac{63}{64}}$,
31. $\sqrt{\frac{27}{14}}$.
32. $\sqrt{\frac{85}{11}}$.
33. $\sqrt[3]{\frac{76}{23}}$.
34. $\sqrt[3]{\frac{85}{47}}$.
35. $\sqrt{\frac{28}{35}}$.
36. $\sqrt{\frac{37}{69}}$.
37. $\sqrt[3]{\frac{29}{78}}$.
38. $\sqrt[3]{\frac{46}{83}}$.
39. $\sqrt{\frac{97}{13}}$.
40. $\sqrt[3]{\frac{92}{29}}$.

N.	Squares	Cubes	Square Roots	Cube Roots	N.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.	1	51	2,601	132,651	7.141	3.708
2	4	8	1.414	1.259	52	2,704	140,608	7.211	3.732
3	9	27	1.732	1.442	53	2,809	148,877	7.280	3.756
4	16	64	2.	1.587	54	2,916	157,464	7.348	3.779
5	25	125	2.236	1.709	55	3,025	166,375	7.416	3.802
6	36	216	2.449	1.817	56	3,136	175,616	7.483	3.825
7	49	343	2.645	1.912	57	3,249	185,193	7.549	3.848
8	64	512	2.828	2	58	3,364	195,112	7.615	3.870
9	81	729	3.	2.080	59	3,481	205,379	7.681	3.892
10	100	1,000	3.162	2.154	60	3,600	216,000	7.745	3.914
11	121	1,331	3.316	2.223	61	3,721	226,981	7.810	3.936
12	144	1,728	3.464	2.289	62	3,844	238,328	7.874	3.957
13	169	2,197	3.605	2.351	63	3,969	250,047	7.937	3.979
14	196	2,744	3.741	2.410	64	4,096	262,144	8.	4.
15	225	3,375	3.872	2.466	65	4,225	274,625	8.062	4.020
16	256	4,096	4.	2.519	66	4,356	287,496	8.124	4.041
17	289	4,913	4.123	2.571	67	4,489	300,763	8.185	4 061
18	324	5,832	4.242	2.620	68	4,624	314,432	8.246	4.081
19	361	6,859	4.358	2.668	69	4,761	328,509	8.306	4.101
20	400	8,000	4.472	2.714	70	4,900	343,000	8.366	4.121
21	441	9,261	4.582	2.758	71	5,041	357,911	8.426	4.140
22	484	10,648	4.690	2.802	72	5,184	373,248	8.485	4.160
23	529	12,167	4.795	2.843	73	5,329	389,017	8.544	4.179
24	576	13,824	4.898	2.884	74	5,476	405,224	8.602	4.198
25	625	15,625	5.	2.924	75	5,625	421,875	8.660	4.217
26	676	17,576	5.099	2.962	76	5,776	438,976	8.717	4.235
27	729	19,683	5.196	3	77	5,929	456,533	8.774	4.254
28	784	21,952	5.291	3.036	78	6,084	474,552	8.831	4.272
29	841	24,389	5.385	3.072	79	6,241	493,039	8.888	4.290
30	900	27,000	5.477	3.107	80	6,400	512,000	8.944	4.308
31	961	29,791	5.567	3.141	81	6,561	531,441	9.	4.326
32	1,024	32,768	5.656	3.174	82	6,724	551,368	9.055	4.344
33	1,089	35,937	5.744	3.207	83	6,889	571,787	9.110	4.362
34	1,156	39,304	5.830	3.239	84	7,056	592,704	9.165	4.379
35	1,225	42,875	5.916	3.271	85	7,225	614,125	9.219	4.396
36	1,296	46,656	6.	3.301	86	7,396	636,056	9.273	4.414
37	1,369	50,653	6.082	3.332	87	7,569	658,503	9.327	4.431
38	1,444	54,872	6.164	3.361	88	7,744	681,472	9.380	4.447
39	1,521	59,319	6.244	3.391	89	7,921	704,969	9.433	4 464
40	1,600	64,000	6.324	3.419	90	8,100	729,000	9.486	4.481
41	1,681	68,921	6.403	3.448	91	8,281	753,571	9.539	4.497
42	1,764	74,088	6.480	3.476	92	8,464	778,688	9.591	4.514
43	1,849	79,507	6.557	3 503	93	8,649	804,357	9.643	4.530
44	1,936	85,184	6.633	3.530	94	8,836	830,584	9.695	4.546
45	2,025	91,125	6 708	3.556	95	9,025	857,375	9.746	4.562
46	2,116	97,336	6.782	3 583	96	9,216	884,736	9.797	4.578
47	2,209	103,823	6.855	3.608	97	9,409	912,673	9.848	4.594
48	2,304	110,592	6.928	3 634	98	9,604	941,192	9.899	4.610
49	2,401	117,649	7.	3.659	99	9,801	970,299	9.949	4.626
50	2,500	125,000	7.071	3.684	100	10,000	1,000,000	10.	4.641

SUPPLEMENTARY TOPIC

96. Cube Roots can be obtained by a division process.

The cube root of 1000 is 10; of 1,000,000 is 100; etc. Hence the cube root of a number between 1 and 1000 is between 1 and 10; the cube root of a number between 1000 and 1,000,000 is between 10 and 100; etc. Hence, if the given number is divided into **periods** of *three* figures each, beginning with the units' figure, for each period in the number, there will be one figure in the cube root. If there are figures to the right of the decimal point, divide them also into periods of three figures each, starting from the decimal point.

Thus, 3457.82653 becomes 3 457.826 530.

The following example illustrates the method of solution.

Example 1. — Find the cube root of 74,088.

Solution. — 1. 74,088 has two periods. Hence there are in the root a tens' and a units' figure.

2. 64,000 is the largest perfect cube in 74,088. $\sqrt[3]{64{,}000} = 40$. Place 40 in the root.

3. Subtract 64,000 from 74,088.

4. Trial divisor. $3 \times 40^2 = 3 \times 1600 = 4800$. $10{,}088 \div 4800 = 2^+$. Place 2 in the root.

5. Complete divisor. Add to 4800: (*a*) $3 \times 40 \times 2$ or 240; and (*b*) 2^2 or 4, making 5044.

$$\begin{array}{r|l} \multicolumn{2}{r}{4\cancel{0}+2} \\ \hline & 74\,088 \\ & 64\,\cancel{0}\cancel{0}\cancel{0} \\ \hline 4800 & 10\,088 \\ 240 & \\ 4 & \\ \hline 5044 & 10\,088 \\ \hline \end{array}$$

6. Multiply 5044 by 2 and subtract from 10,088. Since the remainder is zero, the cube root is $40 + 2$ or 42.

NOTE. — In practice the zeros crossed out are omitted from the computation.

Rule. — To find the cube root of a number:

1. Separate the number into periods of three figures each.

2. Find the greatest number which is a perfect cube that is less than or equal to the first period; write its cube root as the first figure of the cube root of the given number; sub-

tract the cube of it from the first period, and to the remainder annex the next period.

3. **Form the trial divisor by taking three times the square of the part of the root already found and annexing two zeros.**

4. **Divide the remainder in Step 2 by the trial divisor and annex the integral part to the root already found.**

5. **Form the complete divisor by adding to the trial divisor three times the product of the new root figure by the part of the root already found, with one zero annexed, and also the square of the new root figure.**

6. **Multiply the complete divisor by the new root figure and subtract the product from the remainder formed in Step 2.**

7. **Continue in this manner until the cube root or the desired number of decimal places for the root has been obtained.**

NOTE 1.—Note 1, p. 139, applies with equal force to the above rule.

NOTE 2.—If any root figure is zero, annex two zeros to the trial divisor and annex the next period to the remainder.

Example 2. Find the cube root of 8144.865728.

Solution.—1. $\sqrt[3]{8} = 2$, the tens' figure of the root.

2. $3 \times 2^2 = 12$. The trial divisor is 1200. Since 144 is less than 1200, annex 0 to the root, and bring down 865.

3. $3 \times 20^2 = 1200$. Annex 00, forming the new trial divisor 120000.

4. $144865 \div 120000 = 1^+$. Annex 1 to the root.

5. Complete divisor. Add to 120000: (*a*) $3 \times 200 \times 1$ or 600 and (*b*) 1^2, making 120601.

6. Multiply by 1 and subtract from 144865.

7. Bring down 728.

	20.12
	8 144.865 728
	8
120000	144 865
600	
1	
120601	120 601
12120300	24 264 728
12060	
4	
12132364	24 264 728

8. $3 \times 201^2 = 121203$. Annex 00, forming the trial divisor 12120300.

9. $24264728 \div 12120300 = 2^+$. Annex 2 to the root.

10. Add to 12120300: (*a*) $3 \times 2 \times 2010$ or 12060, and (*b*) 2^2 or 4, obtaining the complete divisor 12132364.

11. Multiply the complete divisor by 2 and subtract from the remainder. Since the final remainder is zero, the cube root is 20.12.

EXERCISE 71

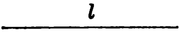
Find the cube root of:

1. 29,791.
2. 19,683.
3. 148,877.
4. 59,319.
5. 681.472.
6. 1860.867.
7. 51,478.848.
8. 517,781.627.
9. 46.268279.
10. 187.149248.
11. 444,194.947.
12. 788.889024.

VI. PRACTICAL GEOMETRY AND APPLICATIONS

97. Geometrical Figures are formed of points, lines, and surfaces. Four kinds of lines are studied in elementary geometry:

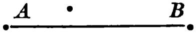
a. **Straight Lines** like line *l*.



One and only one straight line can be drawn through two points.

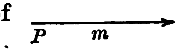
A straight line is supposed to extend indefinitely in each of two directions.

The part of a straight line between two points of it is called a **Segment**; as segment *AB*.

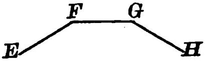


The shortest distance between two points is measured along the segment between them. Segments are measured by units of length, as the yard, the foot, the mile, etc.

The part of a straight line on one side of a point of it is called a **Ray**, as ray *m*.



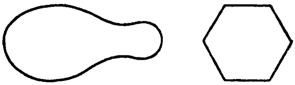
b. **Broken Lines** like *EFGH*. They consist of segments of straight lines.



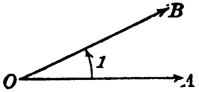
c. **Curved Lines** like *CD*. They do not have any parts that are straight.

d. **Closed Lines**, like those adjoining.

Simple closed lines in a plane surface divide the surface into two parts, one inside the line and one outside.



98. An **Angle** is the figure formed by two rays drawn from the same point; as Angle *AOB*.

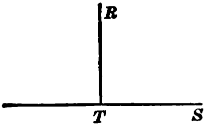


The common point is the **Vertex** of the angle, and the two rays are the **Sides** of the angle.

A common angle is the **Right Angle.**

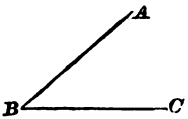
Whenever an object has a "square corner," two of its edges form a right angle.

Two lines that form a right angle are **Perpendicular**; as lines *TS* and *RS*.

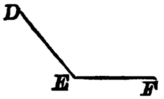


Angles are measured by a unit called a **Degree,**—an angular degree. An angular degree is one ninetieth of a right angle.

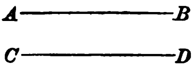
An **Acute Angle** is an angle less than a right angle; as angle *ABC*.



An **Obtuse Angle** is an angle greater than a right angle; as angle *DEF*.



99. Two lines are **Parallel** if they lie in the same plane and do not meet, no matter how far they are extended. The distance between parallel lines is always the same.



EXERCISE 72

1. Mention some objects that have a surface or surfaces that are practically plane surfaces.

2. Mention one or more objects inclosed by a spherical surface.

3. Mention an object that has some other curved surface.

4. Point out in the room some parallel lines.

5. Point out in the room some perpendicular lines.

6. What kind of angle is formed by the hands of a clock at 3 o'clock? How many degrees are there in it?

7. Answer questions like those in Example 6 when the hands point to: *a.* 2 o'clock; *b.* 4 o'clock; *c.* 5 o'clock.

8. How do carpenters mark a straight line between two points by means of a string?

9. Draw a straight line. Through a point on it, draw a perpendicular to it, using the square corner of a card as a pattern right angle.

10. Draw a straight line. Place a point on your paper but not on the line. Through this point draw a perpendicular to the line, using the corner of the card as a pattern right angle.

11. Draw a straight line at least two inches long. On this line, place two points, at least one inch apart. At each of these points draw a perpendicular to the line. What kind of lines do the two perpendiculars appear to be?

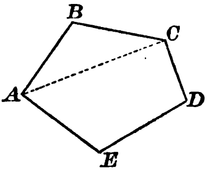
12. Draw two lines that are parallel. At a point of one of them draw a perpendicular. What kind of angles does this perpendicular form with the other parallel?

13. Can you fold a piece of paper so as to form a straight line?

14. Can you now fold the paper so that the first crease will be perpendicular to the second?

15. Following the suggestion of Example 11, can you, by folding a piece of paper three times, make two parallel creases in the paper?

100. A **Polygon** is a closed broken line lying in a plane; as *ABCDE*. *AB*, *BC*, etc., are called its **Sides**; points *A*, *B*, *C*, etc., are called its **Vertices**. A line, as *AC*, joining two non-consecutive vertices is a **Diagonal** of the polygon.



The **Perimeter** of the polygon is the sum of the lengths of its sides.

NOTE. — Formerly it was customary to say that a polygon is a part of a plane bounded by a broken line, and then to call that line the perimeter of the polygon. This statement is still to be found in elementary texts although it does not agree with the best modern mathematical practice.

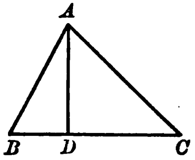
Since a polygon is a closed line in a plane, it incloses a part of the plane called the *interior of the polygon.*

This interior of the polygon is measured by units of surface measure. The **Area** of the polygon is the number of times that the interior of the polygon contains the unit of surface measure.

Thus, if a polygon is such that its interior contains 8 sq. ft., then the area is 8 and the unit of measure is a square foot.

101. Kinds of Polygons.

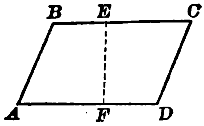
a. A **Triangle** is a polygon having three sides; as triangle *ABC*.



Any side of a triangle may be considered its **Base**; as *BC*. The perpendicular to the base from the opposite vertex is the **Altitude** to that base; as *AD*.

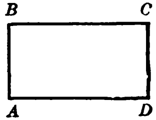
b. A **Quadrilateral** is a polygon having four sides.

c. A **Parallelogram** is a quadrilateral whose opposite sides are parallel; as *ABCD*.



It is proved in geometry that the opposite sides of a parallelogram are equal.

Any side of a parallelogram may be considered its **Base**, and the perpendicular between this side and the one opposite is then the **Altitude**; as base *AD* and altitude *EF*.



d. A **rectangle** is a special kind of parallelogram. Its angles are all right angles. Any side, as *AD*, may be considered its **Base**, and then either of the adjoining sides is its **Altitude**.

e. A **Square** is another special kind of parallelogram. It is also a special kind of rectangle. Its angles are all right angles, and its sides are all equal.

EXERCISE 73

1. Draw as nearly as you can a rectangle whose base is 4 inches and whose altitude is 2 inches.

2. Draw as nearly as you can a large parallelogram. Measure its opposite sides. Are they approximately equal?

3. Draw a triangle, each of whose angles is an acute angle.

a. Select one side as base, and to that side draw the altitude. (See Example 10, Exercise 72.)

b. Similarly draw the altitudes to the second and third sides.

c. What do you observe about the altitudes?

4. Draw a triangle which has for one of its angles a right angle. What sides of the triangle may conveniently be selected for base and altitude?

5. Draw a triangle which has for one of its angles an obtuse angle. Select one of the sides of the obtuse angle as the base, and to that side draw the altitude.

6. Draw a parallelogram. Select one side as base, and to that side draw the altitude of the parallelogram.

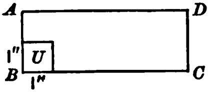
7. Represent a rectangular lot whose length is 120 ft. and width 30 ft. In your drawing, let 30 ft. be represented by 1 in. (This is called drawing to the scale 1 in. = 30 ft.)

8. Represent a square field whose sides are 100 ft. long, letting 1 in. equal 25 ft.

9. Draw to the scale $\frac{1}{4}$ in. = 1 ft. a rectangle representing a kitchen 14 ft. 6 in. wide and 12 ft. 3 in. long.

10. Draw to the scale $\frac{1}{8}$ in. = 1 ft. a rectangle representing the outside foundation walls of a house 33 ft. wide and 38 ft. long, if the walls are 15 in. in thickness.

102. Area of a Rectangle. — In the figure at the right, let the sides of square *U* each represent 1 inch. Then *AB* represents 2 inches and *BC* 5 inches. *U* represents one square inch. *ABCD* contains 2×5 or 10 square inches. That is, the area of *ABCD* is 10.



How many and what kind of square units would there be in *ABCD*:

a. if *AB* were 3 inches and *BC* were 5 inches?

b. If *AB* were 5 feet and *BC* were 8 feet?

c. If *AB* were 7 yards and *BC* were 12 yards?

The base and altitude are measured by the same unit of length; then the interior of the rectangle is measured by a square whose side is this same unit of length. It is clear then that the number of these square units within the rectangle is the product of the number of units of length in the base by the number in the altitude of the rectangle. This fact is *abbreviated* commonly in the

Rule. — The area of a rectangle is the product of its base and altitude.

103. If a represents the number of linear units in the altitude, and b the number in the base, and A the number of square units in the interior of the rectangle, the rule can be expressed in the form

$$A = a \times b.$$

A rule expressed in this manner is called a **Formula.** This is the formula for the area of a rectangle.

Instead of writing the cross, ×, for the sign of multiplication between numbers represented by letters, the letters are written side by side. Thus $a \times b$ is written ab. Therefore the formula for the area of a rectangle is $A = ab$.

Example. — Find the area of a rectangle whose base is 15 ft. and whose altitude is 9 ft.

Solution. — 1. The formula is $A = ab$.

2. Substitute 15 for a and 9 for b. Then

$$A = 15 \times 9 = 135 \text{ sq. ft.}$$

EXERCISE 74

1. What is the area of a square whose side is:

a. 9 ft.? *b.* 12 yd.? *c.* 20 rd.? *d.* 3.75 yd.?

2. What is the area of a rectangle whose:

a. altitude is 16 ft. and whose base is 33 ft.?

b. altitude is $6\frac{2}{3}$ ft. and whose base is $12\frac{1}{2}$ ft.?

c. altitude is 4′ 10″ and whose base is 12′ 4″?

3. What is the altitude of a rectangle whose area is 544 sq. ft. and whose base is 10 rd.?

4. Determine the length of the side of a square whose area will be:

a. 196 sq. yd. *b.* 400 sq. ft. *c.* 625 sq. rd.

5. Determine the approximate length of the side of a square whose area will be:

a. 150 sq. ft. *b.* 200 sq. yd. *c.* 1000 sq. ft.

6. What is the base of a rectangle whose area is $3\frac{1}{9}$ sq. yd. and whose altitude is 3 ft. 6 in.?

7. What is the number of acres in a square field whose side is 396 ft.?

8. What part of an acre is a lot 100 ft. by 300 ft.?

9. What part of an acre is a lot 60 ft. by 120 ft.?

10. What will be the cost of a flexo-tile floor for a hotel kitchen 15 ft. by 35 ft. at $.45 per square foot?

11. What will be the cost for the canvas for the floor of a sleeping porch 10 ft. by 18 ft. at $.75 per square yard?

12. What is the value at $185 per acre of a field 24 rd. wide and 35 rd. long?

13. A firm builds a warehouse four stories high. The interior dimensions of the building are 50 ft. by 200 ft. How many square feet of floor space have they provided for themselves?

14. What will be the expense for a cement sidewalk on the two sides of a corner lot 50 ft. by 160 ft., if the walk is 5 ft. wide, at 12 ¢ per square foot?

15. What will be the cost of covering a bathroom floor 7 ft. 6 in. by 9 ft. 8 in. with flexo-tile at $ .45 per square foot?

16. What will it cost to cover the floor of a sun parlor 10 ft. 10 in. by 18 ft. 4 in. with tile at $ .60 per square foot?

17. Approximately how many bricks 8 inches long and 4 inches wide, will be required to lay a sidewalk 5 ft. wide and 275 ft. long?

18. Determine the cost at $ 13.50 per thousand of the bricks for paving an alley 545 ft. long and 12 ft. wide. The bricks are laid so that the exposed surface is 8 in. by $2\frac{1}{2}$ in. Assume that the contractor can buy only whole thousands of bricks.

19. How many thousand paving stones 9 in. by 4 in. will be needed for 2 miles of road 20 ft. wide?

20. A contractor offers to lay an asphalt pavement at $ 3.60 per square yard. The street is 50 ft. wide. How much must each lot owner be assessed for each foot in the width of his lot?

21. Determine the cost of painting a house 27 ft. wide and 33 ft. long, having an average height of 22 ft. at $ .30 per square yard. (No allowance to be made for windows.)

22. Determine the cost of painting a billboard 20 ft. wide and 10 ft. high at $ .25 per square yard.

23. One gallon of paint is enough for two coats of paint for about 250 sq. ft. of surface. How many gallons of paint must a farmer order for a barn 30 ft. × 50 ft., averaging 23 ft. in height?

104. Problems of the Plastering Contractor. — Plastering, both inside and outside, is figured by the square yard of surface covered. The price charged includes furnishing and placing in position the lath upon which the plaster is spread, as well as the plaster work proper. Usually the contractors do not make any allowance for openings for doors or windows, as it costs as much to work around these openings as it would to cover them.

EXERCISE 75

1. What will it cost to plaster the walls and ceiling of a room 14 ft. by 22 ft., 9 ft. high, at $.40 per square yard?

2. Find the cost of plastering the ceiling and walls of a town hall 30 ft. wide, 45 ft. long, and 16 ft. high at $.45 per square yard.

3. What will be the cost of the stucco finish for a garage 18 ft. wide, 22 ft. long, and averaging 10 ft. in height at $.55 per square yard?

4. What will be the cost of re-plastering the walls of a kitchen 12 ft. wide, 14 ft. long, and 9 feet high at $.30 per square yard?

5. What will be the contract price for the plastering of the walls and ceilings of a house, having the following rooms, at $.45 per square yard?

Kitchen, 14 ft. by 14 ft., 9 ft. high.
Living room, 20 ft. by 14 ft. 6 in., 9 feet high.
Dining room, 13 ft. by 13 ft. 6 in., 9 ft. high.
Hall, 12 ft. by 14 ft. 6 in., 9 ft. high.
Pantry, 5 ft. by 12 ft., 9 ft. high.
Bedroom *A*, 12 ft. by 12 ft., 8 ft. 6 in. high.
Bedroom *B*, 13 ft. by 17 ft., 8 ft. 6 in. high.
Bedroom *C*, 12 ft. 6 in. by 7 ft. 6 in., 8 ft. 6 in. high.
Bathroom *A*, 7 ft. by 8 ft., 8 ft. 6 in. high.
Bathroom *B*, 6 ft. by 11 ft., 8 ft. 6 in. high.

105. Papering. — Wall paper is approximately 18 in. wide and comes in single rolls of 8 yd. or double rolls of 16 yd. The single rolls therefore contain 4 sq. yd. of paper. Paper comes also in other widths, but in each case there are approximately 4 sq. yd. in a single roll of domestic paper. When estimating the quantity of paper needed, allowance must be made for the waste in cutting to match the pattern. It is customary to cut full length strips where there are not openings, — in the expectation that the small spaces over doors and windows can be covered with the pieces of the rolls remaining after the long strips are cut.

A rough estimate of the quantity needed is obtained by assuming that a single roll will cover 33 sq. ft. of surface. The total surface of the room, less the openings, divided by 33 will give the number of rolls.

Another way is to subtract the total width of all doors and windows from the perimeter of the room, and divide the result by the width of the paper. This will give the number of strips needed for the walls. The number that can be cut from a roll depends upon the height of the ceiling.

Example. — What will be the cost at $.60 per roll of the paper for the walls of a room 17 ft. by 12 ft. if the ceiling is 9 ft. high, and for the ceiling at $.45 per roll. The paper is 18 in. wide and is bought in double rolls. There is an 8 in. baseboard running around the room. There is one group of windows 9 ft. wide, another 7 ft. wide, one small door 3 ft. 6 in. wide and one double doorway 7 ft. 6 in. wide.

Solution. — 1. Each strip must be $9' - 8''$ long, or $8'\ 4''$ in length.

2. ∴ 5 strips can be cut from a double roll. $(48' \div 8'\ 4'' = 5^+)$.

3. The total width of all openings is $7' + 7'\ 6'' + 9' + 3'\ 6''$ or $27'$.

4. The perimeter of the room is $17' + 17' + 12' + 12'$ or $58'$.

5. Subtracting the width of all openings leaves 31′.

6. The number of strips needed $= 31' \div 18'' = 20\frac{2}{3}$ or 21 strips.

7. Since only 5 strips can be cut from a double roll, it will take 4 double rolls and one single roll, — all together 9 rolls.

8. The cost will be 9 × \$.60 or \$5.40 for the sides.

9. On the ceiling, if the strips are run the narrow way of the room, the number of strips needed $= 17' \div 18'' = 11\frac{1}{3}$ or 12 strips.

10. Each strip must be 12′ long. 4 such can be cut from a double roll. Hence 3 double or 6 single rolls will be needed for the ceiling.

11. The cost of the paper for the ceiling will be 6 × \$.45 or \$2.70.

12. ∴ the total cost will be \$5.40 + \$2.70 or \$8.10.

EXERCISE 76

1. How many rolls of paper 18 in. wide will be required to paper a room 15 ft. long, 13 ft. wide, and 9 ft. high, no allowance being made for openings? (Buy double rolls.)

2. How much will it cost to paper a room 14 ft. square and 9 ft. high, with paper 18 in. wide, at \$.75 per roll? Allow 80 sq. ft. for doors and windows. (Include ceiling.)

3. How much will it cost to paper a room 15 ft. long, 11 ft. wide, and 8 ft. 6 in. high with paper 18 in. wide, at \$1.10 per roll, allowing 175 sq. ft. for doors, windows, and baseboards?

4. Find the approximate cost of paper for the walls and ceiling of the living room of the plan shown in Example 8, p. 188, if the walls are to be covered with paper at \$.95 per roll and the ceiling at \$.50 per roll. (Assume that the ceiling is 9 ft. high, and do not make any allowance for the baseboards.)

5. For the dining room of the same house, paper for the side walls is to be purchased at \$.70 per roll and for the ceiling at \$.45 per roll. What is the approximate cost?

6. How many rolls of paper 18 in. wide will be required to paper a room 21 ft. long, $15\frac{1}{2}$ ft. wide, and 8 ft. 9 in. high, allowing for two doors each 3 ft. 6 in. wide and 3 windows each 3 ft. 8 in. wide?

7. Find the approximate cost of papering a room 18 ft. long, 14 ft. 6 in. wide, and 9 ft. high with paper 18 in. wide, at $ 1.19 per roll. Allow for two doors 3 ft. 6 in. wide, two windows 2 ft. 9 in. wide, and a group of windows 8 ft. wide.

8. A room 16 ft. long, 12 ft. wide, and 9 ft. high has two doors each 3 ft. 8 in. wide, two windows each 3 ft. wide, and a window group 7 ft. wide. The doors are 7 ft. high, and the windows are 5 ft. 9 in. high. Estimate the number of rolls of 18 in. paper required for the walls by using in turn each of the methods suggested in § 105.

106. Carpeting Rooms is less common than formerly since most people prefer to purchase rugs.

Carpeting comes usually 27 in. wide. If there is a distinct pattern in the carpet, then there is a certain amount of waste in cutting to match the pattern.

Example. — If a room is 15 ft. 4 in. long and 11 ft. 9 in. wide, how much will the carpet cost to cover it, if the carpeting is 27 in. wide and costs $ .63 per yard? Assume that there is a waste of 11 in. on each strip except the first in matching the pattern.

Solution. — 1. The natural way to run the strips is lengthwise of the room. There will be needed as many strips as 27 in. is contained times in 11 ft. 9 in. or in 141 in.

$\therefore$ 6 strips are needed, as $141 \div 27 = 5^+$.

2. The first strip must be 15 ft. 4 in. long.

Each of the others must be 11 inches longer, namely 16 ft. 3 in. There are 5 such strips.

3. $\therefore$ there is required 15 ft. 4 in. + 5 × (16 ft. 3 in.) or all together 96 ft. 7 in.

4. But 96 ft. 7 in. = $32\frac{7}{36}$ yd.

5. $\therefore$ the cost will be $32\frac{7}{36} \times \$.63$ or $20.28.

EXERCISE 77

1. Determine the cost of carpeting the same room as in the preceding example, running the strips crosswise of the room.

2. How many yards of carpeting 27 in. wide are needed for a floor 18 ft. long and 14 ft. wide, assuming that there is not any waste in cutting? (Run carpet lengthwise.)

Make a scale drawing of the room and the strips of carpet, using the scale $\frac{1}{4}'' = 1$ ft.

3. How many yards of carpet 27 in. wide are needed for a floor 20 ft. 10 in. long and 16 ft. 4 in. wide, assuming no waste in cutting?

4. Straw matting comes one yard wide. What will it cost to cover a floor 14 ft. 3 in. wide and 15 ft. 6 in. long with matting costing $ .85 per yard?

5. Find the cost of carpeting a room 16 ft. 9 in. long and 12 ft. 2 in. wide with 27 in. carpet at $ 1.75 per yard. Assume a waste of 10 in. on each strip except the first.

6. How many yards of carpet 27 in. wide will be required for a floor 16 ft. 9 in. long and 13 ft. 4 in. wide if there is a waste of $4\frac{1}{2}$ in. on each strip except the first?

7. Which way should the strips be run to carpet most economically a floor 18 ft. 3 in. by 15 ft. 6 in. with 27 in. carpet, assuming no waste in cutting?

8. Linoleum comes in strips 6 ft. wide or 12 ft. wide. It is sold by the square yard. How many square yards of linoleum are required for a floor 18 ft. long and 14 ft. 3 in. wide, assuming that the floor is covered in the more economical manner?

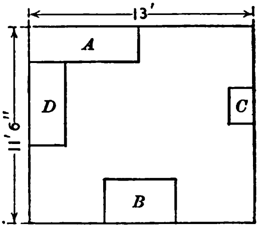
9. A bathroom floor is 7 ft. wide and 11 ft. long. Which width of linoleum will cover it most satisfactorily? How much should be purchased and what will be the cost at $ 1.85 per square yard?

10. Determine the cost of a single color chenille rug, 9 ft. by 12 ft. in size, at $ 11.50 per square yard. (Approximate 1918 price.)

11. Determine the cost of a single tone rug 14 ft. 8 in. wide and 21 ft. 6 in. long at $ 12.25 per square yard.

12. The adjoining figure is the plan of a kitchen. *A* is a built-in cupboard 7′ × 2′. *C* is a stationary radiator extending 18″ from the wall. *B* is a gas stove, on legs but not easily moved; it extends 30 in. from the wall. *D* is the sink, open below, without legs, extending 2′ from the wall.

The owner decides to purchase a piece of linoleum to cover that part of the floor that is actually used. What size piece of linoleum would you order, and what would be the cost at $ 1.75 per square yard?



107. Area of a Parallelogram. — In Fig. 1 below is a parallelogram whose base is *b* units in length and whose altitude is *a* units. Notice that the parallelogram is divided into parts I and II by the altitude *HK*.

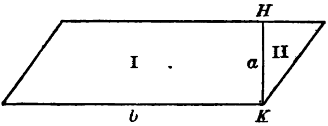


FIG. 1.

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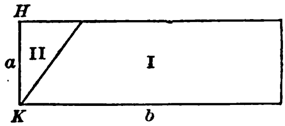


FIG. 2.

If such a figure were cut from paper, and then cut on the line *HK*, the part marked I could be placed at the right of II as shown in Fig. 2, thus forming a rectangular piece of paper. The rectangle and the parallelogram inclose the same amount of surface. In both the altitudes are *a* units in length and the bases are *b* units.

The area of the rectangle is *ab* square units.

Hence the area of the parallelogram is ab square units. Therefore, in abbreviated form (see § 102),

Rule. — The area of a parallelogram equals the product of its base and altitude.

The formula for the area of a parallelogram is

$$A = ab.$$

EXERCISE 78

1. What is the area of a parallelogram whose:

a. altitude is 10 ft. and whose base is 13 ft.?

b. altitude is $12\frac{1}{3}$ ft. and whose base is $6\frac{1}{4}$ ft.?

c. altitude is 5 ft. 4 in. and whose base is 7 ft. 6 in.?

d. altitude 2.3 ft. and base 6.75 ft.?

e. base is 15 rd. and whose altitude is 6 yd.?

108. Area of a Triangle. — In Fig. 1 below, the altitude AD of the triangle is a units in length and the base is b units.

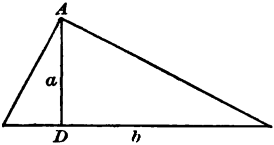


FIG. 1.

= ½ of

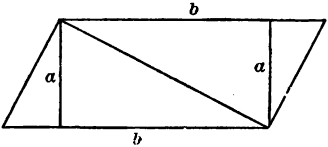


FIG. 2.

If two such triangular pieces were cut from paper and placed together as shown in Fig. 2, the resulting figure would have the form of a parallelogram. The base of the parallelogram would be b units in length and the altitude a units. Hence the area of the parallelogram would be ab square units. Therefore the area of the triangle must be $\frac{1}{2}ab$ square units.

Rule. — The area of a triangle is one half the product of its base and altitude.

The formula is $A = \frac{1}{2}ab$.

EXERCISE 79

1. In the figure above, suppose that the altitude of the triangle is 4 in. and the base is 10 in. in length. How many square units would there be in the parallelogram? Therefore, how many must there be in the triangle?

2. Answer questions like those in Example 1 when the altitude of the triangle is 8 in. and the base is 12 in.

3. Similarly, if the altitude is 10 in. and the base is 15 in.

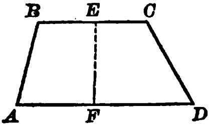
4. What is the area of a triangle whose:

a. base is 12 ft. and whose altitude is 8 ft.?
b. base is 5 yd. and whose altitude is 12 ft.?
c. base is $3\frac{1}{8}$ ft. and whose altitude is $2\frac{1}{2}$ ft.?
d. base is 3.5 ft. and whose altitude is 4.7 ft.?
e. base is 12 yd. 1 ft. and whose altitude is 5 yd. 2 ft.?

5. What is the base of a triangle whose area is 28 sq. ft. and whose altitude is 7 ft.?

6. Determine the altitude of a triangle whose area is 30 sq. ft. and whose base is 6 ft.

109. A **Trapezoid** is a quadrilateral having two sides that are parallel, as trapezoid *ABCD*.



The parallel sides *BC* and *AD* are the bases and the perpendicular *EF* between them is the altitude.

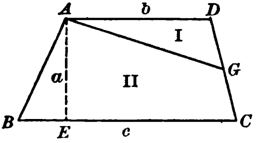


FIG. 1.

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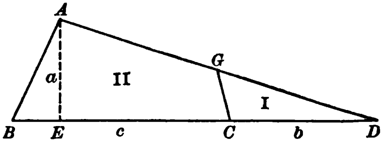


FIG. 2.

In Fig. 1 below, suppose that altitude *AE* is *a* units in length, base *AD* *b* units, and base *BC* *c* units. Let *G* be the middle point of *DC*. If such a figure were cut from paper, the part I could be cut off and placed at the right of the part II as shown in Fig. 2. The result-

ing figure would be triangular in shape, like triangle *ABD*. The altitude would be a units and the base $(b + c)$ units.

The area of the triangle would be $\frac{1}{2}a(b + c)$ square units. But this is also the area of the trapezoid. Therefore

Rule. — The area of a trapezoid is one half its altitude multiplied by the sum of its bases.

The formula is $A = \frac{1}{2}a(b + c)$.

EXERCISE 80

1. Determine the area of a trapezoid whose:

a. bases are 10 in. and 8 in. respectively, and whose altitude is 4 in.

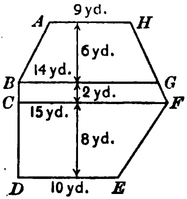
b. bases are 15 ft. and 11 ft. respectively, and whose altitude is 6 ft.

c. bases are 5 yd. and 10 ft. respectively, and whose altitude is 2 ft.

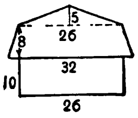
d. bases are $6\frac{2}{3}$ yd. and 4 yd. respectively, and whose altitude is $1\frac{1}{2}$ yd.

e. bases are 5 ft. 8 in. and 11 ft. 4 in. respectively, and whose altitude is 6 ft. 3 in.

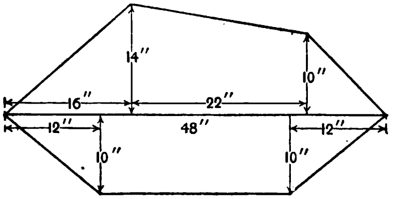
2. Find the area of the figure represented by the adjoining drawing.



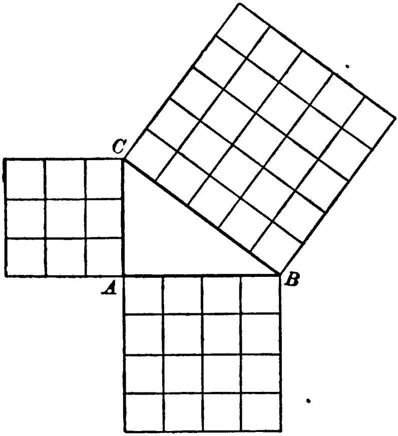
3. The adjoining figure represents the end of a barn. If the barn is 35 ft. long, find the cost of painting the ends and the vertical parts of the sides at 30 ¢ per square yard.



4. Find the area of the figure below.



110. The triangle *ABC*, having a right angle at *A*, is called a **Right Triangle.** The side *BC* opposite the right angle is called the **Hypotenuse** of the right triangle.



Upon the sides of the triangle, squares are drawn. These squares can be divided into the small squares shown in the figure. By counting the small squares, it becomes clear that the square upon the hypotenuse of the triangle equals the sum of the squares upon the other two sides. Since the area of a square is the square of its side,

Rule.—The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.

Example 1.—The sides of a right triangle are 7 in. and 9 in. respectively. Find the length of the hypotenuse.

Solution.—1. Let h = the length of the hypotenuse.

2. $\therefore h^2 = 7^2 + 9^2 = 49 + 81 = 130.$

3. $\therefore h = \sqrt{130} = 11.4^+.$

Since the square of the hypotenuse equals the sum of the squares of the other two sides, then *the square of one of the sides must equal the square of the hypotenuse minus the square of the other side.*

Example 2.—Find the third side of a right triangle whose hypotenuse is 19 in. and whose other side is 12 in.

Solution.—1. Let $x =$ the length of the third side.
2. Then $x^2 = 19^2 - 12^2 = 361 - 144 = 217.$
3. $\therefore x = \sqrt{217} = 14.73^+.$

EXERCISE 81

1. Find the hypotenuse of a right triangle when the sides are:

a. 6 in. and 8 in. respectively.
b. 7 in. and 2 ft. respectively.
c. 13 ft. and 20 ft. respectively.

2. What is the other side of a right triangle whose:

a. hypotenuse is 35 ft. and one side is 21 ft.?
b. hypotenuse is 34 ft. and one side is 17 ft.?
c. hypotenuse is 15 yd. and one side is 15 ft.?

3. Find the length of the diagonal of a square whose sides are 8 in. in length.

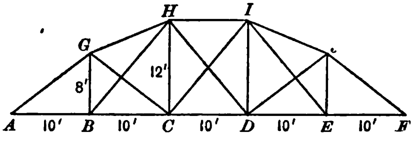
4. Find the length of the diagonal of a rectangle whose base is 42 in. and whose altitude is 16 in.

5. What must be the length of a guy wire to brace a telephone pole 35 ft. high if it is attached to a stake 10 ft. from the foot of the pole?

6. Guy wires are to be attached to the top of a wireless aerial 120 ft. high and at points 75 ft. from the foot of the aerial. How long must they be, exclusive of the part used in making the fastenings?

7. How long must a pole be cut to serve as a brace at a corner post of a fence, if it is to extend from the top of a post 5 ft. high to the bottom of a post distant 12 ft. from the first one?

8. Determine the lengths of the different oblique lines in the adjoining figure.



111. Computing the Area of a Triangle When the Lengths of its Three Sides are Known. — The following rule is proved in geometry.

Rule. — 1. **From one half the perimeter of the triangle subtract in turn each of the three sides, forming three remainders.**

2. **Multiply together one half the perimeter and the three remainders found in Step 1.**

3. **The area of the triangle is the square root of the product found in Step 2.**

Example. — Find the area of the triangle whose sides are 13 in., 14 in., and 15 in. respectively.

Solution. — 1. One half the perimeter $= \frac{1}{2}(13 + 14 + 15) = 21.$

2. $21 - 13 = 8;\ 21 - 14 = 7;\ 21 - 15 = 6.$

3. $\therefore$ the area $= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84.$

EXERCISE 82

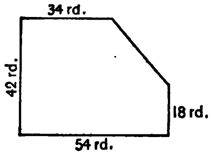
Find the area of a triangle when its sides are:

1. 9 yd., 15 yd., and 18 yd. respectively.

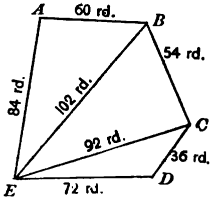
2. 9 ft., 10 ft., and 13 ft. respectively.

3. 13 ft., 13 ft., and 10 ft. respectively.

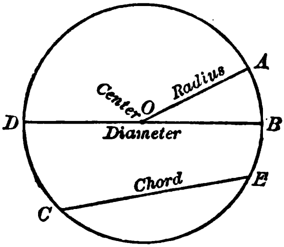
4. A railroad cuts across a corner of a field, leaving outside lines as indicated in the adjoining figure. Determine the number of acres in the field.



5. Find the area of a piece of ground having the dimensions indicated in the adjoining figure.



112. A **Circle** is a closed curved line in a plane, every point of which is equally distant from a point within called the center; as circle *ABD*.



A part of a circle is called an **Arc**; as arc *AB*.

NOTE. — This definition, while differing from that *given formerly* in *all* books and *still* in *some* elementary books, is the accepted one in modern mathematics.

A **Radius** of a circle is the distance from its center to any point of the circle; as *OA*.

A **Diameter** of a circle is a straight line through the center of the circle, having its ends on the circle; as *DOB*.

113. The **Circumference of a Circle** is the length of the circle. (See the note in § 112.) By means of a "tape measure," it is possible to obtain a straight line whose length is approximately equal to that of a circular object. If this length be divided by the length of the diameter, the quotient will be found to be a little more than 3.

NOTE. — A class will find it an interesting experiment to measure several circular objects in this manner and determine the quotients as outlined above.

The following rule is proved in geometry.

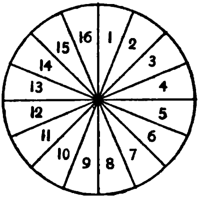
Rule. — To find the approximate circumference of a circle, multiply its diameter by 3.1416.

The multiplier 3.1416 is represented by the Greek letter π, read pi. Hence if C represents the circumference, D the diameter, and R the radius, then

$$C = \pi D = 2\pi R.$$

Another approximate value of π is $3\frac{1}{7}$. This is sufficiently accurate for most practical purposes.

114. Finding the Area of the Interior of a Circle. — The interior of a circle cannot be measured exactly by any unit of square measure.



By drawing radii, the interior of a circle can be divided into sixteen or more equal parts. Each part is approximately triangular in shape, — the base being an arc of the circle and the altitude a radius of the circle.

The area of each part then is, approximately, one half the radius multiplied by $\frac{1}{16}$ of the circumference. When the areas of all sixteen parts are added, thus obtaining the area of the circle, then

Rule I. — The area of a circle equals one half its radius multiplied by the circumference of the circle.

The formula is $A = \frac{1}{2}RC$.

Since the circumference equals $2\pi R$, then

$$A = \tfrac{1}{2}R \times 2\pi R = \pi R^2.$$

Rule II. — The area of a circle equals π times the square of the radius.

Note 1. — Observe that these rules, also, are expressed in the commonly abbreviated form. (See § 102.)

Note 2. — If the radius is expressed in inches, the area is expressed in square inches.

Example 1. — What is the circumference and area of a circle whose radius is 13 inches?

Solution. — 1. The radius = 13.

2. $\therefore C = 2\pi R = 2 \times 3^1 \times 13 = 81\frac{5}{7}$ in.

3. Also $A = \pi R^2 = 3\frac{1}{7} \times 13^2 = 531\frac{1}{7}$ sq. in.

Example 2. — Find the diameter of a circle whose area is 48 sq. ft.

Solution. — 1. Since the area $= \pi \times R^2$, then divide 48 by 3.1416 in order to get R^2;

$$\text{that is, } R^2 = 48 \div 3.1416 = 15.27^+.$$

2. $$\therefore R = \sqrt{15.27} = 3.9^+.$$

EXERCISE 83

1. Find the circumference and the area of a circle whose: *a.* radius is 17 in.; *b.* radius is 3.7 in.; *c.* diameter is 25 ft.; *d.* diameter is $4\frac{1}{4}$ ft.

2. The diameter of a circle is 45 ft. Express its circumference in yards.

3. What is the radius of a circle whose circumference is 278.5 ft.?

4. Find the radius of a circle whose area is 1 sq. yd.

5. Over how many square yards of ground can a horse graze if it is tied to a stake by a rope 30 ft. long?

6. If the diameter of the earth is approximately 7912 mi., what is the length of the equator?

7. How many times must a wheel 34 in. in diameter revolve in going one mile?

8. What is the circumference of a round iron bar 2.5 in. in diameter?

9. What is the circumference of a 15-in. pulley wheel?

10. What is the total pressure on a piston 16 in. in diameter in a steam engine when the pressure is 125 lb. per sq. in.?

11. The diameter of a Ford piston is $3\frac{3}{4}$ in. What is the area of its surface?

12. An acre of ground contains 43,560 sq. ft. How long is the radius of a circle whose area is one acre? What is the circumference of the circle?

13. Two circles having radii of 7 in. and 10 in. respectively are drawn from the same center. What is the area of the ring between them?

14. The number of feet that any point on the rim of a grindstone or emery wheel travels in a minute is called the **surface speed** of the stone. About 800 ft. per minute is considered a good average speed for a grindstone and about 5000 ft. per minute for an emery wheel.

a. Find the surface speed of a 9-in. emery wheel revolving 2100 times per minute.

b. Find the surface speed of a grindstone 24 in. in diameter revolving at the rate of 75 times per minute.

15. Determine the maximum number of revolutions per minute at which a 15-in. emery wheel may revolve so that its surface speed will not exceed 5000 ft. per minute.

16. An emery wheel 15 in. in diameter runs at the rate of 1400 revolutions per minute. What is the surface speed?

17. What is the rim speed of a fly wheel 16 in. in diameter running 500 revolutions per minute?

Solids

115. Closed Surfaces. — A **Spherical Surface** is an example of a closed surface.

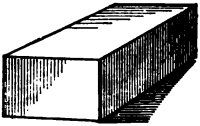
A simple closed surface incloses a definite part of space, separating it from the rest of space.

A **Solid** is the definite part of space inclosed by a simple closed surface. The surface is called the surface of the solid.

The area of the surface is called the **Area** of the solid; the **Volume** of the solid is the number of times that the solid contains a unit of solid measure, such as the cubic inch.

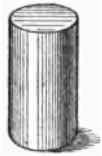
116. The solids most commonly encountered are

(*a*)



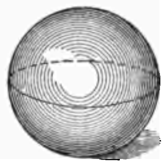
RECTANGULAR PARALLELOPIPED.

(*b*)



RIGHT CIRCULAR CYLINDER.

(*c*)



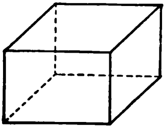
SPHERE.

The Rectangular Parallelopiped

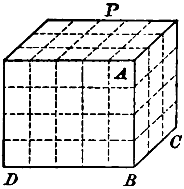
117. The rectangular parallelopiped has six rectangular faces.

A crayon box is a familiar example of a rectangular parallelopiped.

The opposite faces have the same shape and area. Any face may be considered the **Base.** The four edges meeting the base are perpendicular to the base and are of equal length; any one of them is the **Altitude** to that base.



118. Finding the Volume of a Rectangular Parallelopiped. — In the adjoining figure, let *AB* represent 4 in., *BD* 5 in., and *BC* 3 in. Imagine that the figure is divided into cubic inches. There will be four layers of these cubes, each containing 3 × 5 or 15 cubic inches. Hence the volume of the solid is 4 × 15 or 60 cubic inches. This suggests the



Rule. — The volume of a rectangular parallelopiped equals the product of its length, width, and height.

The formula is $V = L \times W \times H$ or LWH.

EXERCISE 79

1. In the figure above, suppose that the altitude of the triangle is 4 in. and the base is 10 in. in length. How many square units would there be in the parallelogram? Therefore, how many must there be in the triangle?

2. Answer questions like those in Example 1 when the altitude of the triangle is 8 in. and the base is 12 in.

3. Similarly, if the altitude is 10 in. and the base is 15 in.

4. What is the area of a triangle whose:

a. base is 12 ft. and whose altitude is 8 ft.?

b. base is 5 yd. and whose altitude is 12 ft.?

c. base is $3\frac{1}{8}$ ft. and whose altitude is $2\frac{1}{2}$ ft.?

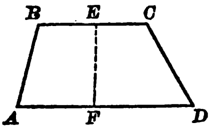
d. base is 3.5 ft. and whose altitude is 4.7 ft.?

e. base is 12 yd. 1 ft. and whose altitude is 5 yd. 2 ft.?

5. What is the base of a triangle whose area is 28 sq. ft. and whose altitude is 7 ft.?

6. Determine the altitude of a triangle whose area is 30 sq. ft. and whose base is 6 ft.

109. A **Trapezoid** is a quadrilateral having two sides that are parallel, as trapezoid *ABCD*.



The parallel sides *BC* and *AD* are the bases and the perpendicular *EF* between them is the altitude.

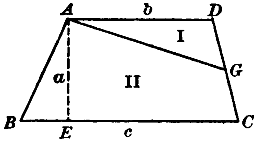


FIG. 1.

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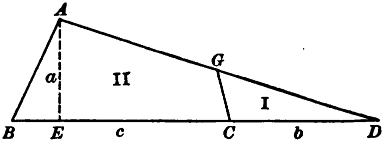


FIG. 2.

In Fig. 1 below, suppose that altitude *AE* is *a* units in length, base *AD* *b* units, and base *BC* *c* units. Let *G* be the middle point of *DC*. If such a figure were cut from paper, the part I could be cut off and placed at the right of the part II as shown in Fig. 2. The result-

ing figure would be triangular in shape, like triangle *ABD*. The altitude would be a units and the base $(b + c)$ units.

The area of the triangle would be $\frac{1}{2}a(b + c)$ square units. But this is also the area of the trapezoid. Therefore

Rule. — The area of a trapezoid is one half its altitude multiplied by the sum of its bases.

The formula is $A = \frac{1}{2}a(b + c)$.

EXERCISE 80

1. Determine the area of a trapezoid whose:

a. bases are 10 in. and 8 in. respectively, and whose altitude is 4 in.

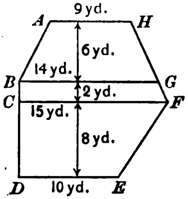
b. bases are 15 ft. and 11 ft. respectively, and whose altitude is 6 ft.

c. bases are 5 yd. and 10 ft. respectively, and whose altitude is 2 ft.

d. bases are $6\frac{2}{3}$ yd. and 4 yd. respectively, and whose altitude is $1\frac{1}{2}$ yd.

e. bases are 5 ft. 8 in. and 11 ft. 4 in. respectively, and whose altitude is 6 ft. 3 in.

2. Find the area of the figure represented by the adjoining drawing.



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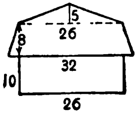
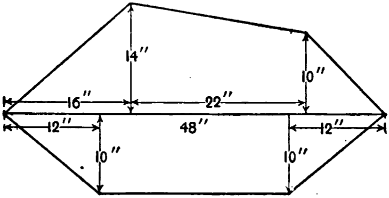
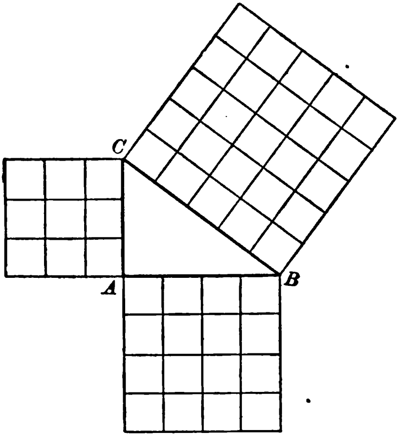


4. Find the area of the figure below.



110. The triangle ABC, having a right angle at A, is called a **Right Triangle.** The side BC opposite the right angle is called the **Hypotenuse** of the right triangle.



Upon the sides of the triangle, squares are drawn. These squares can be divided into the small squares shown in the figure. By counting the small squares, it becomes clear that the square upon the hypotenuse of the triangle equals the sum of the squares upon the other two sides. Since the area of a square is the square of its side,

Rule.—The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.

Example 1.—The sides of a right triangle are 7 in. and 9 in. respectively. Find the length of the hypotenuse.

Solution.—1. Let h = the length of the hypotenuse.

2. $\therefore h^2 = 7^2 + 9^2 = 49 + 81 = 130.$

3. $\therefore h = \sqrt{130} = 11.4^+.$

Since the square of the hypotenuse equals the sum of the squares of the other two sides, then *the square of one of the sides must equal the square of the hypotenuse minus the square of the other side.*

Example 2.—Find the third side of a right triangle whose hypotenuse is 19 in. and whose other side is 12 in.

Solution.—1. Let x = the length of the third side.

2. Then $x^2 = 19^2 - 12^2 = 361 - 144 = 217$.

3. $\therefore x = \sqrt{217} = 14.73^+$.

EXERCISE 81

1. Find the hypotenuse of a right triangle when the sides are:

a. 6 in. and 8 in. respectively.

b. 7 in. and 2 ft. respectively.

c. 13 ft. and 20 ft. respectively.

2. What is the other side of a right triangle whose:

a. hypotenuse is 35 ft. and one side is 21 ft.?

b. hypotenuse is 34 ft. and one side is 17 ft.?

c. hypotenuse is 15 yd. and one side is 15 ft.?

3. Find the length of the diagonal of a square whose sides are 8 in. in length.

4. Find the length of the diagonal of a rectangle whose base is 42 in. and whose altitude is 16 in.

5. What must be the length of a guy wire to brace a telephone pole 35 ft. high if it is attached to a stake 10 ft. from the foot of the pole?

6. Guy wires are to be attached to the top of a wireless aerial 120 ft. high and at points 75 ft. from the foot of the aerial. How long must they be, exclusive of the part used in making the fastenings?

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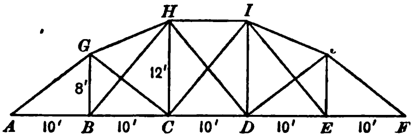

111. Computing the Area of a Triangle When the Lengths of its Three Sides are Known. — The following rule is proved in geometry.

Rule. — 1. From one half the perimeter of the triangle subtract in turn each of the three sides, forming three remainders.

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3. The area of the triangle is the square root of the product found in Step 2.

Example. — Find the area of the triangle whose sides are 13 in., 14 in., and 15 in. respectively.

Solution. — 1. One half the perimeter $= \frac{1}{2}(13 + 14 + 15) = 21.$

2. $21 - 13 = 8;\ 21 - 14 = 7;\ 21 - 15 = 6.$

3. $\therefore$ the area $= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84.$

EXERCISE 82

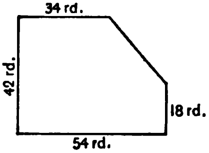
Find the area of a triangle when its sides are:

1. 9 yd., 15 yd., and 18 yd. respectively.

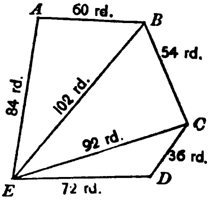
2. 9 ft., 10 ft., and 13 ft. respectively.

3. 13 ft., 13 ft., and 10 ft. respectively.

4. A railroad cuts across a corner of a field, leaving outside lines as indicated in the adjoining figure. Determine the number of acres in the field.



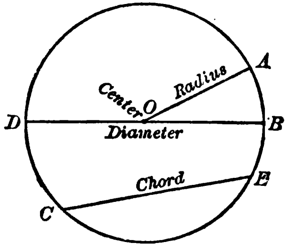
5. Find the area of a piece of ground having the dimensions indicated in the adjoining figure.



112. A **Circle** is a closed curved line in a plane, every point of which is equally distant from a point within called the center; as circle *ABD*.

A part of a circle is called an **Arc**; as arc *AB*.

NOTE. — This definition, while differing from that *given formerly* in *all* books and *still* in *some* elementary books, is the accepted one in modern mathematics.



A **Radius** of a circle is the distance from its center to any point of the circle; as *OA*.

A **Diameter** of a circle is a straight line through the center of the circle, having its ends on the circle; as *DOB*.

113. The **Circumference of a Circle** is the length of the circle. (See the note in § 112.) By means of a "tape measure," it is possible to obtain a straight line whose length is approximately equal to that of a circular object. If this length be divided by the length of the diameter, the quotient will be found to be a little more than 3.

NOTE. — A class will find it an interesting experiment to measure several circular objects in this manner and determine the quotients as outlined above.

The following rule is proved in geometry.

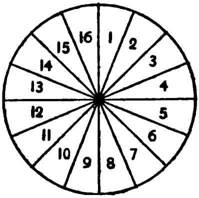
Rule. — To find the approximate circumference of a circle, multiply its diameter by 3.1416.

The multiplier 3.1416 is represented by the Greek letter π, read pi. Hence if C represents the circumference, D the diameter, and R the radius, then

$$C = \pi D = 2\pi R.$$

Another approximate value of π is $3\frac{1}{7}$. This is sufficiently accurate for most practical purposes.

114. Finding the Area of the Interior of a Circle. — The interior of a circle cannot be measured exactly by any unit of square measure.



By drawing radii, the interior of a circle can be divided into sixteen or more equal parts. Each part is approximately triangular in shape, — the base being an arc of the circle and the altitude a radius of the circle.

The area of each part then is, approximately, one half the radius multiplied by $\frac{1}{16}$ of the circumference. When the areas of all sixteen parts are added, thus obtaining the area of the circle, then

Rule I. — The area of a circle equals one half its radius multiplied by the circumference of the circle.

The formula is $A = \frac{1}{2} RC$.

Since the circumference equals $2\pi R$, then

$$A = \tfrac{1}{2} R \times 2\pi R = \pi R^2.$$

Rule II. — The area of a circle equals π times the square of the radius.

NOTE 1. — Observe that these rules, also, are expressed in the commonly abbreviated form. (See § 102.)

NOTE 2. — If the radius is expressed in inches, the area is expressed in square inches.

Example 1. — What is the circumference and area of a circle whose radius is 13 inches?

Solution. — 1. The radius $= 13$.
2. $\therefore C = 2\pi R = 2 \times 3^1 \times 13 = 81\frac{5}{7}$ in.
3. Also $A = \pi R^2 = 3\frac{1}{7} \times 13^2 = 531\frac{1}{7}$ sq. in.

Example 2. — Find the diameter of a circle whose area is 48 sq. ft.

Solution. — 1. Since the area $= \pi \times R^2$, then divide 48 by 3.1416 in order to get R^2;

$$\text{that is, } R^2 = 48 \div 3.1416 = 15.27^+.$$

2. $$\therefore R = \sqrt{15.27} = 3.9^+.$$

EXERCISE 83

1. Find the circumference and the area of a circle whose: *a.* radius is 17 in.; *b.* radius is 3.7 in.; *c.* diameter is 25 ft.; *d.* diameter is $4\frac{1}{4}$ ft.

2. The diameter of a circle is 45 ft. Express its circumference in yards.

3. What is the radius of a circle whose circumference is 278.5 ft.?

4. Find the radius of a circle whose area is 1 sq. yd.

5. Over how many square yards of ground can a horse graze if it is tied to a stake by a rope 30 ft. long?

6. If the diameter of the earth is approximately 7912 mi., what is the length of the equator?

7. How many times must a wheel 34 in. in diameter revolve in going one mile?

8. What is the circumference of a round iron bar 2.5 in. in diameter?

9. What is the circumference of a 15-in. pulley wheel?

10. What is the total pressure on a piston 16 in. in diameter in a steam engine when the pressure is 125 lb. per sq. in.?

11. The diameter of a Ford piston is $3\frac{3}{4}$ in. What is the area of its surface?

12. An acre of ground contains 43,560 sq. ft. How long is the radius of a circle whose area is one acre? What is the circumference of the circle?

EXERCISE 79

1. In the figure above, suppose that the altitude of the triangle is 4 in. and the base is 10 in. in length. How many square units would there be in the parallelogram? Therefore, how many must there be in the triangle?

2. Answer questions like those in Example 1 when the altitude of the triangle is 8 in. and the base is 12 in.

3. Similarly, if the altitude is 10 in. and the base is 15 in.

4. What is the area of a triangle whose:

a. base is 12 ft. and whose altitude is 8 ft.?

b. base is 5 yd. and whose altitude is 12 ft.?

c. base is $3\frac{1}{8}$ ft. and whose altitude is $2\frac{1}{2}$ ft.?

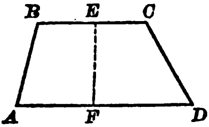
d. base is 3.5 ft. and whose altitude is 4.7 ft.?

e. base is 12 yd. 1 ft. and whose altitude is 5 yd. 2 ft.?

5. What is the base of a triangle whose area is 28 sq. ft. and whose altitude is 7 ft.?

6. Determine the altitude of a triangle whose area is 30 sq. ft. and whose base is 6 ft.

109. A **Trapezoid** is a quadrilateral having two sides that are parallel, as trapezoid *ABCD*.



The parallel sides *BC* and *AD* are the bases and the perpendicular *EF* between them is the altitude.

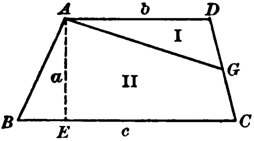


FIG. 1.

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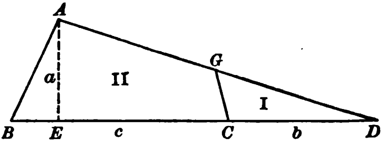


FIG. 2.

In Fig. 1 below, suppose that altitude *AE* is *a* units in length, base *AD* *b* units, and base *BC* *c* units. Let *G* be the middle point of *DC*. If such a figure were cut from paper, the part I could be cut off and placed at the right of the part II as shown in Fig. 2. The result-

ing figure would be triangular in shape, like triangle ABD. The altitude would be a units and the base $(b + c)$ units.

The area of the triangle would be $\frac{1}{2}a(b + c)$ square units. But this is also the area of the trapezoid. Therefore

Rule. — The area of a trapezoid is one half its altitude multiplied by the sum of its bases.

The formula is $A = \frac{1}{2}a(b + c)$.

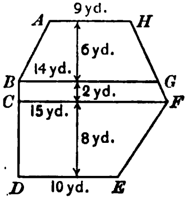
EXERCISE 80

1. Determine the area of a trapezoid whose:

a. bases are 10 in. and 8 in. respectively, and whose altitude is 4 in.

b. bases are 15 ft. and 11 ft. respectively, and whose altitude is 6 ft.

c. bases are 5 yd. and 10 ft. respectively, and whose altitude is 2 ft.

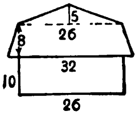


d. bases are $6\frac{3}{8}$ yd. and 4 yd. respectively, and whose altitude is $1\frac{1}{2}$ yd.

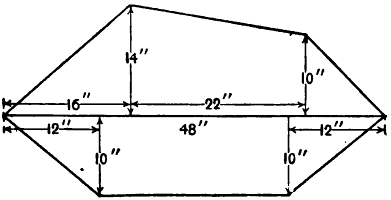
e. bases are 5 ft. 8 in. and 11 ft. 4 in. respectively, and whose altitude is 6 ft. 3 in.

2. Find the area of the figure represented by the adjoining drawing.

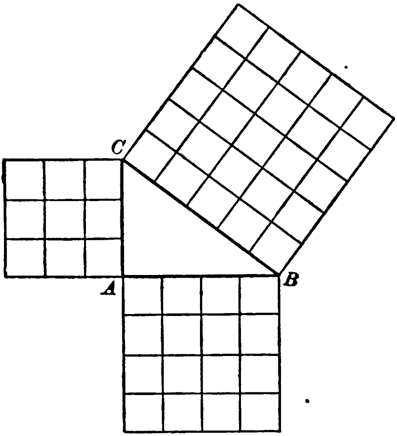
3. The adjoining figure represents the end of a barn. If the barn is 35 ft. long, find the cost of painting the ends and the vertical parts of the sides at 30¢ per square yard.



4. Find the area of the figure below.



110. The triangle ABC, having a right angle at A, is called a **Right Triangle.** The side BC opposite the right angle is called the **Hypotenuse** of the right triangle.



Upon the sides of the triangle, squares are drawn. These squares can be divided into the small squares shown in the figure. By counting the small squares, it becomes clear that the square upon the hypotenuse of the triangle equals the sum of the squares upon the other two sides. Since the area of a square is the square of its side,

Rule.—The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.

Example 1.—The sides of a right triangle are 7 in. and 9 in. respectively. Find the length of the hypotenuse.

Solution.—1. Let h = the length of the hypotenuse.

2. $\therefore h^2 = 7^2 + 9^2 = 49 + 81 = 130.$

3. $\therefore h = \sqrt{130} = 11.4^+.$

Since the square of the hypotenuse equals the sum of the squares of the other two sides, then *the square of one of the sides must equal the square of the hypotenuse minus the square of the other side.*

Example 2.—Find the third side of a right triangle whose hypotenuse is 19 in. and whose other side is 12 in.

Solution.—1. Let x = the length of the third side.

2. Then $x^2 = 19^2 - 12^2 = 361 - 144 = 217$.

3. $\therefore x = \sqrt{217} = 14.73^+$.

EXERCISE 81

1. Find the hypotenuse of a right triangle when the sides are:

a. 6 in. and 8 in. respectively.
b. 7 in. and 2 ft. respectively.
c. 13 ft. and 20 ft. respectively.

2. What is the other side of a right triangle whose:

a. hypotenuse is 35 ft. and one side is 21 ft.?
b. hypotenuse is 34 ft. and one side is 17 ft.?
c. hypotenuse is 15 yd. and one side is 15 ft.?

3. Find the length of the diagonal of a square whose sides are 8 in. in length.

4. Find the length of the diagonal of a rectangle whose base is 42 in. and whose altitude is 16 in.

5. What must be the length of a guy wire to brace a telephone pole 35 ft. high if it is attached to a stake 10 ft. from the foot of the pole?

6. Guy wires are to be attached to the top of a wireless aerial 120 ft. high and at points 75 ft. from the foot of the aerial. How long must they be, exclusive of the part used in making the fastenings?

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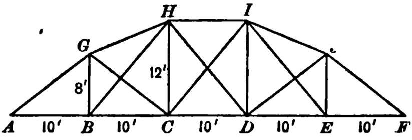

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Rule. — 1. From one half the perimeter of the triangle subtract in turn each of the three sides, forming three remainders.

2. Multiply together one half the perimeter and the three remainders found in Step 1.

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Solution. — 1. One half the perimeter $= \frac{1}{2}(13 + 14 + 15) = 21.$

2. $21 - 13 = 8;\ 21 - 14 = 7;\ 21 - 15 = 6.$

3. $\therefore$ the area $= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84.$

EXERCISE 82

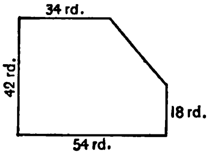
Find the area of a triangle when its sides are:

1. 9 yd., 15 yd., and 18 yd. respectively.

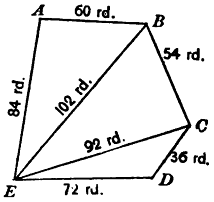
2. 9 ft., 10 ft., and 13 ft. respectively.

3. 13 ft., 13 ft., and 10 ft. respectively.

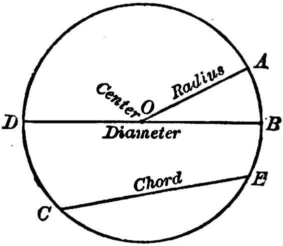
4. A railroad cuts across a corner of a field, leaving outside lines as indicated in the adjoining figure. Determine the number of acres in the field.



5. Find the area of a piece of ground having the dimensions indicated in the adjoining figure.



112. A **Circle** is a closed curved line in a plane, every point of which is equally distant from a point within called the center; as circle *ABD*.



A part of a circle is called an **Arc**; as arc *AB*.

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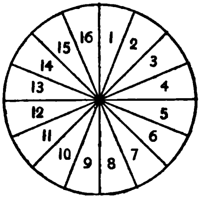

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Rule I. — The area of a circle equals one half its radius multiplied by the circumference of the circle.

The formula is $A = \frac{1}{2}RC$.

Since the circumference equals $2\pi R$, then

$$A = \frac{1}{2}R \times 2\pi R = \pi R^2.$$

Rule II. — The area of a circle equals π times the square of the radius.

Note 1. — Observe that these rules, also, are expressed in the commonly abbreviated form. (See § 102.)

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Solution. — 1. The radius = 13.

2. $\therefore C = 2\pi R = 2 \times 3\frac{1}{7} \times 13 = 81\frac{5}{7}$ in.

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Example 2. — Find the diameter of a circle whose area is 48 sq. ft.

Solution. — 1. Since the area $= \pi \times R^2$, then divide 48 by 3.1416 in order to get R^2;

that is, $R^2 = 48 \div 3.1416 = 15.27^+$.

2. $\therefore R = \sqrt{15.27} = 3.9^+$.

EXERCISE 83

1. Find the circumference and the area of a circle whose:
a. radius is 17 in.; *b.* radius is 3.7 in.; *c.* diameter is 25 ft.; *d.* diameter is $4\frac{1}{4}$ ft.

2. The diameter of a circle is 45 ft. Express its circumference in yards.

3. What is the radius of a circle whose circumference is 278.5 ft.?

4. Find the radius of a circle whose area is 1 sq. yd.

5. Over how many square yards of ground can a horse graze if it is tied to a stake by a rope 30 ft. long?

6. If the diameter of the earth is approximately 7912 mi., what is the length of the equator?

7. How many times must a wheel 34 in. in diameter revolve in going one mile?

8. What is the circumference of a round iron bar 2.5 in. in diameter?

9. What is the circumference of a 15-in. pulley wheel?

10. What is the total pressure on a piston 16 in. in diameter in a steam engine when the pressure is 125 lb. per sq. in.?

11. The diameter of a Ford piston is $3\frac{3}{4}$ in. What is the area of its surface?

12. An acre of ground contains 43,560 sq. ft. How long is the radius of a circle whose area is one acre? What is the circumference of the circle?

c. Three-ply roofing paper at $3.00 per roll. Such paper is 3 feet wide and comes in a roll about 36 feet long.

6. The drawings on p. 187 show the framework of a garage with a gable roof. Find in the drawings:

a. The sill, made of 2 × 4's laid on a broad face.

b. The studding, made of 2 × 4's, doubled at corners and openings of large size, spaced 3′ apart, from center to center, excepting the corner posts.

c. The plate, doubled 2 × 4's.

d. The ridge board.

e. The rafters, nailed to the plate and to the ridge board, extending beyond the plate and spaced 3′ apart, center to center, excepting the two end rafters.

f. The header over the doorway, doubled 2 × 4's.

g. The shingle strips, 1″ × 4″, 2″ apart, one at the ridge on each side of the ridge, and one at the lower end of the roof.

h. Notice the rise of the gable roof, the distance from the top of the plate to the top of the ridge.

i. Notice the span of the roof, the width of the building.

j. Notice that the pitch of the roof is the rise divided by the span.

Then answer the following questions and solve the examples.

a′. What would be the rise of a gable roof if the span is 30 ft. and the pitch is $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$?

b′. What is the pitch of a gable roof if the span is 20′ and the rise is 5′? 8′? 10′?

c′. Determine the length of rafter for the garage pictured, allowing 2 ft. for overhang.

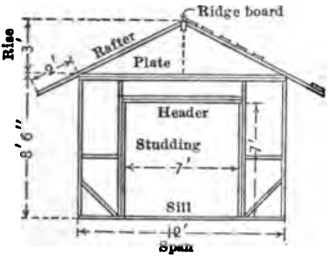
d′. Since 2 × 4's come only in even lengths, what length must be ordered for the rafters, remembering that about 3″ must be wasted at each end in making the slanting cut?

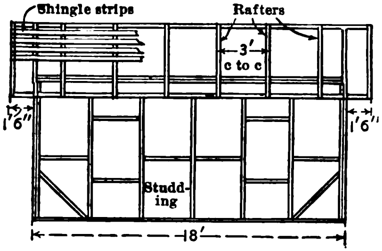
e′. Make out a memorandum for the lumber required for the frame of the garage, as follows:

LUMBER REQUIRED FOR FRAMING OF GARAGE

Purpose	*Number*	*Kind*	*Quantity*
Sills	2	2 × 4 12′	16 B. F.
	2	?	
Studding			
Plate			
Rafters			
Shingle Strips			
Ridge			
Horizontal braces			
Headers			
Diagonal braces			

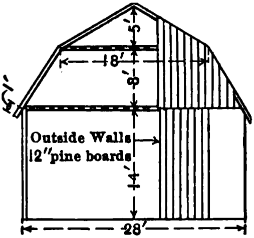
f′. Suppose that the outside of the building is first covered with 1″ × 8″ sheathing, then with paper, and finally with siding. Determine the number of board feet of sheathing, not making any allowance for the doors or windows, allowing 20% for waste in cutting.



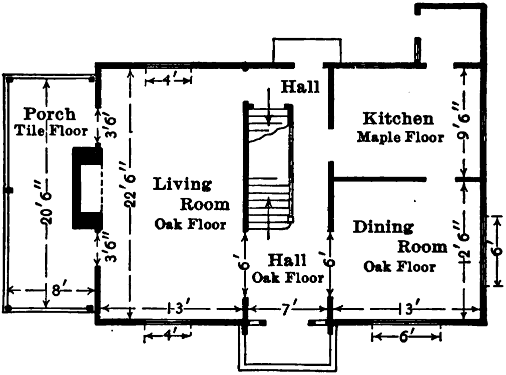


Determine also the number of rolls of building paper needed, if it comes in rolls of 500 sq. ft. Also determine the number of linear feet of 8″ cypress siding needed if it is laid so that 7″ is exposed to the weather. Do not allow for the doors or the windows; add $\frac{1}{8}$ for waste.

7. The adjoining figure is a skeleton view of the end of a gambrel-roofed barn. Suppose that the barn is 45 ft. long. Determine the cost of shingles for the slanting surfaces if they are laid $4\frac{1}{2}''$ to the weather and cost \$ 6.25 per M, assuming that the roof does not project over the ends of the barn. Determine the cost at \$ 60.00 per M of 12″ pine boards for the walls, put on vertically.



8. *a*. From the floor plans below determine the number of linear feet of 3″ oak flooring ($2\frac{1}{4}''$ face) required for the living room, halls, and dining room; also the number of linear feet of 3″ maple flooring for the kitchen. Allow one tenth additional for waste in cutting.



b. Find the cost of the flooring if the oak comes at \$ 85.00 per M, and the maple at \$ 72.00 per M.

c. Find the cost of a tile floor for the porch at \$ .60 per square foot. Do not allow for the fireplace chimney or the places occupied by the posts.

9. Determine the cost of painting the four walls of the shed shown in Example 3, at $.25 per square yard.

10. Determine the cost of painting the vertical walls of the barn shown in Example 7, at $.32 per square yard.

B. Mensuration of Less Common Solids

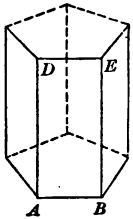
127. A **Polyhedron** is a solid bounded by portions of planes called **Faces** of the polyhedron. The faces intersect in straight lines, called the **Edges** of the polyhedron; the edges intersect in points called the **Vertices** of the polyhedron. The straight lines joining any two vertices which do not lie in the same face are called the **Diagonals** of the polyhedron.

128. A **Prism** is a polyhedron, two of whose faces lie in parallel planes, and whose remaining faces, in order, intersect in parallel lines. The parallel faces are the **Bases**; the other faces are the **Lateral Faces**; the edges which are not sides of the bases are the **Lateral Edges**; the perpendicular between the bases is the **Altitude**; the sum of the areas of the lateral faces is the **Lateral Area.**

129. A **Right Prism** is a prism whose lateral edges are perpendicular to its bases.

The lateral faces are evidently rectangular, and the lateral edges equal the altitude.

130. Finding the Lateral Area of a Prism. — Consider the face *ABED*. It is bounded by a rectangle whose base is *AB* and altitude *BE*, — the same as the altitude of the prism. The area of rectangle $ABED = AB \times BE$. In like manner, the area of each lateral face equals a side of the base multiplied by the altitude of the prism.



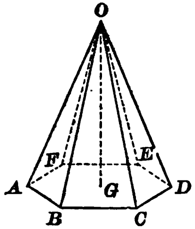
Therefore

Rule I. — The lateral area of a right prism is equal to the sum of the sides of the base multiplied by the altitude of the prism.

Just as the volume of a rectangular parallelopiped is the product of its base by its altitude, so

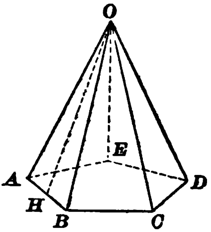
Rule II. — The volume of any prism equals the area of its base multiplied by its altitude.

131. The adjoining figure represents a **Pyramid.** It is a polyhedron having one **Base** bounded by a polygon, face *ABCDEF*, and as many triangular faces with a common vertex, *O*, as there are sides of the base. The common point of the triangular faces is called the **Vertex** of the pyramid; the triangular faces are the **Lateral Faces**; and the sum of their areas is the **Lateral Area**; the triangular faces meet on the **Lateral Edges**, as *AO*, *BO*, etc.; the perpendicular from the vertex to the base is the altitude, as *OG*.



132. A **Regular Pyramid** is a pyramid whose base is inclosed by a polygon whose sides are all equal, and whose angles are all equal, and whose vertex lies on a line perpendicular to the plane of its base at the center of the base.

NOTE. — A polygon having equal sides and equal angles is called a *regular polygon.* Such a polygon has a center.



It is proved in solid geometry that all the lateral edges of a regular pyramid are equal, and that the lateral faces have equal altitudes.

The common length of the altitudes of the lateral faces is called the **Slant Height of the Regular Pyramid,** as line OH.

NOTE. — Only a regular pyramid has a slant height.

The line representing the slant height bisects the side of the base to which it is drawn.

133. Finding the Lateral Area of a Regular Pyramid.

The area of each of the lateral faces is one half its altitude multiplied by its base. But this is one half the slant height of the pyramid multiplied by one of the sides of the base. When the areas of all the faces are added, there results the

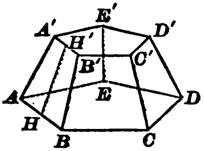
Rule. — The lateral area of a regular pyramid equals one half its slant height multiplied by the perimeter of its base.

Example. — Find the lateral area of a regular pyramid having 8 faces, if one side of the base is 6 ft. and the slant height is 10 ft.

Solution. — 1. The perimeter of the base is 8×6 ft. or 48 ft.
2. $\therefore$ the lateral area $= \frac{1}{2} \times 10 \times 48 = 240$ sq. ft.

134. A Frustum of a Pyramid is that part of a pyramid inclosed between the base and a plane parallel to the base.

The base and the section of the plane parallel to the base are called the **Bases** of the frustum; the perpendicular between the bases is called the **Altitude** of the frustum.



The lateral faces are bounded by trapezoids. If the frustum is the frustum of a regular pyramid, the altitudes of the lateral faces are all equal, and their common length is called the slant height of the frustum; as HH'.

NOTE. — Only frustums of regular pyramids have a slant height.

The line representing the slant height of a frustum of a regular pyramid joins the mid-points of the bases of the face on which it is drawn.

135. Finding the Lateral Area of a Frustum of a Regular Pyramid. — In the figure of § 134, the area of trapezoid $ABB'A'$ is one half its altitude multiplied by the sum of its bases; but this is one half the slant height of the frustum multiplied by the sum of one side of the lower base and one side of the upper base. When the areas of all the lateral faces are found in this manner and added, there results the

Rule. — The lateral area of a frustum of a regular pyramid equals one half its slant height multiplied by the sum of the perimeters of its bases.

136. The following rules for the volumes are proved in solid geometry.

Rule I. — The volume of any pyramid equals one third its altitude multiplied by the area of its base.

Rule II. — The volume of a frustum of any pyramid equals one third its altitude multiplied by the sum of the areas of its lower base and its upper base, plus the square root of the product of these areas.

Example. — Find the volume of a frustum of a regular pyramid whose lower base is a square having a side of 8 in., whose upper base is a square having a side of 5 in., and whose altitude is 6 in.

Solution. — 1. The area of the lower base is 8 × 8 or 64 sq. in.
The area of the upper base is 5 × 5 or 25 sq. in.

2. The volume $= \frac{1}{3} \times 6 \times (64 + 25 + \sqrt{64 \times 25})$
$= \frac{1}{3} \times 6 \times (64 + 25 + 8 \times 5) = 2 \times 129$
$= 258$ cu. in.

EXERCISE 92

1. Find the lateral area and volume of a right prism whose altitude is 14 in., having for its base a right triangle whose sides are 5 in., 12 in., and 13 in.

2. Find the lateral area of a regular pyramid whose base is a square 5 ft. on a side, and slant height 13 ft.

3. What is the volume of a pyramid whose altitude is 24 in., having for its base a right triangle whose sides are 8 in., 15 in., and 17 in.?

4. Find the lateral area of a frustum of a regular pyramid whose lower base is a square 10 ft. on a side, upper base a square 5 ft. on a side, and whose slant height is 12 ft.

5. The volume of a pyramid, whose base is a square, is 1089 cu. in., and its altitude is 27 in. Find the length of each side of the base.

6. Find the volume of a frustum of a pyramid whose lower base is a rectangle 10 in. by 6 in., upper base a rectangle 5 in. by 3 in., and whose altitude is 17 in.

7. The lateral area of a regular pyramid is 1554 sq. in. The base is a triangle whose sides are all equal, and the slant height is 37 in. Find the length of each side of the base.

8. The lateral area of a frustum of a regular pyramid is 936 sq. in. The lower base is a square 18 in. on a side, and the upper base is one 6 in. on a side. Find the slant height of the frustum.

9. A monument is in the form of a frustum of a square pyramid 8 ft. in height, surmounted by a square pyramid 2 ft. in height. If each side of the lower base of the frustum is 3 ft., and each side of the upper base is 2 ft., find the volume of the monument.

10. The volume of a frustum of a pyramid is 210 cu. in. The lower base is a right triangle whose sides are 6 in. and 8 in., and the upper base is a right triangle whose sides are 3 in. and 4 in. Find the altitude of the frustum.

11. The base of a square pyramid is 14 in. on a side, and the altitude is 24 in. Find its lateral area and volume.

12. The volume of a rectangular parallelopiped whose base is a square is 896 cu. ft., and its altitude is 14 ft. Find the length of each side of its base, and its lateral area.

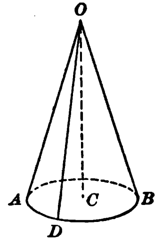
13. The volume of a box is 83.25 cu. ft., and the dimensions of its base are 5 ft. and 4.5 ft. Find its height, and the area of its total surface.

14. The volume of a cube is $4\frac{17}{27}$ cu. ft. What is the area of its entire surface in square inches? (Cube root needed.)

15. A monument whose height is 12 ft. is in the form of a pyramid with a square base, 2 ft. 10 in. on a side. Find its weight if a cubic foot of the stone weighs 180 lb.

CONES

137. Right Circular Cone. — In the adjoining figure, let *ADB* represent a circle with center *C*. Let *OC* be a line perpendicular to **Base** *ADB* at *C*. Assume that line *OA* moves so that point *O* is stationary and point *A* travels on circle *ADB*. This line forms a surface called a **Conical Surface.**



The solid bounded by the base *ADB* and the conical surface is called a **Right Circular Cone.** *O* is the **Vertex** of the cone. Line *OC* is the axis of the cone, and also the **Altitude**; the conical surface is the **Lateral Surface**; the line from *O* to any point of the circle bounding the base is the **Slant Height** as *OA*.

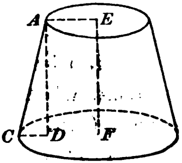
NOTE. — In solid geometry, other cones as well as right circular cones are studied.

138. Measuring the Right Circular Cone. — In solid geometry the following rules are proved:

Rule I. — The lateral area of a right circular cone equals the circumference of its base multiplied by one half its slant height.

Rule II. — The volume of a right circular cone, in fact of any cone, equals the area of its base multiplied by one third its altitude.

139. A **Frustum** of a cone is that part of the cone included between the base and a plane parallel to the base.



The base of the cone and the section of the cutting plane parallel to it are the **Bases** of the frustum; the perpendicular between the bases is the **Altitude**, as line EF; the part of the slant height of the cone included between the bases, line AC, is the **Slant Height** of the frustum of the cone.

In solid geometry are proved:

Rule I. — The lateral area of a frustum of a right circular cone equals the sum of the circumferences of the bases multiplied by one half the slant height.

Rule II. — The volume of a frustum of a right circular cone, or of any cone, equals the sum of the areas of the bases plus the square root of the product of these areas, multiplied by one third the altitude of the frustum.

Example. Find the volume and the lateral area of a frustum of a right circular cone whose slant height is 8 in., whose upper base diameter is 6 in., and whose lower base diameter is 14 in.

Solution. — 1. The circumference of the lower base $= 14\pi$ in., and of the upper base $= 6\pi$ in.

2. $\therefore$ the lateral area $= \frac{1}{2} \times 8 \times (14\pi + 6\pi) = 4 \times 20\pi = 80\pi$ in

3. $= 80 \times 3.1416 = 251.328 = 251.3^+$ sq. in.

4. The area of the lower base $= 7^2\pi$ sq. in. $= 49\pi$ sq. in.
The area of the upper base $= 3^2\pi$ sq. in. $= 9\pi$ sq. in.

5. In the figure of § 139, the altitude $= EF$ or AD.

6. $CD = CF - AE = 7 - 3 = 4.$

7. In right triangle ACD, $\overline{AD}^2 = 8^2 - 4^2 = 48.$
$\therefore AD = \sqrt{48} = 6.9^+$ in.

8. $\therefore$ the volume $= \frac{1}{3} \times 6.9 \times (49\pi + 9\pi + \sqrt{49\pi \times 9\pi})$
$= 2.3 \times (49\pi + 9\pi + 7\pi \times 3)$
$= 2.3 \times 79\pi$
$= 181.7^+$ cu. in.

EXERCISE 93

1. Find the lateral area and the volume of a right circular cone whose altitude is 18 in. and the radius of whose base is 6 in.

2. A tent, in the shape of a right circular cone, has a slant height of 20 ft. and a diameter at its base of 30 ft. How many square yards of material were used in constructing it?

3. Find the volume of a frustum of a right circular cone whose altitude is 5 ft. and the diameters of whose bases are 30 in. and 40 in. respectively.

4. How many cubic feet are there in a column whose length is 24 ft., the diameter of whose larger end is 14 in., and the diameter of whose smaller end is 10 in.?

5. What is the volume of a frustum of a right circular cone whose altitude is 13 in. and the radii of whose bases are 7 in. and 3 in. respectively?

6. Find the lateral area of a frustum of a right circular cone whose slant height is 15 ft. and the radii of whose bases are 5 ft. and 3 ft. respectively.

7. The lateral area of a right circular cone is 188.496 sq. in. and the radius of the base is 6 in. Find the slant height, the altitude, and the volume.

8. The volume of a frustum of a right circular cone is 779.1168 cu. ft. and the radii of its bases are 10 ft. and 2 ft. respectively. What is the altitude?

9. The altitude of a cone is 12 in., and the radius of its base is 7 in. Find the altitude of a cylinder of equal volume, the diameter of whose base is 10 in.

10. The altitude of a frustum of a right circular cone is 6 ft., and the radii of its bases are 3 ft. and 2 ft. respectively. What is the diameter of a sphere of equal volume?

11. The volume of a right circular cone is 2412.7488 cu. in. and its altitude is 12 in. Find the radius of its base.

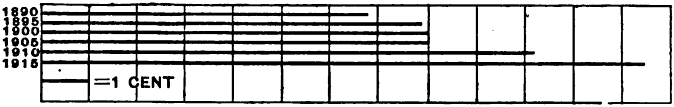
12. Find the radius of a sphere whose surface equals in area the lateral surface area of a right circular cone, whose altitude is 8 ft. and the radius of whose base is 4 ft.

13. The area of a sphere is 314.16 sq. in. What is its diameter and volume?

C. Graphical Representation

140. Numerical relations are often conveyed to the mind advantageously by a **Graph.**

Example. — The figure below represents the average wholesale price in cents per pound of fresh beef in certain years.



The short heavy line at the bottom represents one cent. It is called the **Unit.** The line opposite 1890, compared with the unit, represents the cost of one pound of fresh beef in 1890. Examination of this line shows that it is 6.8

times as long as the unit, and therefore the cost of one pound of fresh beef in 1890 was 6.8 cents.

The figure is drawn on a piece of coördinate paper, merely as a matter of convenience. Such a graph can be made by drawing parallel lines with a ruler. Thus, for this graph, a segment (§ 197) one half inch in length could be used as the unit to represent one cent. Then the line for 1890 would need to be 6.8 times as long or 3⅖ inches in length.

QUESTIONS

1. Determine from the graph the cost of one pound of fresh beef in each of the other years indicated in the graph.

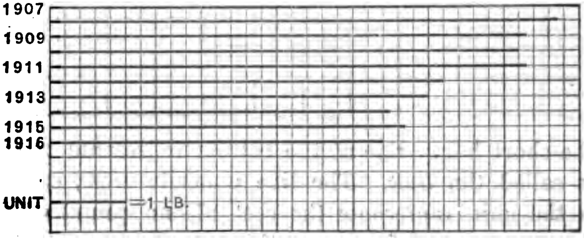
2. In what year was the increase in the cost greatest?

3. Suppose that the unit in the graph were to represent 2 cents. How long, as compared with their present lengths, would the other lines be made then?

4. Suppose that the unit in the graph were to represent ½ cent. How long would the other lines be made then?

141. Selecting the Unit when drawing a graph depends upon the size of the numbers to be represented and upon the size of the paper upon which the graph is to be drawn.

Example. — The graph below represents the number of pounds of round steak that could be purchased for $1.00 in Chicago, in each of certain years. The amount purchasable in 1907 was 7 pounds. That was the greatest number to be represented. It was decided to draw the graph on a



Number of Pounds Round Steak for $1.00

piece of paper about as wide as the printed page of this book. Study of the paper disclosed that the width of seven large squares on a sheet of coördinate paper could be used for the graph. Since the longest line had to represent 7 pounds, evidently the width of one large square could be used to represent 1 pound.

QUESTIONS

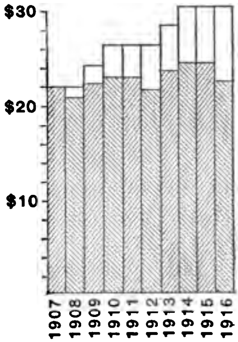
1. Since the side of a large square represents 1 pound, what does the side of a small square represent?

2 How much round steak would $ 1.00 purchase in each of the years represented?

3. Determine the amount of round steak that $ 1.00 will purchase at the present time in your town, and draw the line representing that amount.

142. The adjoining graph illustrates another form of graphical representation.

This graph represents the weekly wages of printers in Chicago in certain years and the purchasing power of those wages as compared with the purchasing power of the wages earned in 1907, owing to the increased cost of the necessities of life. The weekly wages in each year are represented by the rectangle from the horizontal base line to the upper horizontal line above; the purchasing power, by the shaded rectangle.



Thus, the weekly wages in 1908 as well as in 1907, were $ 22.00, but, owing to the increased cost of necessities, these wages had a purchasing power equal only to the purchasing power of $ 20.90 in 1907.

Similarly, the wages in 1916 were $ 30.80, but no more could be purchased with them than was purchasable with $ 22.30 in 1907.

QUESTIONS

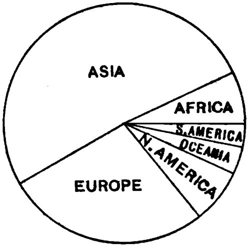
1. About what were the weekly wages in 1910?

2. About what was the purchasing power of the weekly wages in 1910?

3. Without determining the weekly wages in any of the other years or their purchasing power, what facts about the wages and the purchasing power of them do you discover from examination of the graph?

143. The adjoining circle illustrates another form of graph.

In this circle is represented the population of the world and the part of it living in each of the great divisions of the world.



QUESTIONS

Judging from the graph:

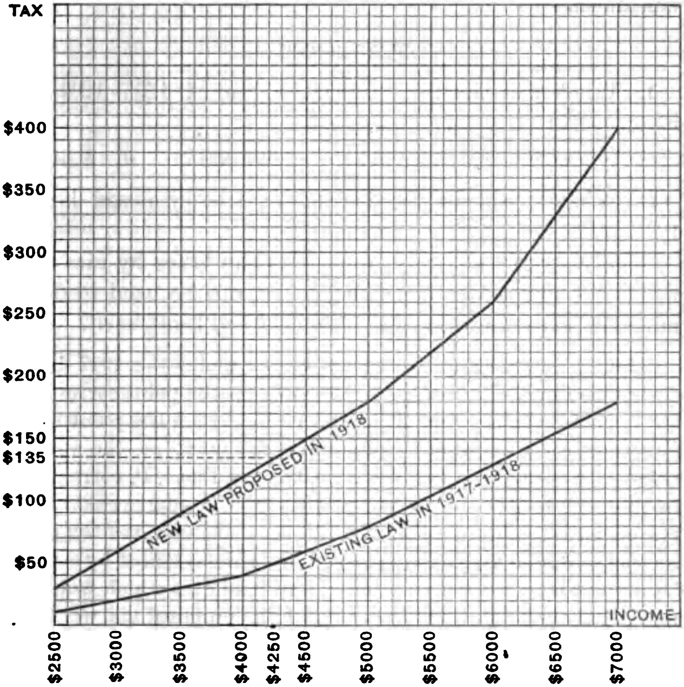
1. How does the population of Asia compare with that of the whole world? With that of Europe?

2. How does the population of Europe compare with that of the whole world? With that of all the other divisions excepting Asia?

3. Similarly compare the population of Africa with that of North America; also with that of South America and Oceania together.

144. Another form of graph is that below, showing the amount of income taxes paid by a married man whose total income is between $ 2500 and $ 7000 inclusive, under the law existing in 1917–1918, and the amount he would pay under the new law proposed in 1918.

Explanation of the Graph.—Along the heavy black horizontal line, at equal spaces, are the numbers $2500, $3000, etc., representing annual incomes of these amounts. An income of $2600 would be represented opposite a point one fifth the distance between the points representing incomes of $2500 and $3000.



On the heavy black vertical line at the left are represented at equal distances the numbers $50, $100, etc. Clearly, the side of a small square represents $10.00.

Notice now the lower broken line, representing the taxes under the law of 1917–1918. Above $2500 is a point opposite which on the vertical axis would be $10. This means that the tax on $2500 was $10. Above $3000 is a dot opposite the point on the vertical axis that corresponds to $20. This means that the tax on $3000 was $20.

After points were located in this manner for the incomes of $2500, $3000, $3500, etc., the broken line was drawn connecting these points. This gave the tax line for the law of 1917–1918.

In the same manner, the line for the proposed tax law of 1918 was drawn.

QUESTIONS

1. What was the tax under the law of 1917–1918 for incomes of:

a. $ 3500? *b.* $ 4000? *c.* $ 5000? *d.* $ 5500?

2. What would be the tax under the proposed law of 1918?

Such a graph not only represents the taxes by means of which the graph was constructed, but can be used to determine the taxes for other incomes between those used.

Thus, the tax on an income of $4250 under the 1917 law is determined by finding on the horizontal axis the point corresponding to that income. (See the graph.) Find then the point above $4250 on the "tax line." Determine the point on the vertical axis which is opposite this point on the tax line. It corresponds in this case to $50. This means that $50 is approximately the tax on an income of $4250. In the same manner, the tax on an income of $4250 under the proposed law is represented by a point. This point is opposite a point on the vertical axis which would be marked about $135. This means that the proposed tax would be about $135. Actually, this is the amount of the proposed tax.

QUESTIONS

1. What was the tax under the old law on an income of:

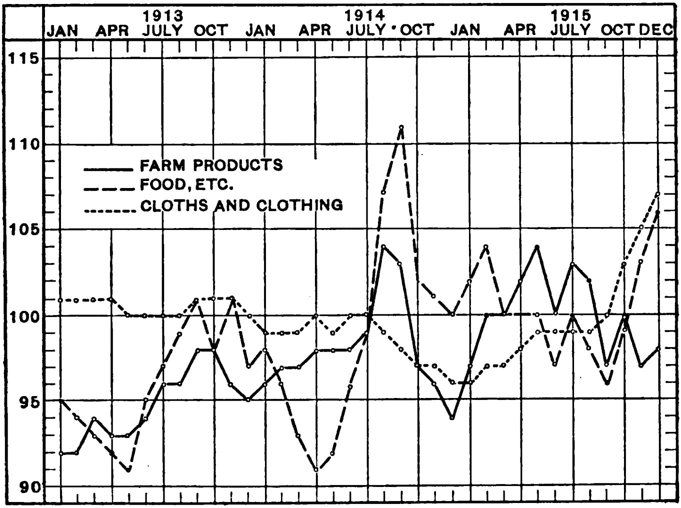
a. $ 5200? *b.* $ 5600? *c.* $ 6400? *d.* $ 6800? *e.* $ 3750? *f.* $ 4750?

2. Approximately, what was the proposed tax under the new law on incomes of:

a. $ 5250? *b.* $ 6300? *c.* $ 6800? *d.* $ 4600? *e.* $ 3700? *f.* $ 2900?

EXERCISE 94

1. The following graph is taken from a Bulletin published by the United States Bureau of Labor. This graph represents the fluctuations in the prices for certain of the necessities of life during certain years. The numerals at the left are called "index numbers." By means of them, the prices during each month of the years represented are compared with the average prices during the year 1915,—the latter being given the value 100. Thus, while the average price of farm products during 1915 was 100, from January to February 1913, the price was 92. This is an indication of the increase in price that occurred between early in 1913 and the year 1915.



Suggested Questions. What was the index number for food for any particular month or months of the table?

Did all three classes of necessities increase or decrease in price at the same time?

2. Represent by lines of different lengths the amounts of butter that could be purchased for $ 1.00, according to the following statistics: 1907, 3.2 lb.; 1908, 3.1 lb.; 1909, 3.1 lb.; 1910, 2.9 lb.; 1911, 3 lb.; 1912, 2.8 lb.; 1913, 2.8 lb.; 1914, 3 lb.; 1915, 3 lb.; 1916, 2.6 lb. (See Example, § 140.)

3. Represent by rectangles of different heights the number of pecks of potatoes that could be purchased for $ 1.00 according to the following statistics: 1907, 4.8 pk.; 1908, 4.2 pk.; 1909, 4.2 pk.; 1910, 4.4 pk.; 1911, 3.3 pk.; 1912, 3.2 pk.; 1913, 4.4 pk.; 1914, 3.9 pk.; 1915, 5.2 pk.; 1916, 2.7 pk. (See Example, § 142.)

4. Represent by lines of different lengths the following average wholesale prices of eggs in New York City: 1905, 27 ¢; 1906, 26 ¢; 1907, 28 ¢; 1908, 28 ¢; 1909, 31 ¢; 1910, 33 ¢; 1911, 30 ¢; 1912, 33 ¢; 1913, 33 ¢.

5. Represent by a graph similar to that in § 144 the following weights of boys and of girls, showing the weights of boys on one broken line and the weights of girls on a second.

AGE	BOYS	GIRLS	AGE	BOYS	GIRLS
6 yr.	45 lb.	43 lb.	12 yr.	76 lb.	78 lb.
7 yr.	49 lb.	47 lb.	13 yr.	84 lb.	88 lb.
8 yr.	54 lb.	52 lb.	14 yr.	95 lb.	98 lb.
9 yr.	59 lb.	57 lb.	15 yr.	107 lb.	106 lb.
10 yr.	65 lb.	62 lb.	16 yr.	111 lb.	112 lb.
11 yr.	70 lb.	69 lb.	17 yr.		

6. (*a*) Represent by a graph like that in § 144 the cost at 22 ¢ per pound of: 1 lb., 2 lb., 3 lb., 4 lb., and 5 lb. of steak.

b. From the resulting graph, determine the cost at 22 ¢ per pound of: (1) $1\frac{1}{2}$ lb.; (2) $2\frac{1}{2}$ lb.; (3) $3\frac{3}{4}$ lb.; (4) $4\frac{1}{8}$ lb.

7. *a*. Draw a graph representing the cost at 27 ¢ per yard of 1 yd., 2 yd., 3 yd., 4 yd., 5 yd., etc., up to 10 yd. of a certain grade of muslin.

b. **From the resulting graph, determine the approximate cost at the same rate of:**

(1) $4\frac{1}{2}$ yd.; (2) $3\frac{1}{4}$ yd.; (3) $5\frac{3}{4}$ yd.; (4) $7\frac{1}{8}$ yd.

8. Obtain the temperature readings for each hour of a day (to-day or yesterday) and draw a graph representing these readings.

D. United States Government Land Survey

145. The land area of the Central and Western States was surveyed by the United States Government.

Surveyors first marked the true meridian of the earth (running north and south) through some prominent point in a district. This is called the Principal Meridian.

The surveyors also marked the parallel of latitude running (east and west) through this point. This parallel is called a **Base Line.** They also marked the parallels of latitude at intervals of twenty-four miles north and south of the base line.

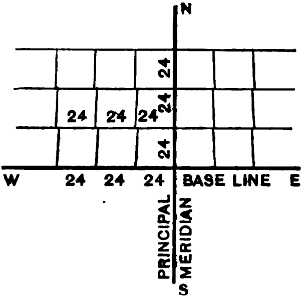


FIG. 1.

On each of these parallels, points were established at intervals of twenty-four miles east and west of the principal meridian. Through these points, meridians were marked between the parallels, as shown in the figure adjoining. Because of the curvature of the earth, these meridians are a little less than twenty-four miles apart at the top, although they are full twenty-four miles apart at the bottom.

As a result, the land was divided into tracts which are approximately twenty-four miles square. (See also Fig. 2.)

Each of these tracts was divided by true parallels of latitude and true meridians into parts approximately six miles square. These parts are called **Townships.** They were meant to be 6 mi. square. The east and west boundaries of a township are full six miles in length; also the south boundary of each township of the bottom row of each 24 mi. square tract is also six miles in length; but the south boundaries of all the others and the north boundaries of all the townships are less than six miles in length.

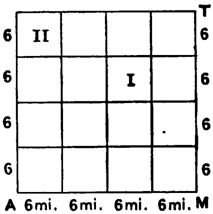
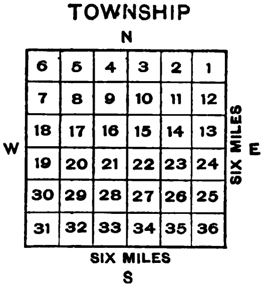


FIG. 2.

The townships in a line north and south form a **Range**; those in a line east and west form a **Tier.**

The ranges are numbered east or west from the principal meridian; the tiers are numbered north or south from the base line.

Thus, in Fig. 2, if *AM* be assumed to be the base line and *MT* the Third Principal Meridian, then township I is 3 N., Range 2 W. of the Third Principal Meridian.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 3.

Each township is divided into thirty-six parts that are approximately one mile square, called **Sections.** These parts are numbered as shown in the adjoining figure, Fig. 3. Actually, because of the fact that most of the townships are not six miles square, and because of the manner of subdividing the townships, the sections in the western range and the northern tier are slightly less than one mile square.

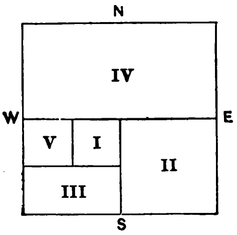


FIG. 4.

The adjoining figure represents a

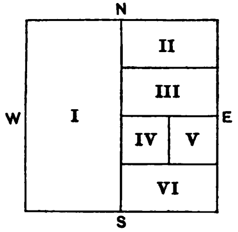
subdivision of a section. Field IV would be described as the north ½ of the section; field II, as the S. E. ¼ of the section; field III, as the S. ½ of the S. W. ¼ of the section; field I, as the N. E. ¼ of the S. W. ¼ of the section.

EXERCISE 95

1. Since a section is approximately one square mile, how many acres does it contain?

2. How many acres are there in each of the fields of Fig. 4, p. 206?

3. Describe the location of field V in Fig. 4, p. 206.



4. The adjoining figure represents Section 22 of Township 3 N., Range 23 E. of the Fourth Principal Meridian. Describe the location of each of the farms shown in the figure. How large is each?

5. Represent by a drawing a farm described as the W. ½ of the N. E. ¼ of a certain section.

6. Represent by a figure a farm that is described as the S. W. ¼ of the N. W. ¼ of a certain section.

7. How many rods of fencing are required to inclose the farms described in Example 5 and Example 6?

8. Find the value of the farm in Example 5 at $ 135 per acre.

9. Find the value of the farm in Example 6 at $ 175 per acre.

VII. PERCENTAGE

146. The expression **Per Cent** is an abbreviation for *per centum*, meaning "in" or "by a hundred."

Thus, 3 per cent means 3 in every hundred.

The symbol for per cent is %; thus, 3 per cent is written 3 %.

Since 3 % means 3 in every hundred, then 3 % of 200 must be 6, 3 % of 500 must be 15, etc. Evidently 3 % of a number is the same as $\frac{3}{100}$ or .03 of the number.

147. The **Rate Per Cent** is the number of hundredths to be found.

The **Base** is the number of which a certain per cent is to be found.

The **Percentage** is the result obtained by taking a certain per cent of a base.

148. A **Rate Per Cent** is usually **Expressed** in **Decimal Form** when it is used.

$$\text{Thus, } 27\,\% = \tfrac{27}{100} = .27.$$

$$5.3\,\% = \frac{5.3}{100} = .053.$$

$$16\tfrac{1}{3}\,\% = \frac{16\frac{1}{3}}{100} = .16\tfrac{1}{3} = .163^{+}.$$

Rule. — To express a rate per cent as a decimal, divide the number of per cent by 100.

EXERCISE 96

Express in decimal form:

1. 6 %.
2. 10 %.
3. 12 %.
4. 15 %.
5. 20 %.
6. 35 %.
7. 16 %.
8. 14 %.
9. 92 %.
10. 83 %.

How many times a number is:

11. 100 % of it? **14.** 175 % of it?
12. 150 % of it? **15.** 300 % of it?
13. 200 % of it? **16.** 250 % of it?

Express in decimal form:

17. 310 %. **19.** 540 %. **21.** 826 %. **23.** 130 %. **25.** 140 %.
18. 260 %. **20.** 375 %. **22.** 350 %. **24.** 225 %. **26.** 1000 %.

27. Four times a number is what per cent of it?

28. Six times a number is what per cent of it?

29. Three and one fourth times a number is what per cent of it?

30. One and four fifths times a number is what per cent of it?

Express in decimal form:

31. $2\frac{1}{2}$ %. **34.** $5\frac{1}{8}$ %. **37.** $\frac{1}{6}$ %. **40.** .06 %. **43.** $3\frac{1}{2}$ %.
32. $4\frac{1}{4}$ %. **35.** $87\frac{1}{2}$ %. **38.** .07 %. **41.** .5 %. **44.** $5\frac{1}{2}$ %.
33. $3\frac{1}{4}$ %. **36.** $5\frac{2}{3}$ %. **39.** $8\frac{1}{2}$ %. **42.** $5\frac{2}{3}$ %. **45.** $4\frac{1}{2}$ %.

Express in decimal form and observe the difference between:

46. 5 % and .05 %. **49.** 25 % and .25 %.
47. 75 % and .75 %. **50.** $1\frac{1}{2}$ % and .015 %.
48. 2 % and .2 %. **51.** $\frac{1}{2}$ and $\frac{1}{2}$ %.

149. Expressing a Common Fraction as a Rate Per Cent.

Example 1.—Express $\frac{5}{11}$ as a rate per cent.

Solution.—1. $\frac{5}{11}$ is $\frac{5}{11}$ of $\frac{100}{100}$ or $\frac{5}{11}$ of 100 %.

2. $\therefore \frac{5}{11} = \frac{5}{11} \times 100\,\% = \frac{500}{11}\,\% = 45\frac{5}{11}\,\%$.

Example 2.—Express $\frac{75}{135}$ as a rate per cent.

Solution.—1. $\frac{75}{135} = \frac{75}{135}$ of $\frac{100}{100}$ or 100 %.

2. $\therefore \frac{75}{135} = \frac{75}{135} \times 100\,\% = \frac{750}{135}\,\% = 55.5\,\%$.

Rule. — To express a common fraction as a rate per cent: Multiply the numerator by 100, and divide the result by the denominator.

NOTE. — If the division is not exact, the quotient may be expressed either as a mixed number, or be carried out to one or more decimal places as in Example 2 above.

EXERCISE 97

1. Determine the rate per cent equal to the fractions in the following table; enter them in the adjoining column in pencil, and then memorize the results.

$\frac{1}{2}$		$\frac{1}{5}$		$\frac{5}{6}$		$\frac{1}{10}$	
$\frac{1}{3}$		$\frac{2}{5}$		$\frac{1}{8}$		$\frac{3}{10}$	
$\frac{2}{3}$		$\frac{3}{5}$		$\frac{3}{8}$		$\frac{7}{10}$	
$\frac{1}{4}$		$\frac{4}{5}$		$\frac{5}{8}$		$\frac{9}{10}$	
$\frac{3}{4}$		$\frac{1}{6}$		$\frac{7}{8}$		$\frac{1}{12}$	

Using the results of the preceding table, give the rate per cent equal to:

2. *a.* $\frac{3}{2}$; *b.* $2\frac{1}{2}$; *c.* $\frac{7}{2}$; *d.* $1\frac{1}{4}$; *e.* $4\frac{1}{2}$.
3. *a.* $\frac{4}{8}$; *b.* $\frac{6}{8}$; *c.* $\frac{5}{10}$; *d.* $\frac{8}{10}$; *e.* $\frac{12}{15}$.
4. *a.* $\frac{11}{8}$; *b.* $\frac{6}{5}$; *c.* $\frac{13}{10}$; *d.* $\frac{7}{6}$; *e.* $\frac{14}{5}$.

Determine the rate per cent equal to:

5. $\frac{4}{7}$.
6. $\frac{1}{11}$.
7. $\frac{7}{12}$.
8. $\frac{17}{20}$.
9. $\frac{6}{25}$.
10. $\frac{7}{40}$.
11. $\frac{15}{16}$.
12. $\frac{9}{15}$.
13. $\frac{32}{55}$.
14. $\frac{24}{15}$.
15. $\frac{27}{68}$.
16. $\frac{29}{32}$.
17. $\frac{18}{22}$.
18. $\frac{43}{56}$.
19. $\frac{275}{340}$.
20. $\frac{642}{825}$.
21. $\frac{16}{88}$.
22. $\frac{95}{40}$.
23. $\frac{322}{450}$.
24. $\frac{29}{72}$.

150. Finding the Percentage when the Base and the Rate are Given.

Example 1. — Find 15 % of $38.

Solution. — 1. 15 % of $38 means .15 of $38.

2. ∴ 15 % of $38 = .15 × $38 = $5.70.

Example 2. — Find 12½ % of $72.

Solution. — 1. 12½ % of $72 is the same as ⅛ of $72.

2. ∴ 12½ % of $72 = ⅛ of $72 = $9.

Rule. — To find the percentage when the base and rate are given, multiply the base by the rate expressed in decimal form, or in fractional form.

EXERCISE 98

1. Find 1 % of 60, of 75, of 126, of 8409, and of 27,536.

2. Find 2 % of each of the numbers in Example 1.

3. Find 10 % of each of the numbers in Example 1.

4. Find 25 % of each of the numbers in Example 1.

5. Find 7 % of each of the numbers in Example 1.

6. Find 15 % of $28, of $92, of $672, of $2344, and of $5476.

7. Find 12½ % of each of the amounts in Example 6. Also find 37½ %, 62½ %, and 87½ %.

8. Find 125 % of $80. Also find 210 %, 120 %, and 62½ %.

9. Find 3½ % of $2500. Also find 4¼ %, and 4½ %.

10. Find 12½ % of $1200. Also find 16⅔ % and 200 %.

11. A merchant sold goods that cost him $275 for 18 % more than they cost. What did he receive?

12. Out of 25 bu. of seed corn, only 75 % germinated properly. How many bushels germinated properly?

How much is:

13. 70 increased by 10 % of itself? By 30 %?

14. $2500 decreased by 20 % of itself? By 50 %?

15. 35,000 sq. ft. of floor space increased by 12½ % of itself?

16. $2650 increased by 12½ % of itself? Decreased by 10 % of itself?

17. The employees in a certain factory were told that their wages were to be increased by 10 %. What would be the new wages of men who had been receiving: *a.* $85? *b.* $125? *c.* $140? *d.* $95? *e.* $175?

18. 25 % of the 1500 trees planted on a real estate subdivision died. How many were left?

19. A man asking $8250 for his house and lot agreed to sell it for 5 % less to any one who bought direct from him instead of through an agent. What would be the difference in cost to the purchaser?

20. A grocer agreed to reduce all prices in his store 5 % for cash purchases and 3 % additional if the purchaser carried away his purchases instead of having them delivered by the grocer. What would be the new prices on goods which were marked: *a.* 75¢? *b.* 40¢? *c.* 82¢? *d.* $1.75? *e.* 25¢?

21. The depositors in a certain bank that failed were promised 35 % of the amount of their deposits. How much should A, B, and C receive whose deposits were $785, $234.50, and $548.76 respectively?

22. One plan of payments for bonds of the Fourth Liberty Loan provided that 10 % should be paid when the subscription was made, 20 % on or before Nov. 21, 1918; 20 %, Dec. 19, 1918; 20 %, Jan. 16, 1919; 30 %, Jan. 30, 1919.

What were the payments for a person who subscribed for a $50 bond? For $3500 worth of bonds? For $400 worth of bonds?

23. A bankrupt paid his creditors 63 % of what he owed them. What was received by a man to whom $1823.70 was due?

24. The salary of an agent was $ 1000 per year. He also received 3 % of the amount of all sales over $ 25,000. What was his total income during a year when he sold $ 73,000 worth of goods?

25. If 52 % of a certain ore is lead and $1\frac{2}{3}$ % of the balance is silver, how much of each metal is there in 10 tons of the ore?

26. A certain fertilizer is guaranteed to contain about 4 % of nitrogen, 10 % of phosphoric acid, and 8 % of potash. How many pounds of each element are there in a ton of the fertilizer?

27. In a certain cow testing association, the three best producing cows and their records were:

Cow No.	Total Pounds of Milk	Per Cent of Butter Fat	No. Pounds of Fat	Value of Fat at 30 ¢ per Lb.
1	8109	4.33 %		
2	5023	5.2 %		
3	4897	5.13 %		

Complete the table.

28. In brass, there is approximately 65 % of copper and 35 % of zinc. How many pounds of each are there in a ton of brass?

29. How many pounds of nickel are required for 10 tons of nickel steel containing 2.8 % of nickel?

30. The employees in a certain company were informed at Christmas that their wages were to be increased by 15 %. What would be the new daily wages of a man receiving: *a.* $ 1.75 per day? *b.* $ 3.25 per day? *c.* $ 6.40 per day?

31. A contractor figures 2850 board feet of studding are required for a certain job. He decides to add 20 % for waste in cutting and for extras. How much should he order?

32. According to the rules of dealers, maple flooring of a certain grade may have not more than 7 % of the quantity ordered, in lengths of 2 ft. to $3\frac{1}{2}$ ft. How many feet of these short lengths may there be in an order for 4350 ft.?

33. A heating contractor observed that current high prices caused an increase in the cost of installing a heating plant over the cost of two years ago of about 45 %. What rough estimate can he give of the cost of a plant similar to one that cost $345.75 two years ago?

34. Architects charge a fee of 5 % for drawing the plans and writing the specifications for an ordinary dwelling. What will be the fee for a house costing $7568.45?

35. 5 % of the energy supplied by a pulley belt to a line shaft by a 3-horse-power engine in a certain plant is lost because of slipping of the belt. How much of the energy produced by the engine is actually delivered to the shaft?

36. Babbitt metal, used in making bearings for machinery, consists of about 80 % of tin, 10 % of copper, and 10 % of antimony. How much of each of these materials are required to make 1500 lb. of Babbitt metal?

37. In the spring of 1918, farmers were asked to increase their acreage planted to spring wheat in 1917 by 10 %. How many acres should a man have planted who had had 25 acres in 1917?

38. Cloth loses $1\frac{1}{8}$ % of its length when it is sponged. What will be the new length of a piece of cloth that measures 12 yards before it is sponged?

39. A foundryman making 47 lb. of bronze wants it to contain 40 % of copper, 10 % of tin, and the balance zinc. How many pounds of each metal are required?

40. In a certain company 1500 workmen are employed at an average wage of $2.75 per day. They ask for an increase of 15 % in their wages. How much would this increase add to the pay roll of the company?

151. Finding what Per Cent one Number is of Another is the most common method of comparing two numbers.

Example 1. — 610 is what per cent of 2745?

Solution. — 1. 610 is the percentage, 2745 is the base, and the rate is unknown.

2. Since the rate times the base equals the percentage, then the percentage divided by the base must give the rate.

3. $\therefore$ the rate $= 610 \div 2745 = \frac{2}{9} = 22\frac{2}{9}\,\%$.

Rule. — To find the rate when the percentage and the base are given, divide the percentage by the base.

NOTE. — Usually it is wise to form a fraction whose numerator is the percentage and whose denominator is the base. If the fraction is easily reduced, it should be reduced, because the result may be one of the fractions given in Example 1, p. 210. If the fraction is not easily reduced, divide, carrying the result out, if necessary, to one or two decimal places, as in § 149.

Example 2. — What per cent of 120 is 15?

Solution. — The rate $= \frac{15}{120} = \frac{1}{8} = 12\frac{1}{2}\,\%$.

Example 3. — 37 is what per cent of 150?

Solution. — The rate $= \frac{37}{150} = 24.6\,\%$.

EXERCISE 99

1. What per cent of 24 is 12? 6? 8? 4? 2? 3?
2. What per cent of 30 is 3? 6? 5? 15? 10?
3. What per cent of 48 is 16? 6? 24? 36?
4. What per cent of 50 is 5? 20? 30? 25? 10?
5. What per cent of 72 is 9? 18? 48? 36? 144?
6. What per cent of 36 is 24? 48? 72? 90? 360?
7. 3 is what per cent of 12? of 9? of 24? of 30?
8. 11 is what per cent of 33? of 66? of 44? of 55?
9. 14 is what per cent of 35? of 42? of 28? of 84?
10. 32 is what per cent of 48? of 64? of 320? of 160? of 16?

11. What per cent of 25 is 5? is 25? is 50? is 100?

12. 40 is what per cent of 60? of 20? of 50? of 90?

13. 22 is what per cent of 11? of 44? of 33?

14. What per cent of 64 is 16? is 64? is 128? is 96?

15. 35 is what per cent of 25? of 350? of 20? of 5?

16. What per cent of 200 is 50? is 140? is 250? is 175?

17. An article is composed of 19 parts of silver and 6 parts of copper. What per cent of the whole is the silver? What per cent is the copper?

18. From a barrel of gasoline containing 50 gal., 12 gal. were lost by leakage. What part was lost?

19. A certain oil company found that its pump for retailing gasoline to motorists was delivering $\frac{1}{2}$ pt. too much on every 5-gallon sale. *a.* The excess delivered was what per cent of the amount sold? *b.* At that rate, what was the loss in gallons in a day when 2468 gallons were sold?

20. Out of a population of 26,832 in a certain town, 3354 were foreign born. What per cent were foreign born?

21. If 675 lb. of silver are obtained from 45 T. of a certain ore, what per cent of that ore is silver?

22. A boy in a machine shop spoiled 8 bolts out of every 200 that he made. What per cent did he spoil?

23. A certain shop that employed 275 men and boys before the war with Germany, employed 605 at one time in 1918. What per cent was this of their former number of employees?

24. Laborers who were earning $ 2.25 per day in 1917 were able to get $ 3.00 per day in 1918. *a.* What per cent of their 1917 wages was their increase in wages? *b.* What per cent of their 1917 wages was their 1918 wages?

25. Round steak sold for 20 ¢ per pound in 1917 and for 32 ¢ per pound during the summer of 1918. *a.* What per cent of the selling price in 1917 was the increase in the

selling price in 1918? *b.* What per cent of the 1917 selling price was the 1918 selling price?

26. Anthracite coal sold for $ 9.00 per ton in 1916 and for $ 11.50 early in 1918. What per cent of the 1916 price was the 1918 price?

27. Milk sold for 12 ¢ per quart in Chicago in 1918 and for 6 ¢ per quart a few years before. What per cent of increase had there been in the selling price?

28. A contractor finds that 225 board feet of flooring out of an order of 2500 board feet is in pieces less than 3½ ft. long. Specifications were that not over 7 % were to be in short lengths. Was the flooring up to specifications?

29. A farmer bought some seed corn, the germination of which was said to be 93 %. He planted three separate lots of 50 seeds each. In pan A, 42 seeds germinated; in pan B, 40 seeds germinated; in pan C, 44 seeds germinated. What was the percentage of germination in each pan? What was the average percentage of germination? Was the seed up to its advertised grade?

30. In a real estate subdivision consisting of 35 lots, one man owns 10 lots. What per cent of the lots does this man own?

31. The following table gives the rate of pay per hour of certain workmen in an eastern city in 1907 and in 1916. Complete the table.

WORKMAN	PAY PER HOUR IN 1907	PAY PER HOUR IN 1916	INCREASE OR DECREASE	PER CENT OF INCREASE OR DECREASE
Bricklayer . . .	60 ¢	65 ¢		
Carpenter . . .	43.8 ¢	57 ¢		
Cement finisher .	50 ¢	62½ ¢		
Gas fitter	37½ ¢	60 ¢		
Painter	39 ¢	60½ ¢		

32. Out of 240 eggs placed in an incubator, 20 were found infertile. What per cent were infertile?

33. Out of 240 eggs placed in an incubator, 190 chicks were hatched. What per cent of the eggs "hatched out"?

34. One mixture of concentrated feed for cows is:

Wheat bran, 100 lb.
Ground oats, 100 lb.
Corn meal, 150 lb.
Gluten meal, 75 lb.
Dried brewers' grain, 75 lb.
Cottonseed meal, 50 lb.

What per cent is each constituent of the total?

35. In recent years, efforts have been made to have "sane" Fourth of July celebrations. The following table shows the results. Complete the table.

	1911	1916	PER CENT DECREASE
Total deaths . . .	57	30	
Loss of fingers . .	83	24	
Total injured . .	1546	820	

36. On Feb. 26, 1916, the French cruiser *Provence II*, engaged in transporting troops in the Mediterranean, was sunk. About 4000 soldiers were on board, and 3000 were lost, — making it the greatest sea disaster in history, to date. What per cent were lost?

37. During the period from Aug. 15, 1914, to July 1, 1916, 1875 vessels passed through the Panama Canal. Of this number, 709 were American owned and 822 were British. What per cent were American vessels and what per cent were British?

38. A molder put up 168 molds in a day, each containing 16 pieces. When he sorted the completed work, 2544 pieces were found to be perfect. What per cent of his work was perfect?

39. The weight of corn on the cob is 72 pounds and of shelled corn is 56 pounds. What per cent of the weight on the cob is the weight of the cobs?

40. Steel is made by heating iron and other metals in a furnace until they melt and fuse. 9 tons of iron and ½ ton of the other metals were placed in a furnace. 8¼ tons of steel were obtained. What per cent of the material was lost in the process?

41. If a man who was earning $ 135 per month had his pay raised to $ 150 per month, what was the per cent of increase?

42. The entire length of the Panama Canal is 50.5 miles. Its length on land is 40.5 miles. What per cent of its length is on land?

43. A contractor took a job for a contract price of $ 2644. He found that the work and material cost him $ 2350. What per cent of the sums he spent were his profits?

44. The approximate production for certain years of anthracite and bituminous coal in millions of tons is given in the following table. Determine the percentage of increase of each year's production over that of the preceding year.

	ANTHRACITE		BITUMINOUS	
	Production in Millions of Tons	Per Cent of Increase	Production in Millions of Tons	Per Cent of Increase
1900	51.3		189.5	
1905	69.4		281.2	
1910	75.5		372.3	
1914	81.1		377.4	

45. The balance of a certain corporation on April 30, 1916, was approximately $ 1,398,000, and on April 30, 1917, it was $ 1,547,000. What was the per cent of increase over the balance of 1916?

46. The urban and rural population of the United States in 1900 and in 1910 are given in the following table. Complete the table.

	1900	1910	INCREASE	PER CENT OF INCREASE
Urban .	30,797,185	42,623,383		
Rural .	45,197,390	49,348,883		

47. Using the following statistics, determine the percentage of increase in the population of the United States in 1910 over that of 1900 for each color or race.

COLOR OR RACE	1910	1900	PER CENT OF INCREASE
White	81,731,957	66,809,196	
Negro	9,827,763	8,833,994	
Indian	265,683	237,196	
Chinese	71,531	89,863	
Japanese . . .	72,157	24,326	

48. The population of Chicago in 1910 was 2,393,325 and the total population of Illinois was 5,638,591. What per cent of the population of Illinois lives in Chicago?

49. The losses of merchant ships from submarine attacks and mines during the first two years of the European War were estimated by French statisticians to have been 2,536,000 tons. Of these, the Entente Allies lost 1,994,000 tons, neutrals 330,000 tons, and the Central Powers 212,000 tons. What per cent of the total did each lose?

152. Finding the Base when the Percentage and the Rate are Given, while less common a problem than those considered in § 150 and § 151, is necessary at times.

Example 1. — 85 is 5 % of what number?

Solution. — 1. 85 is the percentage, 5 % is the rate per cent, and the base is unknown.

2. Since the rate × the base = the percentage,
then the base = the percentage ÷ by the rate.

3. $\therefore$ the base $= \frac{85}{.05} = 1700.$

Check. — Does .05 × 1700 = 85? Yes.

Rule. — To find the base when the percentage and the rate are given, divide the percentage by the rate.

NOTE. — If the rate is easily expressed as a common fraction, the solution may be performed as in Example 3, below.

Example 2. — What number increased by 8 % of itself is 540?

Solution. — 1. The number is 100 % of itself. When increased by 8 % of itself, the result is 108 % of the number.

2. $\therefore$ 540 = 108 % of some number. 540 is the percentage and 108 % is the rate per cent.

3. $\therefore$ the base = 540 ÷ 1.08 = 500.

Check. — Does 1.08 × 500 = 540? Yes.

Example 3. — 126 is $87\frac{1}{2}$ % of what number?

Solution. — 1. $87\frac{1}{2}\,\% = \frac{7}{8}.$

2. $\therefore 126 = \frac{7}{8}$ of some number.

3. $\therefore \frac{1}{8}$ of the number $= \frac{1}{7} \times 126 = 18,$
and $\frac{8}{8}$ of the number $= 8 \times 18 = 144.$

EXERCISE 100

Find the number of which:

1. 20 is: *a.* 2 %; *b.* 5 %; *c.* 8 %; *d.* 40 %; *e.* 50 %.

2. $56 is: *a.* 7 %; *b.* 6 %; *c.* 10 %; *d.* 28 %; *e.* 42 %.

3. 6 % of what number is:

a. 39 mi.? *b.* $45? *c.* 231 T.?

4. 15 % of what number is:

a. 630? *b.* $91.50? *c.* $439.50? *d.* 750?

5. Of what number is:

a. 75 rd. 25 %? *b.* $240 6 %? *c.* 16 T. $66\frac{2}{3}$ %?

6. What number diminished by 36 % of itself is 420?

7. If the expenses of collection of a charity fund are about 8 %, approximately how much should be collected in order that $1500 may be left?

8. It is said that only 60 % of an army actually fight at the front, the remaining 40 % being needed to take care of these 60 %. How large must an army be therefore in order to have 1,500,000 men available for actual fighting?

9. A man said that his taxes for a certain year were $131.25. If this is $1\frac{3}{4}$ % of the assessed valuation of his property, what was the valuation placed upon his property?

10. A man receives from a source other than his salary an income of $2100. Of what sum is this 6 %?

11. A poultryman is getting a good "hatch" if 85 % of the eggs that he places in an incubator hatch out chicks. How many eggs must be placed in incubators in order to obtain about 500 chicks?

12. Since cloth loses about $1\frac{7}{8}$ % of its length when it is sponged, how much cloth must a woman buy in order to have 18 yards full length after sponging?

13. The population of Tacoma, Washington, in 1910, was 83,743. This was said to be 122 % of its population in 1900. What was its population in 1900?

14. The population of Oklahoma City, Oklahoma, in 1910 was 64,205. This was said to be 539.7 % of its population in 1900. What was its population in 1900?

15. There is a shrinkage of about 15 % in the ice placed in an ice house, before it is sold during the following summer. How much ice should a company store in order to have at least 25,000 tons to sell during the summer season?

EXERCISE 101

Miscellaneous Exercises

1. Find the missing numbers in the following table:

P	R %	B
435	15 %	?
508.25	?	2675
?	6¾ %	5278
76	?	294
322	4½ %	?
?	3¾ %	15,275

2. A merchant has floor space of 40 ft. by 70 ft. He decides that he must have about 35 % more floor space. How much floor space will he then have all together?

3. A man paid $273.45 in taxes last year. He learns that taxes are to be increased about 8 %. What is the probable amount of his taxes for the coming year?

4. In the month of May, 18 days were cloudy. What per cent of the days were cloudy?

5. In one class, 14 pupils out of 27 were marked excellent. In another class, 4 pupils out of 23 were marked excellent. Find the per cent marked excellent in each class.

6. A man whose salary was $2750 per year was given an increase of $250. What per cent of increase was this?

7. At one time during the summer of 1918, the available supply of wheat in the United States was enough to furnish 12,000,000 bu. per week until harvest time, whereas the normal consumption was 50,000,000 bu. per week. By what per cent should each person have cut his regular consumption of wheat in order to do his part toward making the available supply last?

8. The following table shows the total population in the United States ten years of age and over, and the total number of illiterates. Find the per cent of the illiterates among each class. (Census 1910.)

CLASS	POPULATION	ILLITERATE	PER CENT ILLITERATE
Native whites . .	50,989,341	1,534,272	
Foreign whites . .	12,944,529	1,650,361	
Negroes	7,317,922	2,227,731	

9. A contractor's bid on a certain street construction job was $ 55,785. It was agreed that 10 % of the amount was to be retained by the town for two years as a guarantee fund, to insure compliance with the specifications. How much would the contractor receive upon completion of the job?

10. In 1909, the population and the number of wage earners in manufacturing plants in the five largest cities of the United States were:

CITY	POPULATION	NO. ENGAGED IN MFG. PLANTS	PER CENT ENGAGED IN MFG. PLANTS
New York City .	4,766,883	554,002	
Chicago . . .	2,185,283	293,977	
Philadelphia . .	1,549,008	251,884	
St. Louis . . .	687,029	87,371	
Boston, Mass. .	670,585	69,637	

Complete the table.

11. When gas is made from coal in a gas factory, coke is obtained as a by-product. The amount of coke obtained is about 70 % of the amount of coal used. How many tons of coke will be produced in a factory which consumes 3750 tons of coal per month?

12. The populations of certain states in 1910 and in 1900 are given in the following table. Find the per cent of increase or decrease in the population of 1910 over that of 1900.

State	1910	1900	Per Cent Increase or Decrease
Idaho	325,594	161,772	
Oklahoma . . .	1,657,155	790,391	
Iowa	2,224,771	2,231,853	
Washington . .	1,141,990	518,103	

13. A contractor figures that the actual cost of a certain job is $13,275. He decides to add 10 % for his profits and 5 % for possible but unforeseen extra expenses. How much will his bid be?

14. In 1909, 7,678,578 persons were employed in manufacturing plants in the United States. Of these, 273,265 were proprietors or members of the firms; 790,267 were salaried employees; and 6,615,946 were wage earners. What per cent was each class of the total? Arrange the given data and your results in tabular form.

15. The city population in 1900 was 31,609,645 and in 1910 it was 42,623,383; the rural population in 1900 was 44,384,930 and in 1910 it was 49,348,883. Find the total population in each of these years and the per cent of increase in 1910 over that of 1900 for the total, the city, and the country populations. Arrange the given data and your results in tabular form.

VIII. BUYING AND SELLING GOODS

153. The advertised price at which an article may be bought is called the **List Price.** Sometimes the article is sold for less than the list price; then the difference between the list price and the actual selling price is called a **Discount.**

A discount is usually stated as a certain per cent of the list price.

Thus, a discount of 2 % from the list price means that 2 % of the list price is subtracted from the list price.

154. Why Discounts are Given. — Wholesalers usually publish a general catalogue describing their goods. In this catalogue, a list price for each article is given. This price is usually made great enough so that it is the maximum price that must be charged even if the price of material and labor should increase. In order to inform their customers about the actual prices at any time, wholesalers issue from time to time a list of discounts from these published list prices. In rare cases, especially recently, wholesalers have had to *add* to their list prices in order to arrive at adequate selling prices.

The following is an excerpt from such a price list, giving prices relating to page 184 of a certain jobber's catalogue.

Page 184. *Hand Power Punches and Shears*

Fig. A 1	List plus 60 %.	Fig. A 10	List plus 60 %
Fig. A 3	List plus 60 %.	Fig. A 42	List plus 60 %
Boilermakers' Screw Punch			Discount of 45 %
Extra parts for same			Discount of 50 %
Fig. 510 B, Simplex Shear			Net
Beading Tool, each			$.90

Sometimes special discounts are given to favored customers, either because they are large purchasers or as a special courtesy to them because of personal friendship.

155. Sometimes **More Than One Discount** is offered. In such cases, the first discount is computed on and subtracted from the list price; the second discount is computed on and subtracted from the first remainder; etc.

Example.—Find the net cost of an automobile tire listed at \$ 32.60, with discounts of 20 % and 10 %.

Solution.—1.	The list price of the tire =	\$32.60
2.	20 % or $\frac{1}{5}$ of \$32.60 =	6.52
	The first remainder =	\$26.08
3.	10 % or $\frac{1}{10}$ of \$26.08 =	2.61
	The net price =	\$23.47

156. When goods are bought, an agreement is made about the time when payment is due. This comes under the **Terms of Sale.** The sale may be for "net cash," or for "net cash in 10 days," or 30 days, or 90 days. When payment is not demanded at once, a discount for cash payment is usually offered.

Thus, the terms of a sale may be "2 % for cash in 10 days, or net cash in 30 days." The cash discount is subtracted from the net cost after all other discounts have been subtracted.

157. Below is a sample **Invoice,** modeled after one used by a well-known manufacturing company. Note in it the manner of indicating the terms of payment, as well as the general form of the invoice.

In this invoice, "2 %/15th Prox." means that 2 % will be allowed if payment is made on or before the 15th of the next month.

At the end of the invoice, the 2 % cash discount had been deducted by the clerk at Wharton and Huber's at the time of making payment, for, as indicated by the Receipt, the payment was made before the 15th of the next month.

THE STOWE MANUFACTURING CO.

Brighton, Mass., Feby. 6, 1918

____Cases________lbs.

Sold to Wharton & Huber,

____Pkgs.__________lbs.

Address 243 Johnson St.,

Mayville, Ill.

Shipped by Parcel P.

2/8 with other goods.

Terms: 2%/15th Prox.

QUANTITY	LIST NO.	DESCRIPTION	PRICE	EXTENSION	TOTAL
2½ dz.	J 364	Desk Locks	$6.38 dz.	15 95	
		Less 16⅔, 7½ %			12 29
1¼ dz.	O 426	Bronze padlock	21.70 dz.	27 13	
		Less 10 %			24 42
		Parcel post			17
		Insurance			05
					36 93
		2 % cash discount			73
		Received Payment			36 20
		Mar. 8, 1918			
		Stowe Man'fg. Co.			

NOTE. — The symbol N/30 or N/60 means "net cash in 30 days" or in 60 days respectively.

EXERCISE 102

Find the net price when the rate of discount is:

a. 10 %; *b.* 20 %; *c.* 50 %; *d.* 25 %;
e. 33⅓ %; *f.* 5 %; *g.* 12½ %; *h.* 16⅔ %;

and the list price is:

1. $15.00. **3.** $2.35. **5.** $336.00. **7.** $47 60. **9.** $4.15.
2. $25.00. **4.** $7.86. **6.** $125.00. **8.** $27.50. **10.** $8.72.

Find the net amount of an invoice for:

11. \$ 60 less $33\frac{1}{3}$ % and 10 %.

12. \$ 80 less 10 % and $33\frac{1}{3}$ %.

13. \$ 5.45 less 20 % and 10 %.

14. \$ 15.84 less $12\frac{1}{2}$ % and 25 %.

15. \$ 27.65 less 20 % and 5 %.

16. \$ 18.50 less $66\frac{2}{3}$ % and 5 %.

17. \$ 250 less $12\frac{1}{2}$ % and 10 %.

18. \$ 563.20 less $12\frac{1}{2}$ % and 15 %.

19. \$ 26.75 less 20 %, 10 %, and 5 %.

20. \$ 73.75 less 40 %, 20 %, and 15 %.

Find the net amount of the following invoices:

Ex. No.	Cost at List Price	Trade Discount	Terms	Date of Invoice	Date Paid
21.	\$54.90	10 %	2/10; N/30	June 8, 1917	June 30, 1917
22.	231.00	20 %, 10 %	5/30; N/90	May 10, 1918	June 1, 1918
23.	19.60	25 %, $12\frac{1}{2}$ %	1/10; N/30	Sept. 12, 1918	Sept. 20, 1918
24.	125.85	$33\frac{1}{3}$ %, 5 %	3/30; N/90	Aug. 6, 1918	Sept. 20, 1918
25.	5265.00	$33\frac{1}{3}$ %, 25 %	5/30; N/60	Apr. 13, 1918	May 10, 1918

NOTE. — Other drill examples can be made by inserting other prices for those given in Examples 21-25.

26. A local hardware dealer gives 5 % discount for cash. Find the amount to be paid on the following purchases for cash:

1 garden spade @ \$ 1.85; 1 garden hoe @ \$ .60; 1 garden rake @ \$ 1.10; 1 hand cultivator @ \$ .20; 1 large cultivator @ \$ 1.45.

The following examples were copied from actual invoices received by a hardware dealer. Find the net amount of each invoice.

27. Item A, amounting to $ 116.40, less 87½ %, 25 %, 25 %, 5 %.

Item B, amounting to $ 79.00, less 70 %, 5 %, 20 %, 10 %.

Item C, amounting to $ 10.95, less 85 %, 10 %, 10 %, 5 %.

28. Item A, amounting to $ 28.00, less 85 %, 25 %, 25 %, 5 %.

Item B, amounting to $ 202.30, less 87½ %, 25 %, 25 %, 5 %.

Paid so as to receive also a cash discount of 2 %.

29. Item A, amounting to $ 4.50, less 85 %, 10 %, 10 %.

Item B, amounting to $ 22.50, less 85 %, 25 %, 25 %, 5 %.

Item C, amounting to $ 209.20, less 87½ %, 25 %, 25 %, 10 %.

30. Item A, amounting to $ 42.50, less 62½ %, 5 %, 25 %, 10 %.

Item B, amounting to $ 7.50, less 85 %, 10 %, 10 %, 5 %.

31. Determine the amount to be paid by a firm for two #82 keys listed at 25 ¢ each, less 30 % and 15 %, assuming that they paid the bill in time to secure an additional discount of 2 % for cash.

32. The following approximate prices are quoted on ordinary garden hose by one dealer:

A grade, in 500-ft. rolls, 35 % and 5 % off list price.
Less than 500 ft., 30 % off list price.

B grade, 25 % off list price.
C grade, 50 % off list price.

Determine the approximate cost of:

1500 ft. $\frac{3}{4}''$ A grade hose listed at 25 ¢ per foot.
1200 ft. $\frac{1}{2}''$ A grade hose listed at 19 ¢ per foot.
1000 ft. $\frac{3}{4}''$ B grade hose listed at 19 ¢ per foot.
2500 ft. $\frac{3}{4}''$ C grade hose listed at 15 ¢ per foot.

33. Determine the cost of the following blacksmith's supplies bought for cash.

One #10 heavy duty anvil listed at $ 110.00 but sold at an advance of 15 % over list price.

One #104 5-lb. sledge listed at $ .30 per pound, less 50 %.

One #103 3-lb. double-faced hammer, listed at $ .40 per pound, less 45 %.

One round blacksmith's punch, $\frac{5}{8}''$, weighing 1 lb. 12 oz. at $ .25 per pound.

One round blacksmith's punch, $\frac{3}{4}''$, weighing 2 lb. 4 oz., at $ .25 per pound.

One 10″ blacksmith's pincers, listed at $ .75, less 10 %.

One 12″ blacksmith's pincers, listed at $ .86, less 10 %.

One 14″ blacksmith's pincers, listed at $ 1.00, less 10 %.

Terms: 5 % cash, or N/30 days.

Make out in full an invoice from each of the following sets of data, patterning after the sample on p. 228.

34. The United States Book Co., Chicago, Ill., sold to Hendricks and Son, Danville, Mich., on Aug. 14, 1918: 45 copies Beginners' Algebra @ $ 1; 45 copies First Course in English @ $ 1.25; 12 copies High School Physics @ $ 1.25. Allow a dealer's discount of 20 %. Terms, 2 % in 10 days, net in 30 days. Assume that Hendricks and Son pay the bill on Aug. 21. How much money should they send?

35. The Union Tire Co. of Akron, Ohio, on July 23, 1918, sold to J. B. Friend and Co. of Summit, Ohio: 12 Clincher casings 30 × 3½ @ $20.45; 10 Clincher casings 30 × 3½ @ $17.67; 12 Dunlop casings 32 × 3½ @ $24.38; 6 Dunlop casings 33 × 4 @ $31.25; 6 Dunlop casings 34 × 4 @ $36.85.

Prices subject to discounts of 12½% and 10%. Terms, 5% off for cash before 10th of next month. Bill paid Aug. 5, 1918. Determine amount paid.

36. The Natenas Enameling Co., Chicago, Ill., on Mar. 16, 1918, sold to David Jenkins and Co., of Hastings, Indiana, 4½ doz. #200 stove pans @ $6 per dozen, less 40, 10, 5, and 5%. They made a cash discount of 25¢ on the whole bill to cover the freight. Terms, 2% for cash in 10 days, net in 60 days. The bill was paid on Mar. 27, 1918. Find the amount.

37. The Acme Screw Co., Providence, R. I., sold to the Kerwin Hardware Co. of Montrose, N. Y., on May 12, 1918, the following: 10 gr. each of the following bright flat-head screws, ¾″ #8 @ $.95; ½″ #6 @ $.80; 1½″ #10 @ $1.40; 1½″ #11 @ $1.50; 1½″ #12 @ $1.65; 1¼″ #10 @ $1.30; 1¼″ #12 @ $1.55. These were sold at list less 87½, 5, 25, and 10%. (These were the actual discounts.) Also 10 gr. each of the following blued round-head screws: 2½″ #14 @ $2.90; 1½″ #12 @ $1.65. These were sold at list less 85, 5, 25, and 10%. Also 10 gr. nickel round-head screws 1½″ #8 @ $1.30 less 77½, 5, 25, and 10%.

The terms were 2% for cash in 10 days, net in 30 days. The bill was paid on May 25, 1918. Find the amount.

38. The Manville Manufacturing Co., Chicago, Ill., sold to A. B. Stephens of Kewauna, Ohio, on Aug. 14, 1918, one 5 H.P. stationary gasoline engine, listed at $178, with $10 extra for a kerosene attachment. It was sold at list less 30%. Terms, 3% for cash in 10 days, net in 30 days. The bill was paid on Aug. 29, 1918. Find the amount.

39. The Winston Garage bought the following supplies from the Central Auto Supply Co. of Cleveland, Ohio, on Feb. 24, 1918: ½ doz. automobile jacks #6, listed at $2, less 25 %; 3 doz. #20, size 93 grease cups @ $.40 each, less 75 %; ½ doz. Ford Tool Sets at $3.73 each, less 20 % and 10 %. Terms, cash in advance.

40. J. R. DeLancey, a street contractor, ordered from the Spencer, Baldwin Co. of Detroit, Mich., on June 5, 1918, the following tools: 2 doz. Fig. 57 C, #4, D handle shovels @ $19.50 per dz., less 10 %; 1 doz. Fig. 50 B, #3, D handle shovels @ $19, less 5 %; 2 doz. Fig. 85, 8 lb. clay picks @ $16, less 35 %; 3 doz. Fig. 95 pick handles @ $11.50, less 50 %; 6 Fig. 52 wheelbarrows @ $66 per dozen, less 5 %; 3 scrapers, size 2, @ $8.25, plus 65 %. Terms, 5 % off for cash in 30 days, net in 90 days. Bill paid July 1, 1918. Find the amount.

41. B. M. Chandler and Co. publish the following changes in prices from those published in their Catalogue #27.

Polished Brass Row Locks, Style 105, list plus 80 %.
Polished Brass Row Locks, Style 103, list plus 40 %.
Spruce Boat Oars, per foot 16¢.
Terms, 2 %/10 days, N/30 days.

Determine the cost to the Lake View Boat Co. of the following goods, bought on April 15, 1918, and paid for on April 22, 1918.

6 pairs #2, Style #105, polished brass row locks, listed at $2; 12 pairs #2, Style #103, polished brass row locks, listed at $1.75; 6 pairs 7 ft. spruce oars; 6 pairs 7½ ft. spruce oars; 6 pairs 8 ft. spruce oars.

42. The Rainey Pipe Co. of Marksville, Penn., sold to the Hastings Heating Co., of Milton, Ohio, on July 2, 1917, the following:

2505 ft. ½″ iron pipe @ 8½¢ per foot, less 45.48 %;
6006 ft. ¾″ iron pipe @ 11½¢ per foot;
3014 ft. 1″ iron pipe @ 17¢ per foot;
3009 ft. 1¼″ iron pipe @ 23¢ per foot;
2007 ft. 1½″ iron pipe @ 27½¢ per foot;
2004 ft. 2″ iron pipe @ 37¢ per foot;
1512 ft. 2½″ iron pipe @ 58½¢ per foot;
810 ft. 3″ iron pipe @ 76½¢ per foot;
These all subject to a discount of 48.48 %.
408 ft. 4″ iron pipe @ $1.09 per foot, less 44.48 %.

Terms 5 % for cash in 30 days, net in 90 days.

Bill paid July 31, 1918. Find the amount.

(This is an actual invoice of a heating company.)

Profits and Losses

158. **Profits** and **Losses** in sales of merchandise or in other business transactions are usually computed and described in per cents.

The **Net Cost** of an article is the actual sum paid for it. If other expenses are incurred in getting possession of the article, such as freight or delivery charges, fees to a purchasing agent, or if expenses are incurred in disposing of the article, such as expense for rent, heat, light, clerk hire, etc., — all these items must be added to the net cost to give the **Gross Cost.**

If an article is sold for more than its cost, then

The Selling Price — The Gross Cost = The Profit.

If an article is sold for less than it cost, then

The Gross Cost — The Selling Price = The Loss.

Profits and losses are usually computed as a certain per cent of the cost. The cost is the base, and the profit or loss is the percentage.

EXERCISE 103

Find the selling price of an article if:

Ex. No.	Cost Was	Gain Was	Ex. No.	Cost Was	Loss Was
1.	$2200	10 %	**6.**	$625	20 %
2.	1650	20 %	**7.**	1040	12½ %
3.	750	5 %	**8.**	6400	50 %
4.	1400	6 %	**9.**	3225	10 %
5.	2100	33⅓ %	**10.**	4500	25 %

Find the rate per cent of gain or loss if:

Ex. No.	Cost Was	Gain Was	Ex. No.	Cost Was	Loss Was
11.	$10,000	$2000.00	**16.**	$625	$125
12.	4500	450.00	**17.**	260	195
13.	2350	352.50	**18.**	1290	430
14.	6800	1700.00	**19.**	3050	1830
15.	720	90.00	**20.**	2200	200

Find the rate of gain or loss if:

Ex. No.	Cost Was	Selling Price Was	Ex. No.	Cost Was	Selling Price Was
21.	$125.00	$150.00	**26.**	$32.50	$35.75
22.	64.00	72.00	**27.**	450.00	375.00
23.	150.00	135.00	**28.**	8.00	11.00
24.	275.00	200.00	**29.**	1275.00	1500.00
25.	1050.00	1200.00	**30.**	2650.00	2500.00

31. A dealer purchased goods for $ 245 and sold them at a loss of 33⅓ %. What was his loss?

32. A tradesman sold at a loss of 15 % goods that cost $ 586.20. What did he receive?

33. A dealer bought 25 barrels of flour at $10.40 per barrel. At what price must he sell them per barrel to make a gross profit of 18 %?

34. A merchant sold some goods for $416.52. If it is assumed that he marks his goods so as to make a gross profit of 17 %, what did the goods cost him?

35. At what price must a lot selling for $500 be bought so as to make 15 % on the cost?

36. A wholesale dealer bought 250 bu. of potatoes at $.93 per bushel. The freight was $8.50, and the cartage was $5.25. At what price must he sell them per bushel so as to gain 15 % on the gross cost?

37. What is the rate per cent of gain on a lot bought at $420 and sold for $595, if the seller had to pay an agent a commission of 5 % of the selling price?

38. What is the rate of gain on peaches bought for $3.75 per bushel, and sold for $.25 per quart?

39. A certain automobile that sold for $1265 in the fall of 1917 was selling for $1490 in the summer of 1918. If a dealer sold during the summer of 1918 an automobile that he had in stock in the fall of 1917, what per cent of profit was there for him in the transaction?

40. A man bought a lot costing $525 and built upon it a house costing $2750. He had to pay for special improvements on the street $235. He sold the house and lot for $3775. What per cent of profit did he make on the investment, assuming that the whole transaction occurred during one year?

41. A man rented an acre of ground for $8. He paid $6 to have it plowed, harrowed, and planted to corn. He paid $1.50 for the seed corn. It cost him $4 to have the corn cultivated and $5 to have the corn harvested. He got 45 bu. of corn worth $.75 per bushel. What per cent of profit did he make on his investment?

Marking Goods

159. Merchants often employ a secret system for marking the cost and selling price of their goods. Often the letters of some word are used to represent the digits with which numbers are written.

Thus, the letters of the word *manuscript*, adjoining, might be used to represent the digits below them respectively.

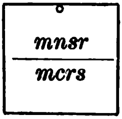
m	a	n	u	s	c	r	i	p	t
1	2	3	4	5	6	7	8	9	0

Then, to represent $ 269, the letters *acp* would be written on the price tag.

To avoid the repetition of one letter where a price like 44 ¢ is to be represented, a repeater such as the letter *z* is used.

Thus, 44 ¢ would be represented by *uz*.

A typical price tag representing a cost of $ 13.57 and a selling price of $ 16.75 is shown at the right.



160. One problem of the merchant is the determination of a selling price so as to gain a reasonable profit on the cost.

Example. — A merchant wished to mark an article costing $ 1.28 at an advance of 25 %. Determine the selling price, and represent it and the cost by the letters of the word *manuscript*.

Solution. — 1. 25 % of $ 1.28 = $ .32.

2. ∴ the selling price = $ 1.28 + $ .32 = $ 1.60.

3. The cost will be represented by *mai* and the selling price by *mct*.

EXERCISE 104

Determine the selling price and represent it and the cost in each of the following examples by the letters of the word *manuscript*, if the cost is:

1. \$.86, to be sold at a profit of: *a.* 15 %; *b.* 20 %.
2. \$1.13, to be sold at a profit of: *a.* 20 %; *b.* 12½ %.
3. \$7.83, to be sold at a profit of: *a.* 25 %; *b.* 30 %.
4. \$.68, to be sold at a profit of: *a.* 35 %; *b.* 25 %.
5. \$1.42, to be sold at a profit of: *a.* 33⅓ %; *b.* 20 %.
6. \$5.35, to be sold at a profit of: *a.* 10 %; *b.* 12½ %.
7. \$12.40, to be sold at a profit of: *a.* 16⅔ %; *b.* 15 %.
8. \$26.25, to be sold at a profit of: *a.* 12½ %; *b.* 10 %.
9. \$32.15, to be sold at a profit of: *a.* 20 %; *b.* 25 %.
10. \$65.35, to be sold at a profit of: *a.* 40 %; *b.* 37½ %.
11. A merchant bought:

½ dz. #200 Aluminum Teapots	@ \$3.00 each, less 15 %.
½ dz. #400 Aluminum Teapots	@ \$3.50 each, less 15 %.
¼ dz. #173 Chafing Dishes	@ \$8.50 each, less 25 %.

a. Find the net cost of these articles, if purchased and paid for so as to receive a discount of 2 % for cash.

b. Determine the selling price for each article, if sold so as to make a gross profit of 40 % over the net cost.

12. A merchant bought the following goods:

1 doz. horse blankets, #3721, size 84 × 90 @ \$8.00, less 45 %.
½ doz. horse blankets, #3945, size 84 × 90, @ \$9.50, less 40 %.
1 doz. rawhide halters, #592, 1½ in., @ \$45.00 per dozen, less 30 %.

a. Determine the total net cost of these articles.

b. Determine the selling price for each single article so as to gain 20 % on the net cost.

Commission and Brokerage

161. Firms and individuals often buy or sell goods or conduct other business transactions through agents. Such agents are also called **Salesmen, Commission Merchants,** or **Brokers.**

A **Commission Merchant** buys and sells farm, garden, and dairy produce, and groceries.

A **Broker** arranges business bargains and contracts between parties. There are real estate brokers, stock brokers, bond brokers, cotton brokers, etc.

The person or persons for whom an agent acts is called his **Principal.**

The fee charged by an agent in general is called a **Commission.** That charged by a broker is called **Brokerage.** In many cases the fee is a certain per cent of the money value of the property involved in the transaction.

162. Goods shipped by a commission merchant to his principal or by a principal to his agent is called a **Shipment**; goods received by either from the other is called a **Consignment.**

A commission merchant, after selling goods for a principal, sends him an **Account Sales**; after buying goods for a principal, he sends him an **Account Purchases.**

The **Gross Proceeds** of a sale is the sum received from the purchaser. The **Net Proceeds** is the sum remaining for the principal after all charges have been deducted.

In case a commission merchant purchases goods for a principal, the **Prime Cost** is the actual sum paid for the goods. The **Gross Cost** is the sum of the prime cost and all other expenses involved in making the purchase and acquiring possession of the goods, such as freight, cartage, etc.

163. The *commission fee,* when expressed as a per cent, *is based upon the gross proceeds of a sale or on the prime cost of a purchase.*

An Account Sales

Milwaukee, Wis., March 26, 1918

M. B. Caldwell & Company
Commission Merchants

For the account of Henry Jordan, Marshland, Wis.

Date Rec'd	Car	Contents	Date Sold	Charges		Quantity	Grade	Price	Amount	
3/12	37592	Rye	3/12			450.89 bu.	3	$2.83	1276	02
		25250 lb.								
		Wheat				915.33 bu.	2 R.S.			
		54920 lb.				45.76	5 %			
						869.57		2.15	1869	57
									3145	59
Freight				$70	18					
Switching										
Weighing and Inspection .				2	00					
Commission (1½ ¢ per bu.)				19	80				91	98
								Net proceeds	3053	61

Note. — Observe that the weight of rye and wheat are given in pounds and are converted into bushels. See Tables for the weights of each per bushel.

Also observe that the wheat suffers a reduction because of impurities. 2 R. S. means # 2 Red Spring Wheat.

EXERCISE 105

1. What are the net proceeds to a farmer whose agent sells for him 1975 bu. of corn, at $ 1.33⅓ per bushel, charging 2 % for commission and $ 47.50 for freight and switching charges?

2. An agent sold 3500 yd. of silk at $2.18 per yard. His charges were: commission, 2½%; freight, $135.00. What were the net proceeds?

3. What were the net proceeds on the sale of 375 bbl. of flour at $10.40 per barrel, if the commission was 3½%, and charges for freight and storage were $.67 per barrel?

4. An attorney charged 10% for collecting a debt of $945.00. He submitted also incidental charges of $15.00. What were the proceeds to his client?

5. What are the net proceeds of a transaction if the gross proceeds are $786.45, the commission rate being 2½%, and other charges $22.65?

6. What rate of commission is an agent charging if the gross proceeds of a transaction are $387.50 and the commission is $9.69?

7. A college student sold during eight weeks of his summer vacation 730 copies of a book at $1.50 each, receiving a commission of 35%. His expenses averaged $15.00 per week. What did he make during his trip?

8. A man bought 1500 bu. of oats at $.69 per bushel, paying a commission fee of 1%, and other charges of $15.00. A few days later, he sold them for $.73½ per bushel, again paying a commission of 1%. What did he gain or lose by the transaction?

9. The gross proceeds of a transaction are $2748.00. There are charges of $12.50. The net proceeds are $2598.10. What rate of commission (on gross proceeds) was charged?

10. An attorney charges $50.00 for his services in collecting a debt of $725.00. What rate of commission did he receive? (Carry result to only one decimal place.)

11. Rule and fill in an *account sales* from the following data: William Parker and Son, Commission Merchants, Chicago, Ill., under date of Oct. 15, 1918, notified W. J. Brown, Park Ridge, Ill., that they had received on Oct. 11, 1918, a car, No. 44620, of barley, weighing 47,520 lb.; that they sold it on Oct. 13, 1918, as grade 4 barley, at $1.35 per bushel. Charges: freight, $39.59; weighing, $.55; inspection, $1.20; commission, 1½¢ per bushel.

12. Rule and fill in an account sales from the following data:

J. C. Benedict, St. Paul, Minn., on Sept. 28, 1918, notified Henry Sanstrom, Big Falls, Minn., of the receipt on Sept. 22 of car No. 60538, containing 50,350 lb. of wheat, containing 5% impurities; that they sold it on Sept. 22 at $2.18 per bushel. Charges: freight, $39.45; weighing, $.80; inspection, $1.35; commission, 1½¢ per bushel.

13. James Barton, Des Moines, Iowa, sold for Oscar Sorenson, Fairfield, Iowa, 27½ T. of hay at $23.75 per ton. He charged a commission of 3%. He paid freight of $42.85. Render an account sales, under date of Nov. 22, 1918.

14. Render to the Midvale Market Co., Midvale, Penn., an account purchase of the following articles, bought for them on June 21, 1918, by M. L. Selden, Fruit and Produce Merchant of Pittsburg, Penn.:

3 tubs containing 50 lb. each of butter, at $.47 per pound; 10 cases, containing 30 doz. each of eggs, at $.34 per dozen; 5 bunches of bananas at $3.25 per bunch; 2 boxes oranges at $7.75 per box; 10 crates of peaches, at $2.50 per crate; 2 crates of pineapple at $4.75 per crate; 5 boxes of Michigan celery, at $2.75 per box; 500 lb. of new potatoes at $2.85 per hundred. Charges: commission, 5%; cartage, $4.25; freight, $6.35.

15. R. L. Burton and Co., Chicago, Ill., sold for the Union Grove Creamery, Union Grove, Ill., the following consignments of butter during one month, charging a commission of 2 %. Make out an account sales, under date of June 30, 1918.

June 8, 15 tubs, averaging 50 lb., at $ .43 per pound; freight, $ 1.62.

June 18, 20 tubs, averaging 50 lb., at $ .45 per pound; freight, $ 2.28.

June 24, 13 tubs, averaging 50 lb., at $ .44 per pound; freight, $ 1.52.

June 29, 12 tubs, averaging 50 lb., at $ .46 per pound; freight, $ 1.33.

Cartage on each consignment, $ .10 per tub.

16. A broker buys for his principal, securities valued at $ 9500.00, charging brokerage of $\frac{1}{8}$ %. What was the brokerage?

17. The Central Realty Company, Melrose, Conn., sold for Hannah Putnam a farm of 84 A., at $ 72.50 per acre. The charges were: for bringing down to date the abstract of title, $ 3.75; incidental expenses, $ 17.75; commission, 5 %. What were the net proceeds?

18. Barker and Davis, Auctioneers, charge a commission of 5 % for their services. What are the net proceeds to the owner of a farm if Barker and Davis sell at auction their farm personal property for $ 3486.38?

The following problems, while not practical, sometimes appear in examinations.

19. An agent received $ 2500 to invest, out of which he must receive enough to pay him his own commission at $2\frac{1}{4}$ %. How much can he invest and what will be his commission?

Suggestion. Since the commission is computed on the prime cost, then $2500 is $102\frac{1}{4}$ % of the prime cost of the goods purchased.

20. A merchant received $ 2194.50, with which to purchase flour at $ 9.37½ per barrel, and pay him his own commission at the rate of 4½ %. How many barrels of flour should he purchase?

21. An agent charges a commission of 4 % for purchasing goods. If his commission was $ 352.60, what was the prime cost of the goods purchased?

22. What sum must be sent to a broker to enable him to buy 15 T. of hay at $ 23 per ton, and pay his commission at the rate of 3½ % ?

23. A commission merchant purchases for a principal goods at a prime cost of $ 2384.65. His fee is at the rate of 2½ %; other charges are $ 73.42. What was the gross cost of the shipment?

24. An agent sold for a grain dealer a carload of wheat, containing 875 bu. of wheat. He sent the dealer an account sales, showing gross proceeds of $ 1006.25, and net proceeds of $ 965.58. If the miscellaneous charges were $ 27.54, what was the charge per bushel for selling the wheat?

25. An agent sold goods amounting to $ 9220. He submitted a bill for $ 404.25, including $ 58.50 for freight and storage, and the balance for his commission. What was his rate of commission?

IX. INTEREST

164. **Interest** is money paid for the temporary use of some other person's money. It is like rent paid for the temporary use of a house, farm, boat, or any other piece of property belonging to another person.

The **Principal** is the money borrowed, on which interest is paid. The **Amount** is the sum of the principal and interest.

Interest is usually described and computed as a certain rate per cent of the principal for one year. This rate per cent is the **Rate of Interest.** It will be understood that the rate is for one year. The year is usually regarded as consisting of 12 months of 30 days each or 360 days.

165. **Simple Interest** is interest computed upon the principal alone.

166. **General Method of Computing Interest at Any Rate.**

Example 1. — Find the interest on $2000 at 5% for 3 years.

Solution. — 1. The interest for 1 yr. = .05 × $2000
= $100.

2. ∴ the interest for 3 yr. = 3 × $100
= $300.

Example 2. — Find the interest on $850 for 2 yr. and 7 mo. at 7%.

Solution. — 1. The interest for 1 yr. = .07 × $850
= $59.50.

2. ∴ the interest for 2 yr. = 2 × $59.50
= $119.00.

3. **The interest for 1 mo. = $\frac{1}{12}$ of $59.50 = $4.958.**

4. **∴ the interest for 7 mo. = 7 × $4.958 = $34.71.**

5. **∴ the total interest = $119.00 + $34.71 = $153.71.**

Example 3. — Find the amount of $775 for 7 mo. and 11 days at 5%.

Solution. — 1. The interest for 1 yr. = .05 × $775 = $38.75.

2. ∴ the interest for 1 mo. = $\frac{1}{12}$ of $38.75 = $3.229.

3. ∴ the interest for 7 mo. = 7 × $3.229 = $22.603, or $22.60.

4. Since 11 days = $\frac{11}{30}$ mo., the interest for 11 days = $\frac{11}{30}$ × $3.229

$$= \frac{\$35.519}{30} = \$1.183^{+} \text{ or } \$1.18.$$

5. ∴ the total interest is $22.60 + $1.18 = $23.78.

6. ∴ the amount = $775 + $23.78 = $798.78.

Rule. — To find the interest on a given principal for a given time at a given rate, multiply the principal by the rate, and that product by the time expressed in years.

Note 1. — In practice, one uses the method which best suits the problem.

a. If the time is a number of years, multiply the interest for one year by the number of years.

b. If the time involves months, find the interest for one month by dividing the annual interest by 12, and then multiply by the number of months.

However, for such a time as 8 mo., take $\frac{8}{12}$ or $\frac{2}{3}$ of the annual interest.

Add the interest for the number of months to the interest for the given number of years.

c. If the time involves days, consider 30 days as forming a month. Divide the monthly interest by 30, or the annual interest by 360, and multiply by the number of days. Add the result to the interest for the given number of years and months.

However, if the time is such as 12 days, note that 12 days is $\frac{12}{30}$ or $\frac{2}{5}$ of a month, hence the interest for 12 days is $\frac{2}{5}$ the monthly interest. Similarly, 12 days = $\frac{12}{360}$ of a year, and hence the interest for 12 days is $\frac{12}{360}$ or $\frac{1}{30}$ of the annual interest.

Note 2. — Carry out computations only to mills, and then increase the number of cents by 1 if the number of mills is 5 or more than 5. Thus, $29.637 will be called $29.64. Otherwise, ignore the number of mills. Thus, $29.633 will be called $29.63.

EXERCISE 106

Find the interest and the amount:

Ex. No.	Princ.	Rate	Time	Ex. No.	Princ.	Rate	Time
1.	$2500	4 %	3 yr. 9 mo.	9.	$1500	7 %	3 mo. 10 da.
2.	8300	5 %	2 yr. 4 mo.	10.	1800	8 %	5 mo. 9 da.
3.	950	6 %	6 yr. 2 mo.	11.	860	5½ %	2 yr. 4 mo. 10 da.
4.	675	8 %	2 yr. 3 mo.	12.	975	4½ %	3 yr. 2 mo. 20 da.
5.	475	5 %	1 yr. 5 mo.	13.	1650	3½ %	4 yr. 3 mo. 9 da.
6.	250	5 %	8 mo. 15 da.	14.	375	4¾ %	2 yr. 5 mo. 21 da.
7.	1200	6 %	9 mo. 18 da.	15.	1350	6½ %	1 yr. 7 mo. 22 da.
8.	750	4 %	7 mo. 12 da.	16.	3275	4¼ %	2 yr. 5 mo. 18 da.

Note. — If more drill examples are wanted, find the interest on each of these sums at the rate and for the time given in Ex. 1, or in Ex. 2, etc.

167. Six per cent is a commonly used rate of interest. This rate, combined with the assumed thirty-day month, leads to the use of the **Six Per Cent Method** of computing interest.

The interest on any principal for a year at 6 % is .06 of the principal. Hence, the interest for two months is .01 of the principal and for one month is ½ of .01 of the principal.

Rule. — To find the interest at 6 % for any number of months:

(1) Find .01 of the principal by pointing off two places.

(2) Then divide by 2 and multiply by the number of months.

Note. — When the number of months is even, multiply the result of Step 1 by ½ the number of months.

Thus, the interest on $175.25 at 6 % for 10 months is

5 × $1.7525 = $8.7625 = $8.76.

And the interest on $175.25 at 6 % for 7 months is

7 × ½ of $1.7525 = 7 × $.87625 = $6.13375 = $6.13.

Since 6 days is $\frac{1}{5}$ of the 30-day month, the interest for 6 days is $\frac{1}{5}$ of $\frac{1}{2}$ of .01 or .001 of the principal.

Rule. — To find the interest at 6 % for any number of days:

(1) Find .001 of the principal by pointing off three places.

(2) Then divide by 6 and multiply by the number of days.

NOTE. — If the number of days is divisible by 6, multiply the result of Step 1 by $\frac{1}{6}$ the number of days.

Thus, the interest on \$350 at 6 % for 18 days is $\frac{18}{6}$ or $3 \times$ \$.350 = \$1.05.

And the interest on \$350 at 6 % for 13 days is $13 \times \frac{1}{6} \times$ \$.350 = $13 \times$ \$.0583 = \$.76.

EXERCISE 107

Find the interest at 6 % on each of the following sums of money for:

a. 8 mo.; *b.* 10 mo.; *c.* 2 mo.; *d.* 5 mo.; *e.* 7 mo.; *f.* 3 mo.

1. \$360
2. \$140
3. \$268
4. \$136
5. \$185
6. \$940
7. \$425.50
8. \$563.80
9. \$147.65
10. \$1632.50
11. \$2143.25
12. \$5063.75

13–24. Find the interest at 6 % on each of the foregoing sums for:

a. 12 da.
b. 18 da.
c. 42 da.
d. 54 da.
e. 14 da.
f. 21 da.
g. 15 da.
h. 9 da.

25. Find the interest at 6 % for 1 yr. 7 mo. and 20 da. on \$2755.

Solution. —

1. The interest for one year = \$165.30
2. The interest for 7 mo. = 96.43 $(= 7 \times \frac{1}{2} \times 27.55)$
3. The interest for 20 da. = 9.18 $(= 20 \times \frac{1}{6} \times 2.755)$
4. ∴ the total interest = \$270.91

Find the interest at 6 % on:

26. $641 for 5 yr. 10 mo.

27. $71.25 for 2 yr. 5 mo.

28. $833 for 3 yr. 2 mo.

29. $10.25 for 1 yr. 4 mo.

30. $2575 for 4 yr. 7 mo.

31. $146.25 for 1 yr. 3 mo. 12 da.

32. $275.50 for 3 yr. 4 mo. 18 da.

33. $735.75 for 5 yr. 2 mo. 27 da.

34. $227.60 for 1 yr. 14 da.

35. $2450 for 1 yr. 7 mo. 11 da.

168. Finding the Interest at any Rate by the Six Per Cent Method.

Since 5 % is $\frac{5}{6}$ of 6 %, then the interest at 5 % is $\frac{5}{6}$ of the interest at 6 %. Hence, to find the interest at 5 %, multiply the interest at 6 % by $\frac{5}{6}$ or subtract from the interest at 6 % one sixth of itself.

$4\frac{1}{2}$ % is three fourths of 6 %. Hence, to find the interest at $4\frac{1}{2}$ %, subtract from the interest at 6 % one fourth of itself.

Example. — Find the interest on $1250 for 3 mo. and 15 da. at: *a.* 5 %; *b.* 4 %; *c.* $4\frac{1}{2}$ %.

Solution. —

1. The interest on $1250 at 6 % for 3 mo. $= \frac{3}{\cancel{2}} \times \$\overset{6.25}{\cancel{12.50}} = \$18.75$.

2. The interest on $1250 at 6 % for 15 da. $= \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{6}}} \times \$\overset{.625}{\cancel{1.25}} = \$3.125$

3. ∴ the interest on $1250 at 6 % for 3 mo. 15 da. = $21.875.

4. *a.* The interest at 5 % $= \frac{5}{\cancel{6}} \times \$\overset{3.645+}{\cancel{21.875}}$

$= \$18.225 = \$18.23$.

b. The interest at 4 % $= \frac{2}{3}$ the interest at 6 %

$= \frac{2}{\cancel{3}} \times \$\overset{7.291+}{\cancel{21.875}} = \$14.582 = \$14.58$.

c. The interest at $4\frac{1}{2}$ % $= \frac{3}{4}$ the interest at 6 %

$= \frac{3}{\cancel{4}} \times \$\overset{5.468+}{\cancel{21.875}} = \$16.404 = \$16.40$.

EXERCISE 108

Find the interest by the 6 % method:

Ex. No.	On	For	At			
			a	b	c	d
1.	$530	8 mo.	5 %	7 %	8 %	4 %
2.	450	5 mo.	9 %	3 %	2 %	1 %
3.	275	4 mo. 18 da.	5 %	8 %	4½ %	7½ %
4.	1250	3 mo. 15 da.	3 %	4 %	9 %	5 %
5.	252.50	1 yr. 5 mo.	4½ %	1½ %	1^{1} %	8 %
6.	250	75 da.	5 %	7 %	9 %	1½ %
7.	375	60 da.	3 %	4 %	4½ %	8 %
8.	750	35 da.	7 %	9 %	2 %	4½ %
9.	1500	40 da.	5 %	4 %	4½ %	1½ %
10.	500	85 da.	5¼ %	2½ %	3½ %	4¼ %

169. When money is loaned, the interest is usually payable semi-annually or quarterly.

Often men must borrow a sum of money for a few days, and do so at such rates as 3⅝ %, 5¼ %, etc.

Computations such as those in the following examples result.

EXERCISE 109

Find the semi-annual interest on:

1. $750: *a.* at 6 % *b.* at 5 % *c.* at 4½ %.
2. $2500: *a.* at 5½ % *b.* at 6 % *c.* at 4¼ %.
3. $4000: *a.* at 6½ % *b.* at 5½ % *c.* at 4 %.
4. $500: *a.* at 4½ % *b.* at 5½ % *c.* at 6 %.
5. $825: *a.* at 4¾ % *b.* at 5½ % *c.* at 6 %.

Find the interest on:

6. $2300 at 5½ %: *a.* for 10 da. *b.* for 12 da. *c.* 15 da.
7. $1500 at 6½ %: *a.* for 9 da. *b.* for 13 da. *c.* 17 da.

8. $675 at 5¼ %: *a.* for 11 da. *b.* for 12 da. *c.* 8 da.

9. $3750 at 4½ %: *a.* for 16 da. *b.* for 10 da. *c.* 20 da.

10. $15000 at 5⅞ %: *a.* for 4 da. *b.* for 9 da. *c.* 15 da.

170. If the time is less than one year, the number of months and days may be computed as in the following example.

Example. — A man bought $175 worth of coal on July 17, 1918, agreeing to pay 6 % interest on this sum until the bill was paid. He paid it on Nov. 10, 1918. How much interest did he have to pay?

Solution. — 1. From July 17 to Oct. 17 is three months; from Oct. 17 to Oct. 31 is 14 days; from Oct. 31 to Nov. 10 is 10 days. Hence the total time is 3 months 24 days.

2. The interest on $175 at 6% for 3 mo. = $2.63. (§ 167)
3. The interest on $175 at 6% for 24 days = $ .70.
4. ∴ the total interest = $3.33.

If the time is more than one year, find first the number of years, and then the number of months and days as in the following example.

Example. — Find the interest at 6 % on $1150 from June 18, 1917, to April 11, 1919.

Solution. — 1. From June 18, 1917, to June 18, 1918, is 1 year.
2. From June 18, 1918, to March 18, 1919, is 9 months.
3. From March 18, 1919, to April 11, 1919, is 24 days.
4. Hence, the time is 1 year, 9 months, 24 days.
5. The interest on $1150 for 1 yr. at 6% is $ 69.00.
6. The interest on $1150 for 9 mo. at 6% is $ 51.75.
7. The interest on $1150 for 24 da. at 6% is $ 4.60.
8. ∴ the total interest is $125.35.

EXERCISE 110

Find the interest:

Ex. No.	On	At	From	To
1.	$685	6 %	June 13, 1917	Jan. 25, 1918
2.	4175	6 %	April 10, 1918	May 12, 1918
3.	5160	5 %	Feb. 21, 1918	Mar. 14, 1919
4.	950	4 %	Nov. 15, 1918	July 26, 1919
5.	498	4½ %	July 14, 1917	May 21, 1919
6.	475	5 %	Nov. 29, 1918	Sept. 9, 1919
7.	550	4¼ %	Jan. 26, 1919	Mar. 12, 1919
8.	8780	3 %	Apr. 4, 1918	Nov. 17, 1919
9.	225	5½ %	Dec. 22, 1917	Feb. 3, 1919
10.	175	4½ %	Apr. 6, 1919	May 12, 1920

171. Since the calendar year consists of 365 days, there is a slight inaccuracy in basing interest computations upon an assumed year of 360 days.

Thus, the interest for 40 days should be $\frac{40}{365}$ of the annual interest instead of $\frac{40}{360}$. The interest therefore should be a little less than that charged by the computations based upon a 360-day year.

Interest calculated by considering the exact number of days in the given time and using 365 days for the year is called **Exact Interest.** It is used by the United States Government in computing interest on its securities. It is also used by some business houses.

Example. — Find the exact interest at 6% on $2500 from June 12, 1916, to Sept. 8, 1918.

Solution. — 1. From June 12, 1916, to June 12, 1918, is 2 years.
2. From June 12, 1918, to June 30, 1918, is 18 days.
From June 30, 1918, to July 31, 1918, is 31 days.
From July 31, 1918, to Aug. 31, 1918, is 31 days.
From Aug. 31, 1918, to Sept. 8, 1918, is 8 days.

3. ∴ the total time is 2 years and 88 days.
4. The interest on \$2500 at 6 % for 2 years is \$300.
5. The interest on \$2500 at 6 % for 1 year is \$150.
∴ the interest for 88 days is $\frac{88}{365} \times \$150 = \$36.16$.
6. ∴ the total interest = \$300 + \$36.16 = \$336.16.

EXERCISE 111

In the first three examples, find the interest by the six per cent method and also the exact interest; in the remaining examples find only the exact interest.

Find the exact interest on:

1. \$ 925 at 6 % for 3 yr. 110 da.
2. \$ 250 at 5 % for 2 yr. 200 da.
3. \$ 550 at 4 % for 75 da.
4. \$ 7150 at 5 % for 92 da.
5. \$ 1230 at 6 % for 1 yr. 15 da.

Find the exact interest:

Ex. No.	On	At	From	To
6.	\$650	6 %	May 5, 1917	Aug. 12, 1917
7.	225	5 %	March 10, 1918	Sept. 22, 1918
8.	1500	4 %	July 15, 1917	June 10, 1918
9.	25,000	4½ %	Aug. 18, 1918	Dec. 2, 1919
10.	12,500	3 %	Sept. 16, 1919	Dec. 30, 1919

172. Finding the Rate when the Principal, Interest and Time are Given.

Example. — What rate of interest has a man received who gets a return of \$ 41.76 on an investment of \$ 384 for 2 yr. and 5 mo.?

Solution. — 1. By the 6 % method, the interest at 1 % on \$384 for 2 yr. 5 mo. is \$9.28.

2. ∴ the required rate is the number of times \$9.28 is contained in \$41.76 or 4½; *i.e.*, the rate is 4½ %.

Rule. — To find the rate when the principal or amount, interest, and time are given:

1. Find the interest at 1 % on the principal at the given rate, for the given time.

2. Divide the given interest by the result of Step 1, thus obtaining the rate per cent.

NOTE. — If the interest and the amount are given, first find the principal by subtracting the interest from the amount.

EXERCISE 112

At what rate per cent of interest will:

1. $ 480 earn $ 56.00 in 2 yr. and 11 mo.?
2. $ 6450 earn $ 123.65 in 4 mo. 18 da.?
3. $ 3780 earn $ 11.76 in 28 da.?
4. $ 750 earn $ 34.38 in 10 mo.?
5. $ 225 earn $ 29.25 in 4 yr. 4 mo.?
6. $ 8500 amount to $ 8780.50 in 8 mo. 24 da.?
7. $ 720 amount to $ 734.70 in 7 mo.?
8. $ 192 amount to $ 202.56 in 1 yr. 10 mo.?

9. What rate of interest has a man earned on an investment of $ 750 if he gets back $ 875 at the end of 15 mo.?

10. A man can buy his coal for $ 105 cash, or $ 110 if he pays for it in 60 days' time. What rate of interest does he pay if he chooses to pay in 60 days' time?

11. A home was purchased for $ 5100, and was sold two years later for $ 5800. The owner reckoned his rent as worth $ 300 per year to him. He paid out $ 40.00 per year for taxes, $ 35.00 per year for insurance and $ 65.00 per year for repairs. What per cent of income on his investment did he make?

12. A farm has a total value of $ 18,500. The net income for the year is $ 1250. What rate of interest is that on the investment?

13. A man borrows $125 and at the end of 60 days returns to the lender $130. What rate of interest has he paid on the borrowed money?

14. A man lends $2500 to a company. During the year he receives $160 interest. What rate of interest is his money earning?

15. From another investment of $725 he receives during the year an income of $60. What rate of interest is this?

173. Finding the Time when the Principal or Amount, Interest, and Rate are Given.

Example. — How long must $1000 be on interest at 6 % to double itself?

Solution. — 1. Since the money doubles itself, it earns $1000 in interest.

2. The interest on the given principal, $1000, for 1 yr. at 6 % is $60.00.

3. Therefore it will take as many years as $60 is contained times in $1000.

4. This is 16 years, or 16 yr. and 8 mo.

Rule. — To find the time when the principal, rate, and interest are known:

1. Find the interest on the principal at the given rate for one year.

2. Divide the given interest by the result of Step 1, thus obtaining the number of years in the required time.

3. Express the time in years, months, and days.

NOTE. — By carrying the division out to three decimal places, sufficiently accurate results are obtained. Thus, if the time should happen to be 2.736 yr., the years are 2.

$$.736 \text{ yr.} = .736 \times 12 \text{ mo.} = 8.832 \text{ mo.}$$

$$.832 \text{ mo.} = .832 \times 30 \text{ da.} = 24.96 \text{ da. or } 25 \text{ da.}$$

Hence the time is 2 yr. 8 mo. 25 da.

EXERCISE 113

Find the time in which:

1. $92 will gain $11.73 at 6 %.
2. $250 will gain $85.00 at 6 %.
3. $696 will amount to $713.40 at 5 %.
4. $637.50 will amount to $1020 at 4 %.
5. $936 will gain $54.60 at 3 %.
6. $295 will gain $17.10 at 4 %.
7. $1060 will amount to $1200 at 5 %.
8. $716.84 will gain $42.40 at 6 %.
9. $476.50 will amount to $489.20 at 4 %.
10. $1816 will gain $19.08 at 6 %.
11. $800 will amount to $1000 at 4½ %.
12. $265 will amount to $500 at 5 %.
13. $57 will gain $13 at 3 %.
14. In what time will $500 double itself at 5 %?
15. In what time will $800 double itself at 5 %?
16. Judging from Examples 14 and 15, in what time will any sum double itself at 5 %?

Verify your conclusion by testing it for some definite sum.

17. In what time will any sum double itself at 4 %?

174. Finding the Principal when the Interest or Amount, Time, and Rate are Given.

Example. — What principal at 6 % will amount to $3872 in 3 yr. 6 mo.?

Solution. — 1. The interest on $1 for 3 yr. 6 mo. is $.21.
Hence $1 will amount to $1.21 in 3 yr. 6 mo.

2. ∴ it will take as many dollars to amount to $3872 as $1.21 is contained times in $3872.

3. This is $3200.

Rule. — To find the principal when the rate, time, and interest are given:

1. Find the interest on $1 for the given time at the given rate.

2. Then divide the given interest by the result of Step 1. This will give the principal.

3. Check the result.

NOTE. — If the amount is given, find the amount of $1.00 at the given rate for the given time. Divide the given amount by this result to get the principal.

EXERCISE 114

What principal at simple interest will:

1. Gain $ 136.40 in 3 yr. 8 mo. at 6 % ?
2. Gain $ 13.82 in 1 yr. 6 mo. at 8 % ?
3. Amount to $ 503.97 in 9 yr. 6 mo. at 6 % ?
4. Gain $ 22.29 in 10 mo. at 3 % ?
5. Amount to $ 431.20 in 2 yr. 6 mo. at 4 % ?
6. Gain $ 132.00 in 1 yr. 3 mo. at 4 % ?
7. Amount to $ 50.14 in 1 yr. 1 mo. 15 da. at 8 % ?
8. Gain $ 109.62 in 11 mo. 18 da. at 6 % ?
9. Amount to $ 385.00 in 9 mo. 6 da. at 7½ % ?
10. Amount to $ 921.93 in 2 mo. 12 da. at 6 % ?
11. Gain $ 42.25 in 1 yr. 1 mo. at 6 % ?
12. Amount to $ 2515.35 in 4 mo. 15 da. at 6 % ?
13. Gain $ 9.60 in 12 da. at 3 % ?
14. Amount to $ 3556.91 in 7 mo. 12 da. at 6 % ?
15. Gain $ 44.85 in 4 mo. 18 da. at 4½ % ?

Compound Interest

175. In some interest computations, the interest earned on the principal during a period of time like three months, six months, or a year is added to the principal and, *with it,* draws interest during the next period of time.

When the interest *is* added periodically to the principal in this manner, the interest computed on the principal and the unpaid interest is called **Compound Interest.** In most cases, interest *is compounded quarterly, semi-annually or annually.*

Example. — Find the compound interest and the amount on $500 invested for 2 yr., at 6% interest, compounded semi-annually.

Solution. —

1. The interest for the first six months = 3% of $500 = $ 15.00.
2. ∴ the amount at the end of 6 mo. = $515.00.
3. The interest during the 2d 6 mo. = 3% of $515.00 = $ 15.45.
4. ∴ the amount at the end of 1 yr. = $530.45.
5. The interest during the 3d 6 mo. = 3% of $530.45 = $ 15.91.
6. ∴ the amount at the end of 1½ yr. = $546.36.
7. The interest during the 4th 6 mo. = 3% of $546.36 = $ 16.39.
8. ∴ the amount at the end of 2 yr. = $562.75.
9. ∴ the compound interest = $562.75 − $500 = $ 62.75.

NOTE 1. — The simple interest at 6% on $500 for 2 yr. = $60.

NOTE 2. — In the following list of examples, carry out all the computations to four decimal places.

EXERCISE 115

1. Find the amount to which $412 will accumulate in 5 years if it was invested on Jan. 2, 1918, at 4% interest, compounded quarterly.

2. What is the compound interest on $850 for 3 years at 6%, compounded annually?

3. What is the amount to which $500 accumulates in 4 years at 5 %, compounded annually?

4. What is the compound interest on $675 for 5 years at 4 %, compounded annually?

5. What is the compound interest on $1050 for 3 years 6 months at 6 %, the interest being compounded semi-annually?

6. What is the amount of $5000 for 2 years at 8 %, compounded quarterly?

7. What is the amount of $3000 for 2 years 6 months at 4 % interest, compounded semi-annually?

8. *a.* What is the compound interest on $6000 for five years at 4 %, compounded quarterly?

b. What is the simple interest on $6000 for five years at 4 %?

9. What is the amount of $1200 for 3 years at 4 %, compounded semi-annually?

10. What is the amount of $1200 for 3 years at 4 %, compounded quarterly?

Savings Bank Accounts

176. A **Savings Bank** receives small sums of money for deposit, paying interest therefor. Interest is computed, as a rule, quarterly or semi-annually on certain specified **Interest Days.** The time between two successive interest days is called the **Interest Period.** Interest is allowed usually only on the smallest cash balance of the depositor during the interest period ending on the day interest is computed. Interest is not allowed on any fractional part of a dollar.

Thus, if the smallest balance during a period is $273.67, interest is computed on $273.

Interest is usually at the rate of 3 % or 4 %.

If the interest is not drawn when payable, it is added to the principal, and draws interest as a new deposit; in this way savings banks pay compound interest.

The following form of record is kept in a savings bank. It is assumed that interest at 4 % is computed on the 1st of January, April, July, and October.

People's Savings Bank

In Account with *Mr. Charles Goss*

Date		Deposits		Withdrawals		Interest		Balance	
1918		$		$		$		$	
Feb.	2	25	00					25	00
Mar.	28	100	75					125	75
Apr.	1							125	75
Apr.	10	75	00					200	75
May	15	50	00					250	75
July	1					1	25	252	00
Aug.	15	20	00					272	00
Sept.	1			75	00			197	00
Sept.	24	150	00					347	00
Oct.	1					1	97	348	97

This record shows that Mr. Goss opened his account on Feb. 2, 1918. On April 1, he did not receive any interest since his smallest balance between Jan. 1 and April 1 is zero. On July 1, he received $1.25 interest since his smallest balance between April 1 and July 1 was $125.75 and the interest on $125 for 3 months at 4 % is $1.25. He withdrew $75.00 on Sept. 1, and deposited $150 on Sept. 24. On Oct. 1, he received $1.97 interest, since his lowest balance was $197 on Sept. 1, and the interest on that amount for 3 months at 4 % is $1.97.

EXERCISE 116

Solve the following examples. Assume that the interest days are the 1st of January, April, July, and October.

1. The South Side Savings Bank pays 4 % interest. Make out an account for Clara Danville.

Deposits: April 1, $ 50; April 12, $ 15.25; April 30, $ 22.00; May 11, $ 16.00; June 20, $ 25.00; July 16, $ 50.00; Aug. 5, $ 18.00; Aug. 27, $ 13.75; Sept. 19, $ 33.42; Oct. 10, $ 15.00; Oct. 23, $ 19.00; Nov. 16, $ 17.35; Nov. 22, $ 43.62; Dec. 3, $ 35.00. Withdrawals: Dec. 22, $ 75.00. Find balance on Jan. 1.

2. The Mapleville Savings Bank pays compound interest at the rate of 3 %. Make out an account for George Kendall, and balance it on Feb. 16, 1919. (Deposits are marked +, and withdrawals -). Dec. 27, 1917, +$ 200; Jan. 22, 1918, +$ 85.32; Feb. 2, -$ 10.00; March 13, +$ 62.75; April 6, -$ 12.00; April 13, +$ 25.64; May 10, +$ 48.52; May 17, -$ 15.00; June 23, +$ 36.00; July 2, -$ 25.00; July 19, +$ 7.82; Aug. 6, -$ 19.00; Sept. 5, +$ 20.00; Sept. 17, +$ 24.00; Sept. 29, -$ 63.00; Oct. 14, -$ 17.00; Oct. 26, +$ 46.85; Nov. 18, +$ 73; Dec. 20, -$ 50.00.

3. The Westport Savings Bank pays 4 % interest. Make out an account for John Clews down to Jan. 5, 1918.

Feb. 16, 1917, +$ 55; Feb. 26, +$ 32.50; Mar. 19, +$ 27.85; Apr. 10, +$ 83; Apr. 17, -$ 25; Apr. 28, +$ 63.75; May 14, +$ 95; June 6, +$ 120.36; June 28, +$ 165.22; July 6, +$ 85; July 24, -$ 50; Aug. 19, +$ 16; Aug. 26, +$ 20; Sept. 4, +$ 32.50; Sept. 16, +$ 29.45; Sept. 29, +$ 63.75; Oct. 6, -$ 35; Oct. 15, +$ 13.50; Oct. 24, +$ 26.72; Oct. 31, +$ 16.48; Nov. 18, -$ 15.00; Nov. 22, +$ 11.75; Nov. 30, +$ 35.25; Dec. 10, +$ 64; Dec. 15, +$ 37.00; Dec. 20, +$ 12.50; Dec. 28, +$ 125; Jan. 5, 1918, -$ 20.

Supplementary Topics

177. The first issue of **War Savings Stamps** was on sale from Dec. 3, 1917, to Dec. 31, 1918. A stamp having $ 5 nominal value could be purchased up to Jan. 31, 1918, for $ 4.12. Thereafter, the cost increased one cent each month until the end of the year 1918. Each $ 5 stamp was to be attached to a **War Savings Certificate** having 20 spaces.

The owner may, if necessary, upon ten days' written notice through a post office, receive for each $ 5 stamp the amount paid for it plus one cent for each calendar month that has elapsed since that in which it was purchased.

On the other hand, if the stamps are retained until Jan. 1, 1923, the owner of each $ 5 stamp will receive $ 5 for it from the government. The increase is based on an interest rate of approximately 4 %, compounded quarterly from Jan. 2, 1918.

EXERCISE 117

1. *a.* What did it cost to purchase enough stamps to fill a War Savings Certificate in January, 1918?

b. For how much can that certificate be redeemed on Jan. 2, 1920?

c. What will be the value of the certificate on Jan. 2, 1920, assuming an increase of 1 ¢ per stamp for the months of February and March, 1918, and 4 % interest compounded quarterly from April 2, 1918, to Jan. 2, 1920?

d. How much value is sacrificed in cashing the certificate?

e. What is the amount of the certificate on Jan. 2, 1923, continuing the computation started in part *c* of this example?

2. Answer questions like those in Ex. 1, for enough stamps to fill a certificate bought on July 2, 1918.

3. A boy bought three $5 stamps in May, 1918, two in June, seven in July, and eight in August, 1918. Assuming an increase of 1 cent per stamp per month until the next interest date, and thereafter 4% interest, compounded quarterly on all stamps owned for a full quarter or more, find the value of his stamps: *a.* On July 2, 1918; *b.* Oct. 2, 1918; *c.* Jan. 2, 1919; *d.* Jan. 2, 1920, 1921, 1922, and 1923.

NOTE. — Remember the difference between the *value* at compound interest and the value at which the stamps are redeemable on any date before Jan. 1, 1923.

178. In practice, the solution of problems in compound interest is abridged by the use of a table like that on p. 265. In this table is given the amount of $1.00 compounded annually for any number of years from 1 to 40, for certain rates per cent.

Example. — What is the amount of $375 in 5 yr. at 4%, compounded annually?

Solution. — 1. The amount of $1.00 in 5 yr. at 4%, compounded annually, is $1.216653.

2. ∴ the amount of $375 is 375 × $1.216653 or $456.24.

179. The complete usefulness of such a table depends upon a fact brought out by the following problems.

I

Find the amount of $1.00 at 1%, compounded *annually* for *four* years.

1. Interest during 1st yr. = .01
2. Amount end of 1st yr. = 1.01
3. Interest during 2d yr. = .0101
4. Amount at end of 2d yr. = 1.0201
5. Interest during 3d yr. = .010201
6. Amount end of 3d yr. = 1.030301
7. Interest during 4th yr. = .01030301
8. Amount end of 4th yr. = 1.04060401

II

Find the amount of $ 1.00 at 4 %, compounded *quarterly* for *one* year.

1. Interest during 1st quarter = .01
2. Amount end 1st quarter = 1.01
3. Interest during 2d quarter = .0101
4. Amount end 2d quarter = 1.0201
5. Interest during 3d quarter = .010201
6. Amount end 3d quarter = 1.030301
7. Interest during 4th quarter = .01030301
8. Amount end 4th quarter = 1.04060401

Examination of these solutions suggests the

Rule.— 1. The compound interest on and the amount of $ 1.00 at a certain rate compounded *semi-annually* for a certain number of years is the same as the compound interest on and the amount of $1.00 at one half that rate compounded *annually* for double the number of years.

2. The compound interest on and the amount of $ 1.00 at a certain rate compounded *quarterly* for a certain number of years is the same as the compound interest on and the amount of $ 1.00 at one fourth that rate compounded *annually* for 4 times the number of years.

Example.—Find the amount and the compound interest on $ 250, at 6 %, compounded quarterly for 5 yr.

Solution.—1. Since the interest is compounded quarterly, find the amount and the compound interest on $ 1.00 at ¼ of 6 % or 1½ % compounded annually for 4 × 5 or 20 yr.

2. By the table, the amount is $ 1.346855 (page 265, col. 4).

3. ∴ by Rule 2, the amount of $ 250 = 250 × 1.346855 = \$ 336.71$^+$.

4. ∴ the compound interest = $ 336.71 − $ 250 = $ 86.71.

Compound Interest Table

Giving the Amount of $1 Compounded Annually at:

No.	¾%	1%	1½%	2%	3%	4%
1	1.007500	1.010000	1.015000	1.020000	1.030000	1.040000
2	1.015056	1.020100	1.030225	1.040400	1.060900	1.081600
3	1.022669	1.030301	1.045678	1.061208	1.092727	1.124864
4	1.030339	1.040604	1.061364	1.032432	1.125509	1.169859
5	1.038067	1.051010	1.077284	1.104081	1.159274	1.216653
6	1.045852	1.061520	1.093443	1.126162	1.194052	1.265319
7	1.053696	1.072135	1.109845	1.148686	1.229874	1 315932
8	1.061599	1.082857	1.126493	1.171659	1.266770	1.368569
9	1.069561	1.093685	1.143390	1.195093	1.304773	1.423312
10	1.077583	1.104622	1.160541	1.218994	1.343916	1.480244
11	1.085664	1.115668	1.177949	1.243374	1.384234	1.539454
12	1.093807	1.126825	1.195618	1.268242	1.425761	1.601032
13	1.102010	1.138093	1.213552	1.293607	1.468534	1.665074
14	1.110276	1.149474	1.231756	1.319479	1.512590	1.731676
15	1.118603	1.160969	1.250232	1.345868	1.557967	1.800944
16	1.126992	1.172579	1.268986	1.372786	1.604706	1.872981
17	1.135445	1.184304	1.238020	1.400241	1.652848	1.947901
18	1.143960	1.196147	1.307341	1.428246	1.702433	2.025817
19	1·152540	1.208109	1.326951	1.456811	1.753506	2.106849
20	1.161184	1.220190	1.346855	1.485947	1.806111	2.191123
21	1.169893	1.232392	1.367058	1.515666	1.860295	2.278768
22	1.178667	1.244716	1.387564	1.545980	1.916103	2.369919
23	1.187507	1.257163	1.408377	1.576899	1.973587	2.464716
24	1.196414	1.269735	1.429503	1.608437	2.032794	2.563304
25	1.205387	1.282432	1.450945	1.640606	2.093778	2.665836
26	1.214427	1.295256	1.472710	1.673418	2.156591	2.772470
27	1.223535	1.308209	1.494800	1.706886	2.221289	2.883369
28	1.232712	1.321291	1.517222	1.741024	2.287928	2.998703
29	1.241957	1.334504	1.539981	1.775845	2.356566	3.118651
30	1.251272	1.347849	1.563080	1.811362	2.427262	3.243398
31	1.260656	1.361327	1.586526	1.847589	2.500080	3.373133
32	1.270111	1.374941	1.610324	1.884541	2.575083	3.508059
33	1.279637	1.388690	1.634479	1,922231	2.652335	3.648381
34	1.289234	1.402577	1.658996	1.960676	2.731905	3.794316
35	1.298904	1.416603	1.683881	1.998890	2.813862	3.946089
36	1.308645	1.430769	1.709140	2.039887	2.898278	4.103933
37	1.318460	1.445076	1.734777	2.080685	2.985227	4.268090
38	1.328349	1.459527	1.760798	2.122299	3.074783	4.436813
39	1.338311	1.474123	1.787210	2.164745	3.167027	4.616366
40	1.348349	1.488864	1.814018	2.208040	3.262038	4.801021

EXERCISE 118

Find the amount when compounded: *a.* annually; *b.* semi-annually; *c.* quarterly.

1. Of $ 100 for 6 yr. at 3 %.

2. Of $ 1000 for 5 yr. at 4 %.

3. Of $ 250 for 8 yr. at 4 %.

4. Of $ 350 for 10 yr. at 4 %.

5. A man made a bequest of $ 25,000 to a college, with the condition that it should be invested for 10 yr. in securities, paying 4 % interest, compounded quarterly. What will be the amount at the end of that time?

6. Suppose that a savings bank pays 4 % interest quarterly on January, April, July, and October 1st. Suppose that $ 1.00 is placed in the bank at least three days before each interest payment date, starting with January, 1918, and continuing until January, 1923.

a. Make a table showing in one column the date of the deposits, in a second column, the value of each deposit, and in a third column, the amount to which each deposit accumulates at compound interest by January 2, 1923.

b. Find the total amount for all the deposits.

c. Suppose that the quarterly deposit had been $ 5 instead of $ 1. Find the amount of all such deposits by multiplying the result of part *b* by 5.

7. *a.* Obtain as in Example 6, the amount to which $ 1.00 per quarter will accumulate in ten years at 4 % interest, compounded quarterly, each deposit being made before the interest date, until forty deposits have been made.

b. Using the result of part *a*, find the amount if the deposits had been $ 10.00 per quarter instead of $ 1.00.

8. In 1918 a boy was promised a Christmas gift of $ 50 in each of the ten years from 1918 to 1927 inclusive, pro-

vided he deposited it in a savings bank within three days and allowed it to accumulate at 4 % interest, compounded quarterly until Oct. 1, 1928, at which time he could begin drawing upon it for college expenses. How much money will be available at that time?

Suggestion. — This example, like Ex. 6 and Ex. 7, can be simplified by finding first the amount if the deposit were only $ 1.00 and then multiplying this amount by 50.

9. A girl's birthday is March 10th. In 1918 she is 9 years old. She is promised a gift on each birthday, beginning with 1918, until she is 18 years old, of as many times $ 5 as she is years of age, provided she will agree to deposit the money within three days in a savings bank paying 4 % interest, compounded quarterly. What will be the amount of her account on Oct. 1 of the year in which she is 18 years old?

10. A man and his wife, desiring to provide for their later years, decide to deposit $ 50 per quarter in a savings bank paying 4 % interest quarterly. Assume that they started in September, 1915, always making their quarterly deposit at least three days before the next interest day. How much money will be to their credit on Jan. 2, 1925?

Periodic Interest

180. If interest which is due quarterly, or annually, is unpaid, some contracts provide that interest shall be paid upon unpaid interest as well as on the principal until all is paid. This form of interest is called **Periodic Interest.** This form of interest is illegal in some states.

Example. — What amount is due at the end of three years and six months on a debt for $ 500, with interest at 6 % payable semi-annually, on which at maturity no interest payments have been made?

The solution can be arranged in the following form:

	Interest Payments	Due at End of	Overdue	Interest on Payment	Amount of Payment and Interest	
First	$15	6 mo.	36 mo.	$2.70	$17	70
Second	15	12 mo.	30 mo.	2.25	17	25
Third	15	18 mo.	24 mo.	1.80	16	80
Fourth	15	24 mo.	18 mo.	1.35	16	35
Fifth	15	30 mo.	12 mo.	.90	15	90
Sixth	15	36 mo.	6 mo.	.45	15	45
Seventh	15	42 mo.	0		15	00
Principal due at end of 42 mo.					$500	00
Total amount due at end of 42 mo.					$614	45

EXERCISE 119

Assume periodic interest allowable in each of these examples.

1. What amount is due at the end of 2 yr. 6 mo. on a debt of $ 900, with 6 % interest, payable annually, on which no payments have been made?

2. What amount was due on Aug. 5, 1918, on a debt of $ 360 dated Feb. 5, 1914, with 6 % interest, payable annually, on which no interest payments had been made?

3. What is the amount due at the end of 5 years on a debt of $ 1200 with interest at 6 %, payable annually, on which the first two interest payments were made when due?

4. What is the amount due at the end of 4 years on a debt of $ 2200 bearing 5 % interest, payable semi-annually, on which the first five interest payments were made when due?

5. What amount was due Nov. 12, 1918, on a debt of $ 875 dated Aug. 12, 1914, with interest at 6 %, payable semi-annually, on which the first three interest payments were made when due?

Installment Payment of Debts

181. Installment payment of debts bearing interest are quite common.

Example 1. — A man purchased a lot on March 1, 1918, for $ 550, paying $ 50 cash and agreeing to pay $ 25 on the first of each month thereafter, together with the accrued interest on that payment at 6 % from the date of purchase until the date of payment. The payments may be recorded in the following manner.

Payment				Interest on Payment	Payment Plus Interest	Balance Due
Date	Debt	Due	Time			
3/1/'18	$ 550	$ 50	—	—	$ 50	$ 500
4/1/'18	500	25	1 mo.	$ .12½	25.13	475
5/1/'18	475	25	2 mo.	.25	25.25	450

a. Rule a sheet as above and complete the record until the debt is entirely paid.

b. How much will the man pay all together?

Example 2. — Assume that the agreement for the purchase of the lot in the preceding example had been that the man should pay $ 50 cash and, thereafter, $ 25 per month on the first of each month, with the interest that has accrued on the balance of the debt during the preceding month. Rule off and complete a schedule like the following, continuing it until the debt is fully paid.

DATE	DEBT	TIME	INTEREST ON BALANCE	PAYMENT DUE	TOTAL DUE	NEW BALANCE OF DEBT
3/1/'18	$ 550	—	—	$ 50	$ 50	$ 500
4/1/'18	500	1 mo.	$ 2.50	25	27.50	475
5/1/'18	475	1 mo.	2.37	25	27.38	450
6/1/'18	450	1 mo.	2.25	25	27.25	425

How much will the man pay all together?

EXERCISE 120

Rule off and complete a schedule of payments for:

1. A debt of $3000 at 6% starting May 1, 1918, payments of $100 to be made on the first of each month, together with accrued interest from May 1 on each payment to the date of payment. Payments are to be made until the debt is canceled.

2. A debt of $1500 with interest at 6%, dated Aug. 1, 1918, payments of $50 to be made on the first of every month thereafter, until the debt is paid. On the first of each month, there is also to be paid the interest on the unpaid balance of the debt for the preceding month.

3. A debt of $1325 for an automobile was to be paid as follows: The date of purchase was May 15, 1918; the initial payment was $125; the future payments were to be $100 on June 1, 1918, and a like amount on the first of each month thereafter until the debt was paid, together with interest at 6% on each of said payments from May 15, 1918, until the date on which the payment was made.

4. A street assessment of $650, dated Sept. 1, 1915, was to be paid as follows: on April 1, 1916, and on each succeeding April 1, until the assessment and interest is fully paid, there was to be paid one tenth of the assessment, together with accrued interest at 6% on the unpaid balance of the assessment for the preceding year.

5. Assume a debt of $1000, dated March 15, 1918, bearing interest at 6%, to be paid as follows: a payment of $100 to be made semi-annually; out of each payment enough was to be taken to pay the accrued interest for the preceding half year on the unpaid balance of the debt; the rest of the payment was to be used to decrease the balance of the debt.

6. A man bought a piano for $425, paying down $25, and agreeing to pay $10 per month. He also agreed to

pay each month the interest at 6 % that had accrued on the payment made from the date of purchase, May 20, 1917, to the date on which he made the payment. Assume that the payments were made on the 20th of each month. Find the total of all payments plus interest.

7. A man borrowed $ 4000 on Aug. 15, 1916, agreeing to pay interest at 6 % semi-annually. He was given the privilege of paying on the face of the debt on interest payment dates $ 100 or multiples thereof. On Feb. 15, 1917, he paid his interest and $ 300 on the debt; on Aug. 15, 1917, he paid his interest on the balance, and $ 200 on the debt; on Aug. 15, 1918, he paid the interest on the balance of the debt, and $ 600 to decrease the amount of the debt. Make out a schedule of interest and principal payments, assuming that the man plans to pay semi-annually the interest on the balance of the debt and also $ 250 toward the principal, after Aug. 15, 1918, until the debt is fully paid. Find the total of all payments made when the debt is fully paid.

X. INSURANCE

182. Property is often destroyed by fire, wind, and accidents; individuals lose wages as a result of sickness, accidents, and other causes; families lose their source of support through the death of some member. All persons are exposed to these risks, some more and others less, but only a small number actually suffer losses of any one kind in a limited period of time.

Insurance is the business devised for distributing these losses so that each member of a group bears a share determined by the protection he receives. The business of furnishing insurance is conducted by insurance companies who operate through agents to be found in all towns.

Thus, 100 farmers each owning a $3000 barn might agree that they would rebuild the barn of any one of their number if it should be destroyed by fire. If one barn is destroyed, $3000 is lost. Each farmer would then contribute $30, including the man whose barn was lost. Each is willing to make this expenditure to protect himself against the possible loss of $3000.

An **Insurance Policy** is the contract between the company and the insured. It states the amount of protection, called the **Face of the Policy**, and the conditions under which the insurance is afforded. Every one taking out an insurance policy should read it with great care, and should insist upon having the meaning of all its conditions fully explained.

An **Insurance Premium** is the sum which the insured pays the company in return for the insurance.

Fire Insurance

183. The **Standard Fire Insurance Policy** provides insurance "*to an amount not exceeding*" a specified sum for any direct loss or damage by fire. This loss is the "actual cash value" of the property damaged at the time of the fire.

NOTE. — In case some insurance is paid an insured because of damage to his property by fire, the face of the policy is then automatically decreased by the amount of insurance already paid by the company.

By agreement with the companies, an insured may divide his total insurance among two or more companies. Then, in case of loss, each company pays as much of the loss as *its* insurance is of the *total* insurance.

Thus, if the total insurance is $5000, one company insuring $3000, and another $2000, then the first company would pay $\frac{3}{5}$ and the second $\frac{2}{5}$ of any loss by fire.

184. The fire insurance premium depends upon many factors, — such as, the kind of fire protection provided by the city or town, the kind of building occupied by the insured, the business conducted by the insured, the number of occupants in the building, etc. As a rule, fire insurance companies are members of one of two associations of such companies. These associations employ men whose business it is to decide upon rates which will be adequate and just. These rates are recommended to the companies. Specific rates are recommended for each risk in a business district in a city, and a schedule of rates for all other kinds of insurable property.

The rate is usually expressed as a number of cents per $100 of insurance for one year.

185. Upon some kinds of property, a policy can be taken out for a term of three years or of five years. When a policy *is* taken out for a term of three years, the premium is *two and one-half times*, and for five years the premium is *four times* the annual premium. (Western Union rates.)

NOTE. — Formerly the three-year premium was twice, and the five-year premium was three times the annual premium.

Below is part of the dwelling house schedule used in a certain city.

RATE PER $100 OF INSURANCE FOR ONE YEAR

DWELLING HOUSE	BRICK — SHINGLE ROOF		FRAME — SHINGLE ROOF	
Number of Occupants	Building alone	Contents alone	Building alone	Contents alone
One family . . .	$.18	$.22	$.22	$.26
Two family . . .	.20	.24	.24	.28
Three family . . .	.22	.26	.28	.32
Barns and garages .	.30		.40	

186. The agent of the company receives a **Commission** for his services.

The commission is a per cent of the premium charged by the company.

One group of companies pays its agents:

25 % on dwelling houses and contents and most public buildings; 20 % on mercantile buildings; 15 % on other buildings.

EXERCISE 121

Determine the premium and the agent's commission at 25 % of the premium for each of the following policies, using the schedule above.

Ex. No.	Face of Policy	Term of Policy	Property Insured	Kind of Building	Number of Families
1.	$2750	1 yr.	building	brick	two
2.	3200	3 yr.	building	frame	three
3.	1200	5 yr.	contents	frame	one
4.	500	3 yr.	contents	brick	two
5.	4500	5 yr.	building	frame	one
6.	1500	5 yr.	contents	frame	three
7.	750	3 yr.	garage	frame	
8.	650	5 yr.	contents	frame	three
9.	1500	5 yr.	garage	brick	
10.	3750	3 yr.	building	frame	one

11. A factory valued at $ 25,000 is insured for 80 % of its value at an annual rate of 70 ¢ per $ 100.

a. What is the premium?

b. What is the agent's commission if he receives 15 % of the premium?

c. How much does the company actually get?

12. A store owner asks an agent to write up for him $ 30,000 worth of insurance on his store, — the insurance to be in three companies.

The agent gives company A $ 12,000, company B $10,000, and company C, $ 8000. The rate in each company is 55 ¢ per $ 100. Determine:

a. The premium the owner must pay;

b. The commission earned by the agent at 20 % of the premium;

c. The amount each company receives.

13. A factory valued at $ 25,000 is insured for 80 % of its value at an annual rate of 70 ¢ on $ 100.

a. Determine the premium for three years.

b. What is the agent's commission if he receives 15 % of the premium?

c. What does the company receive?

14. Assume that fire causes actual cash loss of $ 5000 to the property insured in Ex. 12. How much must each of the companies pay the insured?

15. A corporation insures its $ 30,000 plant for 75 % of its value at 45 ¢ per $ 100 in three companies. Company A takes $ 10,000, company B $ 5,000, and company C, the balance.

a. What is the face of the policy with company C?

b. What premium is paid each company?

c. The companies each allow the agent 15 %. What are his commissions?

d. How much does each company actually receive?

e. A fire causes a $ 10,000 loss. How much must each company pay the corporation?

16. The owner of a house valued at $ 7500 insured it for three years for $ 5000 at the rate of 25 ¢ per $ 100 for one year.

a. What premium did he pay?

b. The agent's commission was 25 %. What did he receive?

c. The house was completely destroyed by fire. How much should the owner receive from the company?

17. A man insured his house for $ 8000 at the annual rate of 28 ¢ per $ 100, and his household goods for $ 2500 at 36 ¢ per $ 100.

a. Determine his total premium.

b. His house is completely destroyed by fire, but $ 850 worth of the household goods are saved. How much insurance did he receive?

18. Find the premium for a $ 1000 policy on a farmer's barn at the rate of 60 ¢ per $ 100 per year, the policy to run for 5 years.

19. If a farm is occupied by a tenant, the annual rate on a farm policy is 15 ¢ higher than that in Ex. 18. Find the premium for the policy in Ex. 18, if the occupant is a tenant.

187. Short Term Rates. — Policies are sometimes written for less than a year. In such cases the following table of short term rates is used.

These rates are used also when a policy is canceled. *If the insured asks* to have the policy canceled, and he always has the right to do so, the company is allowed to charge the short term rate for that part of the term of insurance which has already expired, and must return to the insured the balance of the original premium. *If the insurance company* cancels the policy, and it has the right to do so on five days' notice, then it is allowed to charge for that part of the term of insurance which has already expired as much of the original premium as the expired term is of the original term.

Thus, if a one-year policy has run for seven months, the company can retain only seven-twelfths of the original premium. This is called pro-rating the premium for the short term.

SHORT RATE TABLE FOR ONE-YEAR POLICIES

TIME IN DAYS	CHARGE	TIME IN DAYS	CHARGE	TIME IN DAYS	CHARGE	TIME IN DAYS	CHARGE
1	2 %	14	13 %	55	29	180	70 %
2	4 %	15	13 %	60	30 %	195	73 %
3	5 %	16	14 %	65	33 %	210	75 %
4	6 %	17	15 %	70	36 %	225	78 %
5	7 %	18	16 %	75	37 %	240	80 %
6	8 %	19	16 %	80	38 %	255	83 %
7	9 %	20	17 %	85	39 %	270	85 %
8	9 %	25	19 %	90	40 %	285	88 %
9	10 %	30	20 %	105	46 %	300	90 %
10	10 %	35	23 %	120	50 %	315	93 %
11	11 %	40	25 %	135	56 %	330	95 %
12	11 %	45	27 %	150	60 %	345	98 %
13	12 %	50	28 %	165	66 %	360	100 %

Example. — If the annual rate is 65¢, then the short rate for 85 days is 39 % of 65¢, or 25.35¢.

If the number of days does not appear in the table, take the per cent opposite the next greater number of days that does appear in the table.

Thus, the per cent for 87 days is that opposite 90 days, namely 40 %.

A similar table of short rates is provided for term policies.

EXERCISE 122

Using the table of short term rates, find the premium:

Ex. No.	If the Face Is	If the Rate Is	If the Term Is			
			a	b	c	d
1.	$3500	$.40	1 mo.	3 mo.	7 mo.	9 mo.
2.	7200	.55	20 da.	2 mo.	5 mo.	75 da.
3.	850	.75	15 da.	4 mo.	35 da.	225 da.
4.	1300	.90	25 da.	80 da.	200 da.	11 mo.
5.	9500	.65	10 da.	92 da.	165 da.	260 da.

Determine the original premium and the refund to the insured in each of the following cases:

Ex. No.	Face of Policy	Term	Rate for the Term	Canceled at End of	By Whom Canceled
6.	$1200	1 yr.	$.60	4 mo.	Company
7.	750	1 yr.	.88	7 mo.	Company
8.	2000	1 yr.	1.12	160 da.	Company
9.	3200	1 yr.	.45	210 da.	Insured
10.	1350	1 yr.	.65	185 da.	Insured
11.	650	1 yr.	.72	96 da.	Insured
12.	4200	1 yr.	.68	163 da.	Insured
13.	5500	1 yr.	.88	155 da.	Company
14.	2250	1 yr.	.95	225 da.	Company

15. A man who has mortgaged his house and lot as security for a loan of $ 3000 for 6 months agrees with the lender to insure the house for $ 3000 for the six months. The annual rate is $ .60. Find the premium for six months.

16. The owner of a store insured the building and contents for $ 12,500 at $ .75 per $ 100 for one year.

a. What was the premium?

b. At the end of three months the company canceled the policy. How much were they required to refund to the insured?

c. Suppose that the insured had asked to have the policy canceled, what would have been the refund?

17. A man insured his household goods for $ 2500 for three years at the rate of $ .22 per $ 100 per year.

a. What was the premium?

b. How much did the agent receive if his commission was 15 %?

c. How much did the company receive?

d. At the end of fifteen months, he asks to have the policy canceled. If the short rate for fifteen months of a three-year term policy is 48 % of the whole premium, how much must the company refund to him?

18. A man has his property insured for $ 1800 for three years at the rate of $ .28 per $ 100 per year.

a. What was his premium?

b. At the end of one year, he wants the face of the policy changed to $ 2500 and is told by the agent that there is now a new rate of $ .32 per year. The agent agrees to cancel the old and write the new policy, *prorating* the premiums. How much additional must the insured pay for the balance of the three-year term?

19. A family insure their household goods for three years for $ 2000 at the annual rate of $ .22. At the end of one year, they move into a house where the annual rate is $ .26. How much additional premium must they pay if the company prorates each of the premiums?

20. On Sept. 11, 1917, a man insured his household goods for one year for $ 1500 at the rate of $ .44.

a. What was his premium?

b. On Feb. 26, 1918, the man moved into a house where the rate was $ .38 per year. How much refund should he have received from the company? (Prorate premiums.)

188. Co-insurance. — Some policies provide that the insured should insure his property for a specified per cent of the value of the property. The owner, however, may not take out the specified amount of insurance. In such cases, the policy provides that only as much of any loss suffered will be paid as the actual insurance is of the specified insurance, not exceeding, however, the face of the policy.

Thus, suppose that a piece of property should be insured for $ 8000 and that the owner carries only $ 6000 worth. Then the owner will receive only $\frac{3}{4}$ of any loss he suffers by fire. If the fire damage is $ 4000, he will receive $ 3000.

This provision is contained in a *co-insurance clause* of the policy, which states the per cent of insurance that the insured should carry. The idea is that the insured becomes a *co-insurer* with the company if he does not take out the specified amount of insurance. Co-insurance is never applied to dwellings or household goods. Usually a lower rate is charged when the policy contains a co-insurance clause.

EXERCISE 123

1. How much insurance will a man receive whose $ 20,000 property is insured for $ 10,000 under a policy containing an 80 % co-insurance clause when the loss by fire is: *a.* $ 1200? *b.* $ 2500? *c.* $ 6500?

2. How much insurance should a man receive whose $ 8000 property is insured for $ 4000 under a policy containing a 75 % co-insurance clause if the damage by fire is: *a.* $ 2100? *b.* $ 1200? *c.* $ 7000?

3. How much would the owner in Example 2 receive under a standard policy in each of these cases?

4. A store, valued at $ 6000, is insured for $ 3500 at $ .65 per year, and the contents, valued at $ 15,000, is insured for $ 7500 at $ .80, both in companies whose policies contain 80 % co-insurance clauses. The term of the policy is three years. A fire causes $ 500 damage to the store and $ 2000 damage to the contents.

a. What was the total premium?

b. How much insurance is paid the owner on the store and on the goods?

5. Property valued at $ 1000 is insured for $ 600 with a 90 % co-insurance clause. A fire causes a loss of $ 875. How much insurance does the owner receive from the company?

EXERCISE 124

Supplementary Exercises

Before doing the following examples, obtain from some fire insurance agent in your own community the rates needed in solving the examples. Also learn whether the policy is a standard policy or one containing a co-insurance clause. Then find the premium for:

1. A $ 5000 policy for three years on a two-story brick building valued at $ 7200 if there are adjacent to it other brick buildings, if the first floor is occupied by a grocer and the second floor by doctors' offices.

2. A policy for $ 15,000 for five years on a frame church valued at $ 17,500. Assume that there are not any other buildings within fifty feet of it and that the church is heated by a furnace.

3. A policy for $ 25,000 for three years on a two-story brick school building valued at $ 40,000, heated by steam.

4. A policy for $ 5000 for three years on a two-story frame dwelling having a stucco exterior, valued at $ 7250.

5. Suppose that the church is damaged to the extent of $ 800. How much insurance must the company pay?

Other Property Insurance

189. Property is insured against loss by wind, called **Tornado Insurance**; against loss at sea, called **Marine Insurance**; against theft, called **Burglary Insurance**, etc. The following examples are some of the simple ones that occur. The rates are in every case bona fide, having been taken from recent agents' manuals..

Automobile Insurance

The following rates apply when a car is insured by the original purchaser of it against loss by fire or theft.

Value of Car		Class C $1400 to $2499	Class D $800 to $1399	Class E $799 and Under
New car	Maximum of insurance	90 % of the value	90 % of the value	90 % of the value
	Rate	2 %	3 %	3¾ %
1-yr.-old car	Maximum of insurance	70 % of the value	60 % of the value	60 % of the value
	Rate	2¼ %	3¼ %	4 %
2-yr.-old car	Maximum of insurance	50 % of the value	40 % of the value	40 % of the value
	Rate	2¾ %	3½ %	4¼ %

EXERCISE 125

Find the maximum amount of insurance that the owner could get and the premium he had to pay when:

Ex. No.	The Car Was	Age of Car Was	Original Price Was
1.	Ford	New	$ 585
2.	Maxwell	One year	785
3.	Buick	Two years	1350
4.	Dodge	New	1535
5.	Cadillac	Two years	1850
6.	Oakland	One year	1085
7.	Studebaker	New	1450
8.	Studebaker	One year	1450
9.	Hudson	Two years	1650
10.	Paige	New	1495

11. The owner of a bookstore takes out $ 3000 of burglary insurance at the rate of $2\frac{3}{4}$ %. Find the premium.

12. The owner of an auto supply store wants $ 4000 worth of burglary insurance. He is told that the annual rate is 6.6 % with a discount of 5 % if there is a watchman in the building but not necessarily in his store. Find his rate if he can claim the discount.

13. A bank, because of the construction of its vault and safe, is offered the rate of 3 % for burglary insurance with a discount of 15 % because of the size of the town, and 15 % additional because of the services of a watchman. Find the annual premium for $ 15,000 worth of insurance. (The two discounts are treated as successive discounts as in § 155.)

14. A suburban home is insured against damage by wind for $ 1500 at the rate of $\frac{3}{4}$ % for three years. Find the premium.

15. A farmer's barn is insured for \$ 2500 against loss or damage by fire, lightning, or tornado, at the rate of $2\frac{1}{2}\%$. Find the premium.

16. The owners of some greenhouses take out insurance against loss by wind, or hail, for \$ 3500 for three years with a rate of $\frac{3}{4}\%$ for the whole term for the tornado insurance alone and $\frac{2}{10}\%$ additional for the hail insurance. Find the premium.

17. A farmer insures his three horses for \$ 150 each and his five cows for \$ 75 each for one year, against injury or loss due to fire, wind, or lightning, at the rate of $1\frac{1}{2}\%$. Find the premium.

18. A farmer insures his frame silo against loss due to wind for \$ 1750 for five years at 6 % for the term. Find the premium.

19. A large ranch owner insures his new tractor for \$ 1500 against damage by fire for 1 % for one year, and against damage by tornado for $\frac{1}{4}\%$ for one year. What is the combined premium?

Life Insurance

190. **Life Insurance** premiums are based upon very elaborate computations in which percentage in one form or another plays an important part. These computations, however, are all made in the actuarial departments of the companies. The computations which either the agent or the insured have to make are quite elementary. The premiums depend upon the kind of policy the insured desires and upon the age of the insured. Below are given the rates of a certain company for certain ages for each of three kinds of policies. Similar rates for applicants of all ages between 20 and 60 inclusive are obtainable from the agents of a company. The rates of different companies vary a little but not much from these.

LIFE INSURANCE RATES

AGE	ORDINARY LIFE POLICY	20-PAYMENT LIFE POLICY	20-YEAR ENDOWMENT POLICY
20.	$18.69	$27.66	$48.93
21.	19.09	28.08	49.00
22.	19.52	28.54	49.08
23.	19.95	29.02	49.18
24.	20.40	29.51	49.28
25.	20.80	30.02	49.37
26.	21.41	30.55	49.48
27.	21.93	31.10	49.60
28.	22.50	31.66	49.72
29.	23.09	32.26	49.85
30.	23.72	32.87	49.98

In the first column is the nearest age of the applicant. Opposite the age are the rates for the three different policies.

Under the ordinary life policy, the insured pays premiums as long as the policy is in force; the face of the policy is payable to the *beneficiary* at the death of the insured.

Under the twenty-payment life policy, the insured pays premiums for twenty years *at most.* At the end of twenty years, the policy is fully paid. The face of the policy is payable to the beneficiary of the insured at the death of the latter, whether that occurs before or after twenty years.

Under the twenty-year endowment policy, the insured pays premiums for twenty years *at most.* The face of the policy is payable to the insured at the end of twenty years, if he is living, or to the beneficiary of the insured if the latter should die before twenty years.

If the premium of a life insurance policy is paid semi-annually, the semi-annual premium is found by adding to the annual premium six per cent of itself and dividing the result by two.

If the premium is paid quarterly, the quarterly premium is found by adding to the annual premium four per cent of itself and dividing the result by four.

EXERCISE 126

What is the annual premium for:

Ex. No.	Kind of Policy	Amount	At Age
1.	Ordinary life	$3000	28
2.	20-payment life	5000	27
3.	20-year endowment	2500	25
4.	20-payment life	4500	30
5.	Ordinary life	10,000	29
6.	20-year endowment	3500	22
7.	20-payment life	2500	26
8.	Ordinary life	1500	23
9.	20-year endowment	4500	30
10.	20-payment life	7500	29

11–20. Determine the semi-annual premiums for each of the policies.

21–30. Determine the quarterly premiums for each of the policies.

191. Dividends. — Most life insurance policies now issued are participating policies. Annually the insured receives from the company what is incorrectly called a dividend. Actually, it is a refund from the company of the insured's share of savings made by the company on estimated expenses, or of savings from estimated losses, or of certain interest earnings of the company in excess of those anticipated. These dividends may be used by the insured to decrease the amount of the premium due the company, may be allowed to accumulate at compound interest in the care of the company, or may be disposed of in other ways with the consent of the insured and of the company.

XI. TAXES

192. Village, city, town, county, and state taxes include some or all of the following:

a. **Real Estate Tax,** levied upon the assessed value of the real estate (including improvements) of an individual.

b. **Personal Property Tax,** levied upon the removable personal property, such as household goods, automobile, animals, money, securities, etc.

c. **Special Taxes** for special purposes, such as the construction of sidewalks, sewers, streets, etc.

d. **Income Tax,** levied upon the assessable income of an individual.

e. **Inheritance Tax,** levied upon wealth inherited from a deceased resident of the taxing community.

f. **Poll Tax,** a fixed amount levied upon each (male) resident of a community.

Of these taxes, the first three are quite general; the next two are increasingly becoming sources of revenue for governmental agencies.

Property Taxes

193. The assessed value of property is the valuation placed upon it for taxing purposes by the properly constituted officers, called **Assessors.**

The value is sometimes definitely assumed to be only one third or one half the actual value; in other cases, it is assumed to be the actual value of the property.

Any individual, dissatisfied with the assessed valuation of his property, may appeal to the **Board of Review,** at an agreed-upon time, for a revaluation.

194. The tax itself is computed as a certain rate per cent of the assessed value; this rate is called the **Tax Rate.**

The following example shows how one community determined its tax rate in a recent year.

Example 1. —

THE ASSESSABLE PROPERTY

Assessed value of real property	$57,791,875
Assessed value of personal property	12,911,365
Total assessed value of property	$70,703,240

TOTAL TAXES REQUIRED

The city's share of state taxes	$ 87,952.82
The city's share of general county taxes	99,862.89
City taxes for educational purposes	281,987.05
City taxes for indebtedness	172,035.00
City taxes for street improvements	85,761.94
City taxes for general purposes	262,245.66
Total taxes required	$989,845.36

The tax rate was determined by regarding the total taxes, $ 989,845.36, as the percentage and the total assessable property, $ 70,703,240, as the base.

What per cent of $ 70,703,240 is $ 989,845.36? The result is the tax rate.

By § 151, the tax rate is 1.4 %.

Example 2. — What was the real estate tax and the personal property tax of a man whose real estate was valued at $ 3575 and personal property at $ 1450?

Solution. — 1. The realty tax = 1.4 % of $ 3575
= $ 50.05

2. The personal property tax = 1.4 % of $ 1450
= $ 20.30.

Where the assessed valuation of the property is very great, as in the case of railroads and public utility corporations, it is necessary to carry out the tax rate to several decimal places.

Thus, in a recent year, one state published as its rate for state tax purposes 1.317260878 %.

EXERCISE 127

Determine the total property taxes of the following persons:

Ex. No.	Name of Person	Real Property	Personal Property	Where the Tax Rate Is
1.	Henry Radley	$ 1525	$ 780	1.5 %
2.	Martin Fosdick	—	550	1.8 %
3.	Samuel Girard	4650	20,500	1.3 %
4.	Fred Thompson	7275	6,825	1.7 %
5.	William Peterson	19,725	32,650	1.4 %
6.	Frank O'Malley	6230	1245	11 mills on $1
7.	James Sargent	19,750	3235	16 mills on $1
8.	Ajax Stone Co.	12,450	2850	13 mills on $1
9.	Edwards Grocery Co.	8750	7450	19 mills on $1
10.	Peter Schmidt	2385	675	18 mills on $1

11. A certain town whose taxable property is $ 1,150,000 wishes to raise $ 15,100 by taxation. What will be the rate of taxation expressed in mills? What will be the entire tax of an individual whose property is valued at $ 8500?

12. If a town whose taxable property is $ 75,400 wishes to raise $ 1055.60, what is the rate of taxation? Express the result as a per cent, carried out to one decimal place as in Examples 1–5.

13. A town whose taxable property is $ 165,000 wishes to raise $ 3630. What is the rate of taxation?

14. A man's house is valued at $ 3640. What will be his tax, at the rate of $ 14.50 on $ 1000?

15. *a.* A town whose real estate is valued at $ 724,600 and personal property at $ 561,400 wishes to raise $ 16,000 in taxes. What must be its tax rate? (Carry out to two decimal places as in Ex. 15.)

b. Find the taxes of a man whose total property is valued at $ 7365.

16. In one community the assessed valuation of property was about $\frac{1}{4}$ the real value and the tax rate was 5.8 %; in another, the assessed valuation was about 90 % of the real value, and the tax rate was 1.4 %. Compare the taxes of two men, one living in each community, each of whom had property of actual value $ 8000.

17. *a.* In a certain state, the assessed valuation of railroad property in 1916 was $ 360,960,000 and the tax rate was 1.3172^{+} %.

Determine the total taxes contributed by the railroads.

b. The valuation of all general property in the state was $ 3,426,797,220.

Determine the aggregate of all the property taxes of the state if the average rate of taxation was 1.3172^{+} %.

18. In this state 85 % of the taxes collected by the state from street railways is returned to the community in which the street railway is located, and 15 % is retained by the state.

One street railway was assessed at $ 850,000. Determine its total tax (rate = 1.3172^{+} %), the amount retained by the state, and the amount returned to the community.

19. A man bought a house and lot for $ 5500. It was assessed for 85 % of its value and taxed at the rate of 16 mills on $ 1. His insurance and repairs amounted to $ 50 per year. He rented the house for $ 40 per month. What per cent of income on his investment did he receive?

20. A man owns a two-family building that cost him $ 7350, on a lot which cost $ 1150. His property is assessed at about 60 % of its value and taxed at the rate of $17\frac{1}{2}$ mills on $ 1. He estimates insurance and repairs at $ 100 per year. What total rent must he receive per month to make 8 % annually on his investment?

21. A man paid $ 5200 for a house and lot. It was valued by the assessor at $ 3850. The tax rate was 1.4 %. He estimates his annual charge for insurance at $ 5.30, and for repairs and painting, $ 40.

a. What was his annual "rent," including interest at 6 % on his investment, taxes, insurance, and repairs?

b. What was his monthly rent?

c. Before purchasing the house, he rented it at $ 40 per month. Did he gain or lose annually by purchasing the house? How much?

d. After owning it four years, he sold the property for $ 5800. What percentage of annual profit on the $ 5200 investment do his profits under parts *c* and *d* represent?

e. Since he has included in part *a* interest at 6 % as part of his rent, what is his total rate of income per year on his $ 5200 investment?

22. What tax will be paid by a man whose real estate is valued at $ 5200, and whose personal property is valued at $ 3850 if he also pays for two polls at $ 1.50 each, the rate of taxation being $ 16.50 on $ 1000.

23. The assessed valuation of real estate in a certain town is $ 825,000 and of personal property $ 394,625. There are 465 citizens who pay a poll tax of $ 1.50 each. The town wishes to raise $ 16,475 in taxes. What must be the rate of taxation?

195. Special Assessments are levied at times upon the owners of property which will be benefited by some special improvement. They are levied particularly in villages and cities to obtain income with which to pay for the construction of sidewalks, streets, sewers, etc. The computation of the assessments for street construction is usually based upon a specified amount per front foot. These and the other special assessments rarely present any arithmetical difficulties.

Supplementary Topics

196. Several states in the United States now derive revenue from **Income Taxes.** While the laws of the several states vary in minor particulars, all agree in specifying that a certain part of the income of a resident of the state shall be exempt from taxation, and secondly that income derived from certain sources shall be exempt, — such as income from corporations that pay taxes to the state. The balance of the income of a resident is called his *taxable income.*

In one state, $ 800 of annual income of every resident is exempt, and $ 1200 of income of every married man whose wife is living; besides, an additional $ 200 for every child under 18 years of age, and for every dependent of the resident, is exempt. The rates of taxation for taxable income follow:

Taxable income between $ 1 and $ 1000 is taxed 1 %; between $ 1000 and $ 2000, 1¼ %; between $ 2000 and $ 3000, 1½ %; between $ 3000 and $ 4000, 1¾ %; between $ 4000 and $ 5000, 2 %; between $ 5000 and $ 6000, 2½ %; and each additional $ 1000 is taxed an additional ½ % until $ 13,000 is reached. All income above $ 13,000 is taxed 6 %.

NOTE. — Observe carefully that a certain rate applies to the first $ 1000; then a new rate to the second $ 1000; a new one to the third $ 1000; etc., until $ 13,000 is reached.

Example. — What is the annual income tax of a man in this state whose taxable income is $ 2500?

Solution.

1. The tax on the first $ 1000 is 1 % of $ 1000 or $ 10.
2. The tax on the second $ 1000 is 1¼ % of $ 1000, or $ 12.50
3. The tax on $ 500 in excess of $ 2000 is 1½ % of $ 500 or $ 7.50
4. ∴ the total income tax is $ 20.00

EXERCISE 128

1. Make out a table giving the annual tax on a taxable income of $ 1000, $ 2000, $ 3000, $ 4000, etc., to $ 13,000 inclusive.

2. Henry Smith, unmarried and without dependents, earns $ 95 per month of taxable income. What is his annual income tax?

3. *a.* Charles Williams, unmarried, but supporting his mother and ten-year-old brother, earned $ 95 per month before the war. What, if any, income tax did he pay annually?

b. During the year 1918, his pay was increased to $ 165 per month. What was his income tax for 1918?

4. Mr. Hughes, supporting his wife and two children, had in 1918 an annual income from all sources of $ 7350. Of this, $ 675 was exempt, since it was derived from business outside the state. What was his income tax?

5. Edward Evans and his wife, jointly, have a total income of $ 23,750. They do not have any children under 18 years of age. What is their annual income tax?

197. Corporations in that same state also pay an income tax according to the following general provisions.

On the first $ 1000 of taxable income, 2 %; on the second $ 1000 or part thereof, $2\frac{1}{2}$ %: on each succeeding $ 1000 or part thereof an additional $\frac{1}{2}$ %, until $ 7000 is reached. On all taxable income above $ 7000, 6 %.

EXERCISE 129

What is the income tax of a corporation in this state whose taxable income in 1918 was:

1. $ 2575
2. $ 3500
3. $ 4200
4. $ 5765
5. $ 16,000
6. $ 35,275
7. $ 142,650
8. $ 325,000
9. $ 175,000
10. $ 432,500
11. $ 522,850
12. $ 738,750

State Inheritance Tax

198. Some states derive income from inheritance taxes. The laws regulating these taxes also vary but have some common elements. The rate of taxation depends upon the amount of the inheritance, and secondly, upon the relationship of the beneficiary to the deceased. The following table gives the rates in one state, for near relatives.

BENEFICIARY	AMOUNT EXEMPT	RATE OF TAXATION ON:				
		First $ 25,000	$ 25,000 to $ 50,000	$ 50,000 to $ 100,000	$ 100,000 to $ 500,000	Amounts over $ 500,000
Husband or wife	$ 10,000	1 %	1½ %	2 %	2½ %	3 %
Children (each)	$ 2,000	1 %	1½ %	2 %	2½ %	3 %
Brother, sister, nephew, niece	$ 500	1½ %	2¼ %	3 %	3¾ %	4½ %
Uncle, aunt, cousin	$ 250	3 %	4½ %	6 %	7½ %	9 %

The tax is collected by the county in which the deceased lived. Out of the tax, 7½ % is retained by the county, and the balance is turned in to the state treasury.

EXERCISE 130

1. Out of an estate valued at $ 85,000, the following bequests were made: to wife, $ 40,000; to daughter and son, each $ 20,000; to brother and niece each $ 2000; to aunt, $ 1000.

a. How much tax was levied against each beneficiary?

b. What amount was retained by the county and what amount was turned in to the state treasury?

2. Assume that the following bequests are made to near relatives out of an estate:

To wife, $ 125,000; to son, $ 75,000; to daughter, $ 75,000;

to sister, $ 10,000; to nephew, $ 5000; to niece, $ 7500; to uncle, $ 3500; to aunt, $ 5000.

a. What tax was levied against each beneficiary?

b. What amount was retained by the county?

National Taxes

199. The money for the support of the national government is derived chiefly from **Duties** levied upon the most recent **Tariff Law** and taxes levied upon the most recent **Revenue Act of Congress.**

Duties

200. **Duties** or **Customs** are taxes levied upon goods imported into the United States.

A **Specific Duty** is a fixed tax upon an article without regard to its value. If the weight or quantity of the goods is expressed in metric units, these must be changed to equal English units.

Thus, the duty on maple sugar is 3 ¢ per pound.

An **Ad Valorem Duty** is levied at a certain rate per cent of the cost of the goods in the country from which they are imported. If this cost is expressed in units of foreign money, the cost must be expressed in units of our money.

Thus, the rate on silk clothing is 50 % ad valorem.

If a silk dress costs 500 fr. in France, the equivalent price in our money is 500 × $.193 or $96.50. The duty on it would be 50 % of $96.50 or $48.25.

Ad valorem duties are not computed on fractions of a dollar.

201. When the duty is specific, certain allowances are made.

Tare is an allowance made for the weight of the container in which the goods is imported.

Leakage is an allowance for the loss of any liquids imported in casks or barrels.

Breakage is an allowance for the loss of any liquid imported in bottles.

202. The **Tariff Law** provides rates of taxation on all articles likely to be imported.

NOTE. — A copy of the latest tariff law can be obtained through the representative in Congress from your district.

There is a list of articles on which duties are levied, — called the **Dutiable List**; there is also a list of articles on which duties are not levied, — called the **Free List.**

Thus, pure bred animals for breeding purposes, leather boots and shoes, coffee, enter free of duty.

203. Administration of the Tariff Law. — Goods purchased abroad must enter the United States through certain ports called **Ports of Entry.** At each such port, is located a United States Customs House. Proper officers at the customs house receive a description of important merchandise, examine some or all of a shipment, decide upon the duties that must be paid, and, after receiving the duties, issue the necessary permits to have the merchandise landed.

NOTE. — Importers may store goods in government warehouses, after giving bond for the payment of the duties. Later, upon payment of storage charges, duties, etc., the importer may withdraw his goods from the warehouse.

Individuals entering the United States from abroad must declare whether they have in their baggage any dutiable goods. Their baggage and persons are subject to inspection.

EXERCISE 131

What is the duty on:

1. 20 doz. watches @ 1275 fr. per doz., duty 30 % ad valorem?

2. 1 gr. gold rings @ 14 sh. for each ring, duty 60 % ad valorem?

3. 1 fur cape valued at £ 35, duty 40 % ad valorem?

4. An invoice of linen goods valued at £ 462, duty 40 % ad valorem?

5. 150 meters silk dress goods @ 15 fr. per meter, duty 45 % ad valorem?

6. 100 doz. table knives @ £ 1 per doz., duty 30 % ad valorem?

7. 300 gal. wine @ 20 fr. per gallon, duty 45 % ad valorem?

8. 500 T. sugar, duty $\frac{78}{100}$¢ per pound?

9. On 65 small oriental rugs having an average value of 1000 fr. each, duty 45 % ad valorem?

10. On 15 silk handkerchiefs valued at 3 sh. each, duty 45 % ad valorem?

11. On a diamond ring valued at £ 53, duty 60 % ad valorem?

12. On 100 meters lace @ 3 fr. per meter, duty 60 % ad valorem?

13. On 5 doz. pairs hose @ £ 1 per dozen, duty 40 % ad valorem?

14. On 150 pieces plate glass 20″ × 22″, 100 pieces 24″ × 28″, and 100 pieces 24″ × 32″, duty 8 ¢ per square foot?

15. On 750 lb. camphor, duty 1 ¢ per pound; 50 lb. benzoate of soda, duty 5 ¢ per pound; and 75 lb. menthol, duty 50 ¢ per pound?

16. On a telescope valued at 375 fr., duty 25 %?

17. On a marble statuette valued at £ 69, duty 45 %?

18. On an automobile valued at 12,500 fr., duty 45 %?

19. On 15 lb. of tobacco, duty $ 2.50 per pound?

20. On a suit of clothes valued at £ 5 8 sh., duty 35 %?

Internal Revenue

204. Revenue taxes for the Government derived from sources within the country are levied at present (1919) under a Revenue Act of Congress which was enacted in February, 1919. This act levies taxes upon the manufacture, sale, and consumption of various products; upon various forms of amusement and business; upon incomes of individuals and corporations; and upon inheritances.

NOTE. — A copy of this Act of Congress can be obtained from the Collector of Internal Revenue of your district, or through the Congressman from your district.

205. United States Income Taxes. — One schedule of rates is provided for the taxation of the incomes of individuals, and another for the incomes of corporations. The kind of income that is taxable is described in the revenue act.

An **Income Tax Return** must be filed by every unmarried person whose net income for the year is more than $1000, and by every "head of a family" whose net income is more than $2000. For the former, $1000 is exempt from taxation, and for the latter $2000 is exempt, and also for the latter $200 additional for each child under 18 years of age and for each dependent who is actually supported by the head of the family.

The law provides for a **Normal Tax** and a **Surtax** on the net annual incomes of an individual.

Normal Tax Rates. — On the first $4000 of net taxable income the rate is 4%; on the balance of the net income the rate is 8%.

Thus, a single man, having a net income of $8500 is allowed an exemption of $1000, so that his taxable income is $7500. On the first $4000 of this, the tax is 4% or $160; on the remaining $3500, the tax is 8% or $280. Hence his total normal tax is $440.

Surtax Rates. — If the net taxable income of an individual exceeds $ 5000, an additional tax is levied, at different rates for different parts of the income.

On All Income		The Rate Is	For Other Incomes Up to $ 100,000	On All Income		The Rate Is
Between	and			Between	and	
$5,000	$6,000	1 %		$100,000	$150,000	52 %
6,000	8,000	2 %	the rate is 1 % more for each additional $2000 until it is 48 % for incomes between $98,000 and $100,000.	150,000	200,000	56 %
8,000	10,000	3 %		200,000	300,000	60 %
10,000	12,000	4 %		300,000	500,000	63 %
12,000	14,000	5 %		500,000	1,000,000	64 %
14,000	16,000	6 %				
16,000	18,000	7 %		Over $1,000,000		65 %
18,000	20,000	8 %				
20,000	22,000	9 %				

The following examples show how to compute the surtax and the total tax.

Example 1. — The normal tax for a single man having a net income of $ 8500 was found to be $ 440. His surtax is computed as follows:

On the $1000 between $5000 and $6000, he pays 1 % or $10
On the $2000 between $6000 and $8000, he pays 2 % or 40
On the $500 between $8000 and $8500, he pays 3 % or 15
His total surtax is $65
His total income tax is $440 + $65 or $505.

Example 2. — Find the total income tax of a man and wife with one child if their net annual income is $ 8000.

Solution. — 1. The total exemption is $2000 + $200 or $2200. This makes the net taxable income upon which the normal tax is computed $5800.

2. On the first $4000, the tax is 4 % or $160
On the remaining $1800, the tax is 8 % or 144
3. Hence the total normal income tax is $304

4. The surtax is computed on that part of the income in excess of $5000.

On the first $1000 (*i.e.* up to $6000) the tax is 1 % or $10
On the next $2000 (*i.e.* up to $8000) the tax is 2 % or 40
5. Hence the total surtax is $50
6. Hence the total income tax is $304 + $50 or $354.

Example 3.—What is the income tax of a man and wife having an income of $25,000?

Solution.—1. Their exemption is $2000, so that their taxable income subject to the normal tax is $23,000.

2. Normal Tax. On the first $4000, the tax is 4 % or $ 160
On the remaining $19,000, the tax is 8 % or 1520
Hence the total normal tax is $1680

3. Surtax. The surtax is computed on all the income in excess of $5000.

On $1000 between $5000 and $6000, the tax is 1 % or $10
On $2000 between $6000 and $8000, the tax is 2 % or 40
On $2000 between $8000 and $10,000, the tax is 3 % or 60
On $2000 between $10,000 and $12,000, the tax is 4 % or 80
On $2000 between $12,000 and $14,000, the tax is 5 % or 100
On $2000 between $14,000 and $16,000, the tax is 6 % or 120
On $2000 between $16,000 and $18,000, the tax is 7 % or 140
On $2000 between $18,000 and $20,000, the tax is 8 % or 160
On $2000 between $20,000 and $22,000, the tax is 9 % or 180
On $2000 between $22,000 and $24,000, the tax is 10 % or 200
On $1000 between $24,000 and $25,000, the tax is 11 % or 110
Hence the total surtax is $1200

4. Hence the total income tax is $1680 + $1200 or $2880.

EXERCISE 132

Determine the total annual income taxes under the law of 1919 for:

1. A single man having an income of $3000 annually.
2. A man and wife having an income of $5000 annually.
3. A single man supporting his mother, having an annual income of $8500.

4. A man, wife, and child having a total annual income of $ 10,000.

5. A man and wife having a total annual income of $ 100,000.

6–10. What per cent of the income in each of the foregoing examples is the annual income tax?

NOTE.—The class will be interested in the tax on an annual income of $150,000, $200,000, $300,000, $500,000, $1,000,000, or $2,000,000. It is suggested that the class be divided into six groups and that the computations for these six annual incomes be distributed, one to each group.

206. United States Inheritance Taxes at the present time (1919) are levied under the Revenue Act of Congress enacted in February, 1919.

NATIONAL INHERITANCE TAX RATES

ON THAT AMOUNT OF THE NET ESTATE	1917 RATE	1919 RATE
Less than $50,000	0 %	0 %
Between $50,000 and $100,000	2 %	1 %
Between $100,000 and $200,000	4 %	2 %
Between $200,000 and $300,000	6 %	3 %
Between $300,000 and $500,000	8 %	4 %
Between $500,000 and $800,000	10 %	6 %
Between $800,000 and $1,050,000	10 %	8 %
Between $1,050,000 and $1,550,000	12 %	10 %
Between $1,550,000 and $2,050,000	12 %	12 %
Between $2,050,000 and $3,050,000	14 %	14 %
Between $3,050,000 and $4,050,000	16 %	16 %
Between $4,050,000 and $5,050,000	18 %	18 %
Between $5,050,000 and $8,050,000	20 %	20 %
Between $8,050,000 and $10,050,000	22 %	22 %
Exceeding $10,050,000	25 %	25 %

Two sets of rates are specified. The "1917" rates apply to the estates of persons who died between October 3, 1917, and the date when the present law became effective; the "1919" rates apply to estates of persons who have died since the present law became effective.

Example.—What is the national inheritance tax on an estate having a net taxable value of $235,000?

Solution.—1. The first $50,000 is not taxable.	
2. On the next $50,000, the tax is 1 % or	$ 500
3. On the next $100,000, the tax is 2 % or	2000
4. On the next $35,000, the tax is 3 % or	1050
5. Hence the total tax is	$3550

EXERCISE 133

Determine the national inheritance taxes which must be paid out of an estate having a net value of:

1. $ 85,000.
2. $ 133,500.
3. $ 265,800.
4. $ 372,500.
5. $ 437,900.
6. $ 643,000.
7. $ 875,000.
8. $ 962,000.
9. $ 1,350,000.
10. $ 6,840,000.
11. $ 13,520,000
12. $ 33,740,000.

XII. BANKING

207. Uses of Banks. Banks afford places of safe deposit for money. Money deposited in a *checking account* ordinarily does not draw interest. Such deposits give the depositors a corresponding amount of **Credit.**

Banks lend money or credit.

Banks furnish convenient means of transferring money or credit from one person or firm to another.

Promissory Notes

208. A **Promissory Note** is a written promise of one person (or firm) to pay a specified sum of money to another specified person (or firm).

The **Maker** of the note is the person who signs it, — the one promising to pay.

The **Payee** of a note is the person to whom payment is promised. One of the following forms is used.

a. Pay to John Davis.

He alone may demand payment.

b. Pay to John Davis or order.

He or any one whom he designates may demand payment.

c. Pay to bearer.

Any one legally possessing the note may demand payment.

The **Face** of a note is the sum to be paid.

The place of payment is usually specified. If it is not, then the note is payable at the office of or else at the home of the maker.

209. The **Date of Maturity** is the date when payment is due.

a. A **Demand Note** is payable "on demand," — payable whenever payment is requested *during business hours of a working day.*

b. A **Time Note** is payable at a future time that can be determined, — such as "thirty days after date."

NOTE. — If payable a certain number of months after date, then *calendar months* are meant.

If payable one month after the last day of any month, it is payable on the *last day* of the next month.

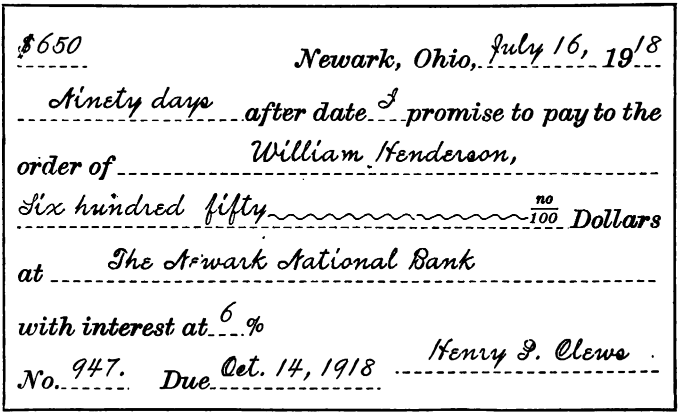
If payable a certain number of days after date, the actual number of days, including Sundays and holidays, is meant.

If a note becomes payable on Sunday, or on a holiday, or on a Saturday when Saturday is a half-holiday, the note is payable on the next business day.

Three additional days, called **Days of Grace**, in which payment might be made were formerly quite commonly allowed. In a few states they are still allowed.

210. Interest-bearing Notes. — If interest is to be paid on the note, then the words "with interest" or "with interest at a number of %" are added after the place of payment.

Value Received is commonly written on the face of the note, but is unnecessary.



$650 *Newark, Ohio,* July 16, *19*18

Ninety days *after date* I *promise to pay to the*

order of William Henderson,

Six hundred fifty ~~~~~~~~~~ $\frac{no}{100}$ *Dollars*

at The Newark National Bank

with interest at 6 *%*

No. 947. *Due* Oct. 14, 1918 Henry J. Clews

211. If a note is made payable to order or to bearer, it can be sold by the payee to another person. Such a note is **Negotiable.** Otherwise a note is **Non-negotiable.** The payee of a negotiable note may surrender his rights by **Indorsing** it on the back of the note, at its left end, in one of three forms.

a	*b*	*c*
William *Henderson*	*Pay to* *Martin Pratt* *or order* *William* *Henderson*	*Pay to* *Martin Pratt* *or order* *without recourse.* *William* *Henderson*

The form *a* is called a "blank indorsement"; form *b* a "full indorsement"; form *c* a "qualified indorsement." When using form *a* or *b* the indorser assumes responsibility for the payment of the note if the maker fails to pay. When using form *c*, the indorser does not guarantee the payment of the note.

212. Joint and Several Notes. A note reading "We promise to pay" and signed by two or more persons is a **Joint Note.** A note reading "I promise to pay" and signed by two or more persons is a **Several Note.**

In either case, the payee may collect the whole payment if necessary from any one of the makers. When the note is a joint note, then each of the makers can hold the others for their share of the face of the note.

213. To obtain payment of a note, the holder must present it to the maker on the date of maturity. If the maker refuses to make the payment, the holder may turn the note over to a notary public, who also presents it to the maker for payment. If payment is again refused, the notary makes formal **Protest**, and sends notice of the facts to all who have indorsed the note. Then, each indorser in turn becomes responsible for the payment of the note.

EXERCISE 134

Write the following notes, dated to-day at your home town, payable at a fictitious bank:

Ex. No.	Face of Note	Due	Payee	Maker	Interest
1.	$ 250	In 3 mo.	Henry Carroll or order	Charles Darrow	6 %
2.	575	In 90 da.	The bank	Stanley Fielding	None
3.	625	On demand	The bank	Frederick Burton	5 %
4.	1250	In 60 da.	James Davis or order	Samuel Aldrich	6 %
5.	1575	In 6 mo.	The bank	Fred Allison	6 %

6. Write a blank indorsement by the payee for the note in Example 1.

7. Write a full indorsement by the payee of the note in Ex. 4, transferring his rights to James Hanson, and the final indorsement by Hanson when he presented the note for payment at maturity. Also find the amount of the note at maturity.

8–10. Find the amount at maturity of the notes in Examples 1, 2, and 5.

214. In order to **Guarantee the Payment of a Note** the maker may:

a. Get some other person with financial credit to indorse it;

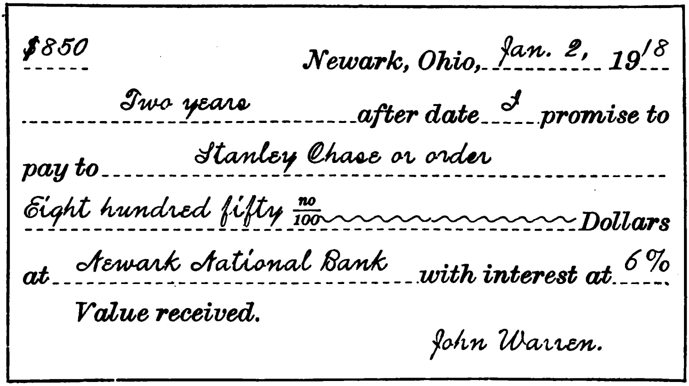
b. Offer security such as a mortgage on real property or certificates of ownership of other property. Such notes are called **Collateral Notes.** In form, they are similar to ordinary promissory notes except for additional clauses temporarily transferring title to the property offered as collateral security.

Partial Payments

215. Sometimes the holder of a note permits the maker of it to pay part of the face of the note before the date of maturity. Such payments are called **Partial Payments.**

In many cases, the holder of a note will not accept any partial payments except on interest payment dates, and then only if the partial payment is a specified sum such as a multiple of $ 50. Thus, if a note is dated on June 5, 1918, with interest payable semi-annually, partial payments might be accepted only on Dec. 5, and June 5, and in multiples of $ 100.

Whenever partial payments are accepted, they are recorded on the back of the note as below:



$850 Newark, Ohio, Jan. 2, 1918

Two years after date I promise to

pay to Stanley Chase or order

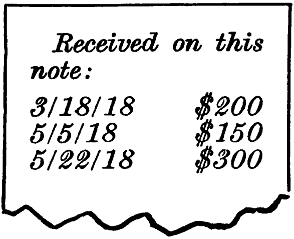
Eight hundred fifty no/100 Dollars

at Newark National Bank with interest at 6%

Value received.

John Warren.

Back of Note



Received on this note:

3/18/18 $200
5/5/18 $150
5/22/18 $300

216. One of two rules is employed to determine the amount still due on the note at maturity, when partial payments have been made.

The **Merchant's Rule** is usually employed when final settlement is made within a year from the date of the note.

Rule. — 1. To the face of the note, add the interest upon it from the date of the note to the date of the settlement.

2. To each partial payment, add the interest upon it from the date when it was made to the date of settlement.

3. From the result of Step 1, subtract the sum of the results in Step 2. The remainder is the balance due.

This rule will be applied to determine the amount due on Dec. 8, 1918, on the note with indorsements shown in § 215.

Solution. — 1. The following form may be used to advantage.

Note and Payments	Date Made	Date of Maturity	Time Elapsed	Face	Interest on	Amount of
Note	1 /2/'18	12/8/'18	11 mo. 6 da.	$850	$47.60	$897.60
Payments						
First	3/18/'18	12/8/'18	8 mo. 20 da.	200	8.67	208.67
Second	5/ 5/'18	12/8/'18	7 mo. 3 da.	150	5.33	155.33
Third	8/22/'18	12/8/'18	3 mo. 16 da.	300	5.30	305.30
Amount of all payments						669.30
Balance due on note						228.30

In columns 1, 2, and 4 are recorded the date of the note and of the payments, the date of final settlement, and the face of the note and of the payments. In column 3, is entered the difference in time between the corresponding dates in columns 1 and 2; thus, the first payment draws interest for 8 mo. and 20 da. The interest is then computed and entered in column 5, and the amount in column 6. Then the sum of the amounts of the three payments is found and subtracted from the amount of the note. This is the amount still due at the time of the final settlement.

EXERCISE 135

Using either the form suggested on p. 308 or some other neat form, determine the amount due at maturity on the following notes:

1. A note for $ 900, with interest at 6 %, dated March 20, 1918, and due October 20, 1918, has the following payments on it: April 12, 1918, $ 200; July 18, 1918, $ 350.

2. A demand note for $ 1050, with interest at 6 %, dated Jan. 8, 1918, was presented for settlement on Dec. 20, 1918. Partial payments as follows were recorded on it: March 15, 1918, $ 175; July 9, 1918, $ 325; Oct. 21, 1918, $ 250.

3. A note for $ 2500 with interest at 6 %, dated Aug. 23, 1918, was presented for settlement on May 12, 1919, after preliminary payments as follows: Nov. 17, 1918, $ 800; Jan. 25, 1919, $ 460; Feb. 16, 1919, $ 1200.

4. On a note for $ 980 given May 18, 1918, and drawing 5 % interest, three payments were made: June 10, 1918, $ 360; Sept. 3, 1918, $ 300; Oct. 17, 1918, $ 180. What was due April 27, 1919?

5. On a note for $ 2600, dated Sept. 13, 1917, and bearing interest at 6 %, the following indorsements were made: Nov. 28, 1917, $ 300; Jan. 20, 1918, $ 450; March 12, 1918, $ 625; May 18, 1918, $ 520. How much was due July 28, 1918?

6. A note for $ 1698, dated Jan. 14, 1918, is indorsed as follows: Feb. 27, 1918, $ 215; Aug. 15, 1918, $ 820; Aug. 28, 1918, $ 120; Oct. 11, 1918, $ 275. At 6 % interest, what is due Nov. 15, 1918?

7. On a note for $ 3750, dated Nov. 17, 1918, and bearing interest at 4 %, the following payments were made: Dec. 20, 1918, $ 1000; May 25, 1919, $ 550; June 21, 1919, $ 375; Sept. 22, 1919, $ 700. How much was due Oct. 27, 1919?

8. A demand note for $ 1500, with interest at 6 %, dated Feb. 24, 1917, was presented for settlement Feb. 8, 1918. Partial payments as follows were recorded on it: May 15, 1917, $ 150; Aug. 5, 1917, $ 175; Nov. 8, 1917, $ 225; Jan. 2, 1918, $ 325. How much was due at the time of settlement?

9. On a note for $ 3875, given July 2, 1918, and drawing 5 % interest, three payments were made: Sept. 5, 1918, $ 875; Dec. 1, 1918, $ 500; March 15, 1919, $ 375. What was due on June 15, 1919?

10. A note for $ 2775, with interest at 6 %, dated Oct. 12, 1918, was presented for settlement on Aug. 15, 1919, after the following payments had been made: Nov. 26, 1918, $ 175; Dec. 24, 1918, $ 225; Feb. 25, 1919, $ 125; April 25, 1919, $ 475; June 15, 1919, $ 775. How much was due?

217. The United States Rule. If the final settlement is made more than one year after the date of the note, the amount due is found by the following rule, adopted by the Supreme Court of the United States.

Rule. — 1. Find the amount of the face of the note to the time when the payment or the sum of the payments made equals or exceeds the accrued interest.

2. From this amount, subtract the payment or the sum of the payments.

3. Consider the remainder the new face of the note and proceed as before up to the date of the settlement.

Notice that the interest to date must be paid before any partial payment or payments can be used to decrease the face of the note. The justification for this rule is that the lender has a right to expect his interest on the debt before consenting to apply a payment to a reduction of the debt.

Example. — What amount was due on Jan. 5, 1919, on a note for $ 850 dated Oct. 10, 1916, bearing interest at 6 %, on which the following indorsements had been made: Dec. 23, 1916, $ 325; Aug. 11, 1917, $ 15; July 16, 1918, $ 150.

Solution. — 1. Face of note	$ 850.00
2. Interest on $ 850 from Oct. 10, 1916, to Dec 23, 1916, 2 mo. 13 da.	10.34
3. Amount to Dec. 23, 1916,	$ 860.34
4. First partial payment	325.00
5. Subtracting, the new principal is	$ 535.34
6. Interest on $ 535.34 from Dec. 23, 1916, to Aug. 11, 1917, 7 mo. 19 da.	20.44
(Since the second payment is less than this interest, no reduction of the principal or interest is made.)	
7. Interest on $ 535.34 from Aug. 11, 1917, to July 16, 1918, 11 mo. 5 da.	29.89
8. Amount of $535.34 to July 16, 1918	$ 585.67
9. The sum of the 2d and 3d partial payments	165.00
10. Subtracting, the new principal is	$ 420.67
11. Interest on $ 420.67 from July 16, 1918, to Jan 5, 1919, 5 mo. 20 da.	11.92
12. Amount due on Jan. 5, 1919	$ 432.59

EXERCISE 136

Find the amount due on the following notes on the date of settlement, using the United States Rule.

1. What amount was due July 17, 1919, on a note for $ 1395, dated Jan. 27, 1917, and bearing interest at 6 %, on which the following indorsements had been made: March 15, 1918, $ 325; Oct. 25, 1918, $ 200.

2. On Nov. 19, 1916, a demand note was issued for $ 680, with interest at 5 %. On Aug. 29, 1917, $ 20 was paid; on Nov. 16, 1917, $ 400. What was due Sept. 16, 1918?

3. On a note for $ 2500, dated Feb. 21, 1915, and drawing interest at 6 %, there was paid, Feb. 17, 1916, $ 850; July 12, 1917, $ 50; and Feb. 4, 1918, $ 750. What was due May 14, 1919?

4. A note for $ 1750, dated Aug. 17, 1917, and drawing 6 %, is indorsed as follows: April 19, 1918, $ 75; July 22, 1918, $ 880; Nov. 12, 1918, $ 550. What was due Dec. 13, 1918?

5. A note for $ 540, dated April 16, 1916, and bearing interest at 4 %, had indorsements as follows: April 30, 1916, $ 100; Nov. 26, 1916, $ 220; Aug. 13, 1918, $ 25. How much was due Nov. 29, 1919?

6. On a note for $ 800, given May 25, 1915, and bearing interest at 6 %, four payments were made: Feb. 13, 1916, $ 30; July 18, 1916, $ 200; June 24, 1917, $ 24; Oct. 3, 1917, $ 150. What was due Feb. 27, 1918?

7. A note for $ 480, dated March 16, 1917, is indorsed as follows: Aug. 29, 1917, $ 15; Sept. 12, 1917, $ 150; Sept. 12, 1918, $ 120. At 7 % interest, how much was due April 14, 1919?

8. On a note for $ 1925, dated Dec. 12, 1914, and bearing 6% interest, the following payments were made: June 17, 1915, $ 110; Sept. 23, 1916, $ 300; Nov. 11, 1917, $ 75; Oct. 17, 1918, $ 460; and March 27, 1919, $ 275. How much was due Aug. 21, 1919?

9. A note for $ 575, dated July 14, 1918, was indorsed as follows: Aug. 17, 1918, $ 50; Oct. 29, 1918, $ 100; Jan. 15, 1919, $ 130. How much was due on it on Sept. 15, 1919, if the rate of interest was 6 %?

10. The following payments were made on a note for $ 2340, bearing 6 % interest, dated Oct 19, 1918: Jan. 15, 1919, $ 250; July 10, 1919, $ 375; Dec. 20, 1919, $ 525; May 18, 1920, $ 600. What is the sum due July 15, 1920?

Discounting Notes

218. Banks, when lending money, may deduct the interest in advance.

Thus, a man borrowing $1000 from a bank for 60 days, with interest at 6 %, may be asked to pay in advance $10, the interest on $1000 at 6 % for 60 days.

The bank, in this case, is said to **Discount the Note.**

A person or firm, having accepted from an individual his note, and wanting the money at once, may ask a bank to purchase the note. Again, the bank is said to **Discount the Note.**

Example. — The Acme Manufacturing Co. holds a non-interest-bearing note for $ 1000, payable in 60 days. They ask a bank to discount it. What are the proceeds?

Solution. — The bank may offer $990 for the note. In doing so, the bank deducts $10 from the face of the note, this being the interest on $1000 at 6 % for 60 days. The proceeds are $990.

219. Bank Discount is a charge made by a bank for the payment of a negotiable note before the note is due.

The day on which the note is discounted *may not* be the day on which the note was written. *The time from the day of discount to the date of maturity is the* **Term of Discount.**

The **Discount** is a certain per cent of the *amount* of the note at maturity. The rate per cent used is called the **Rate of Discount.** This *may not be the same* as the rate of interest payable on the face of the note.

The **Proceeds** of a discounted note is the **amount at maturity less the discount.**

Example 1. — Find the proceeds of a 90-day note for $ 500 discounted on the date of making at 6 %.

Solution. — **1. Since the note is non-interest-bearing, its amount at maturity is $ 500.**

2. Being discounted on the date of making, the term of discount is 3 mo.

3. The discount = the interest on $ 500 at 6 % for 3 mo. = $7.50.

4. ∴ the proceeds = $ 500 − $ 7.50 = $ 492.50.

Example 2. — Henry Sanders asked his bank to discount for him on Oct. 25, 1918, a note for $ 1200, payable in 90 days, without interest, dated Oct. 15, 1918. What were the proceeds?

Solution. — 1. The note did not bear interest, hence the amount at maturity was $ 1200.

2. Being a note for 90 days, it matures 90 days after Oct. 15. Hence on Oct. 25 it still has 80 days to run, — the term of discount.

3. The bank offers to discount it at 6 %. The discount, therefore, is the interest at 6 % for 80 days on $ 1200, or $ 16.

4. The proceeds are $ 1200 − $ 16, or $ 1184.

Example 3. — Find the proceeds of a note for $ 750 dated May 12, 1918, payable 90 days after date, bearing interest at 5 %, if discounted June 23, 1918, at 6 %.

Solution. — 1. The interest on $ 750 at 5 % for 90 days is $ 9.38. Hence the amount of the note at maturity = $ 750 + $ 9.38 = $ 759.38.

2. The note matured 90 days after May 12, or on Aug. 10.

3. The *term of discount* was from June 23 to Aug. 10, or 48 days.

4. The *discount* was the interest on $ 759.38 for 48 days at 6 %, or $ 6.08.

5. The proceeds were $ 759.38 − $ 6.08, or $ 753.30.

EXERCISE 137

Find the bank discount and the proceeds of the following notes. Assume that the rate of discount is 6 % in each case.

Each example provides material for three parts.

Part *a*, assume the date of discount to be the same as the date of the note; parts *b* and *c*, the dates of discount are given.

If the teacher desires, part *a* for all may be done before parts *b* and *c* of the first example. Or, all three parts for examples 1, 2, and 3 may be done before any of the parts for the remaining ones. If additional examples are desired, the sum in any example may be changed to any other sum.

Ex. No.	Face of Note	Date of Note	Time of Note	Rate of Interest	Date of Discount		
					a	*b*	*c*
1.	$ 600	2/ 7/18	60 da.	none	2/7/18	2/20/18	3/ 5/18
2.	250	5/11/18	90 da.	none	5/11/18	6/ 1/18	7/15/18
3.	875	3/16/19	6 mo.	none	3/16/19	5/20/19	7/ 2/19
4.	1000	4/20/18	30 da.	6 %	4/20/18	4/25/18	5/ 1/18
5.	1600	7/12/18	20 da.	6 %	7/12/18	7/15/18	7/25/18
6.	2400	9/18/18	18 da.	5 %	9/18/18	9/20/18	9/30/18
7.	3000	11/22/18	15 da.	5 %	11/22/18	11/25/18	11/30/18
8.	375	7/29/18	90 da.	4 %	7/29/18	8/30/18	9/15/18
9.	560	8/24/18	1 yr.	6 %	8/24/18	12/20/18	3/ 5/19
10.	1350	12/15/18	60 da.	5 %	12/15/18	1/10/19	2/ 1/19

11. What are the proceeds of a 60-day note for $ 950, with interest at 5 %, discounted at 6 % ten days after the date of the note?

12. Find the bank discount on a note for $ 625, with interest at 6 %, payable in 4 months, if discounted 2 mo. 9 da. after the date of the note.

13. What was the bank discount on a note for $ 850, dated July 13, 1918, and payable, with interest at 6 %, 90 days after date if discounted at 6 % on Aug. 25, 1918?

14. What will be the proceeds of discounting at 7 % on the date of making a 30-day note for $ 3000, without interest?

15. Find the bank discount on a note for $ 910, dated Aug. 26, 1918, payable 90 days after date, with interest at 5 %, if discounted Nov. 3, 1918, at 6 %.

16. Write a 90-day note for $ 738 of which you are the payee and Henry Anthony is the maker, dated Aug. 27, 1918, with interest at 6 %, payable at the Farmer's Bank of Janesville, Ohio. Assume that you discounted the note on Sept. 15, 1918, at 6 % at some bank in your town. Properly indorse it over to that bank, and determine the proceeds.

17. On Jan. 16, 1919, S. L. Payne made out in favor of the Anderson Automobile Co., of Anderson, Ky., the following notes to apply on the purchase price of an automobile:

One for $ 250, due in 30 days, interest at 6 %.
One for $ 300, due in 60 days, interest at 6 %.
One for $ 450, due in 90 days, interest at 6 %.

a. Write the notes.

b. On Jan. 20, 1919, the automobile company discounted all three notes at 6 %, at the Anderson State Bank, using the full form of indorsement in transferring them to the bank. Write the indorsements and determine the total proceeds of the notes.

Find the proceeds of the following notes.

Ex. No.	Face	Date of Note	Time of Note	Rate of Interest	Date of Discount	Rate of Discount
18.	$ 475	5/11/18	30 da.	6 %	5/20/18	7 %
19.	1250	7/15/18	90 da.	5½ %	8/10/18	6 %
20.	360	6/28/18	45 da.	8 %	7/15/18	6 %
21.	980	12/20/18	60 da.	6 %	1/5/19	7 %
22.	125	11/16/18	15 da.	8 %	11/20/18	8 %
23.	1450	9/30/18	2 mo.	5 %	10/20/18	6 %
24.	3500	8/24/18	4 mo.	5½ %	10/15/18	6 %
25.	2675	12/19/18	6 mo.	5 %	2/1/19	6 %

Exchange

220. The process of transferring credit between individuals and firms is called **Exchange.**

a. **Domestic Exchange** is the process of transferring credit between individuals in this country.

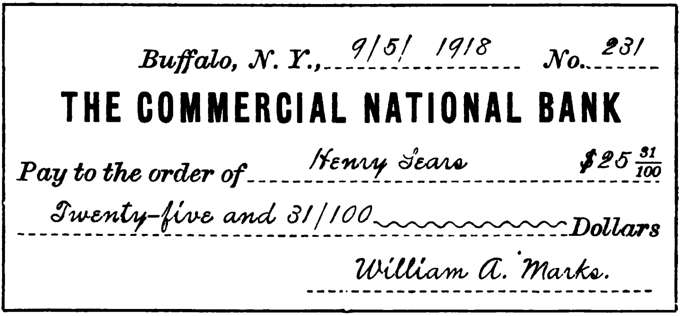
b. **Foreign Exchange** is the process of transferring credit between individuals in this country and those of another country.

Exchange of credit is accomplished by means of **Checks, Money Orders,** and **Drafts.**

Checks

221. A **Check** is a written order upon a bank made by one of its depositors, to pay a specified sum of money to a specified individual or his order.

The bank is the **Drawee**; the person signing the check is the **Drawer**; the one to whom it is payable is the **Payee**; the sum to be paid is the **Face of the Check.**



Buffalo, N. Y., 9/5/ 1918 *No.* 231

THE COMMERCIAL NATIONAL BANK

Pay to the order of Henry Sears $25 $\frac{31}{100}$

Twenty-five and 31/100 *Dollars*

William A. Marks.

Henry Sears may cash the check himself or he may surrender his right by using one of the three forms of indorsement discussed in § 211.

222. If Henry Sears as well as William Marks is a depositor of the Commercial National Bank, and Henry Sears presents the check for deposit at this bank, the clerks there merely transfer credit of $25.31 from the account of Mr. Marks to the account of Mr. Sears. This is *the simplest form of the problem of exchange.*

Mr. Sears, or some one to whom he indorses the check, may deposit or cash the check in a *different* bank.

Banks are in this way daily cashing or accepting for deposit checks drawn upon other banks. In a small community, each bank is likely to send a messenger daily to each of the other banks of the community whose checks have been accepted, to ask for money to the value of these checks. Thus, credits in one bank are transferred to another bank. This also is *a simple problem of exchange.*

In a larger community, the settling up between banks is accomplished through a **Clearing House.** Clerks representing each of the members of the clearing house association gather at a specified time, exchange with one another the checks of their respective banks, recording the amounts thereof, and report to the manager of the association the totals of these exchanges. By suitable procedure, the indebtednesses between the member banks are all paid within a few hours. This is the most complex problem of exchange in a given community.

223. Banks in small communities usually have funds on deposit in at least one bank of a near-by larger community. In particular, banks of the country are grouped in twelve districts in each of which is a **Federal Reserve Bank**, with which most of the banks of that district are associated.

224. Payments made by Checks to persons in other towns.

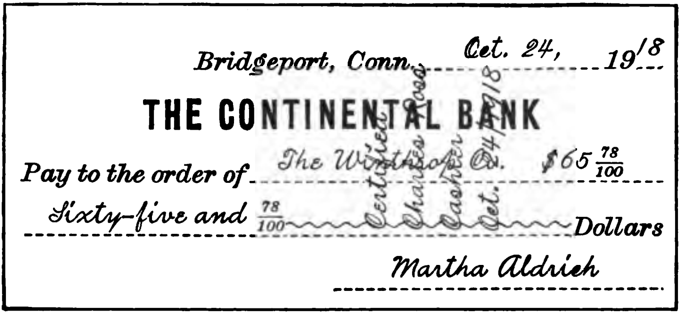
Example. — A man in Texas may send a check to his son in college at Cambridge, Massachusetts. His son may deposit the check in a Cambridge bank. The bank will send the check, probably, to the Federal Reserve Bank at Boston, receiving credit for it. The Federal Reserve Bank at Boston will send it to the Federal Reserve Bank at Dallas, Texas, receiving credit for it. If the bank on which the check is drawn is a member of the Dallas Federal Reserve Bank, the latter, deducting the amount of the check from the account of the issuing bank, sends the canceled check to that bank. The issuing bank, in turn, deducts the amount of the check from the balance of the drawer. Thus, credit has been transferred from the father in Texas to the son in Massachusetts. Sometimes a small fee, $\frac{1}{10}$ % of the face of the check, is charged as exchange by the bank cashing the check. The other banks handle the check as a matter of business courtesy between themselves.

This process represents *a complicated form of Domestic Exchange.*

Other Ways of Transferring Credit

225. Since a check is signed only by an individual, persons unacquainted with him, or the payee, often do not wish to accept the check. Hence, for transfers of funds to a considerable distance certain other means are often employed.

226. A **Certified Check** is a personal check of a depositor in a bank, across the face of which an officer of the bank has written over his signature the word "Good" or "Certified."



Bridgeport, Conn. Oct. 24, 1918

THE CONTINENTAL BANK

Pay to the order of The Winthrop Co. $65 $\frac{78}{100}$

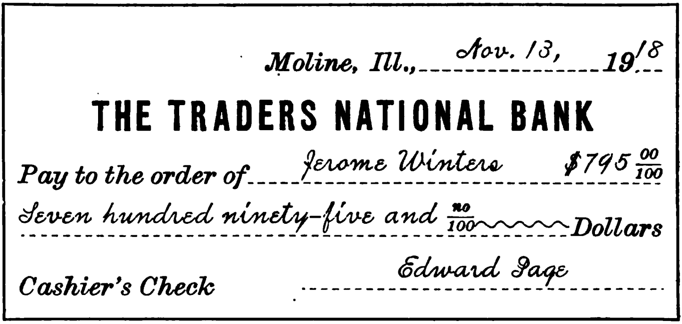
Sixty-five and $\frac{78}{100}$ Dollars

Martha Aldrich

Certified
Charles Todd
Cashier
Oct. 24/1918

Such certification dispels any doubt about the value of the check, for it is a criminal offense for a bank to certify a check unless the face of it is actually on deposit in the bank.

227. A **Cashier's Check** is a check drawn by the cashier of a bank, making a sum of money payable to an individual. There is not any charge, usually, for issuing such a check to a regular depositor of a bank.



Moline, Ill., Nov. 13, *19*18

THE TRADERS NATIONAL BANK

Pay to the order of Jerome Winters $\$795\frac{00}{100}$

Seven hundred ninety-five and $\frac{no}{100}$ *Dollars*

Cashier's Check Edward Page

To obtain such a check, Mr. Winters must pay the bank the equivalent of it. The check is readily accepted in exchange, for it has behind it the reputation of the bank.

228. There are other methods of transferring money from one community to another, which should be mentioned because of their connection with the preceding sections.

Post Office Money Orders may be used to transfer credit either to a point in this county or abroad. They are especially useful for sending small sums of money.

The members of the class should each obtain or be given an Application for Domestic Money Order. They may be obtained at any post office. Upon the back of this Application will be found the rates for money orders.

Money may be Transferred by Telegraph. The only advantage in so transferring credit is the speed with which it can be done. The rates and provisions are obtainable at any telegraph office.

Travelers' Checks are used very commonly by persons who are traveling. Express companies and banks charge $\frac{1}{2}$% of the face of the check as commission.

EXERCISE 138

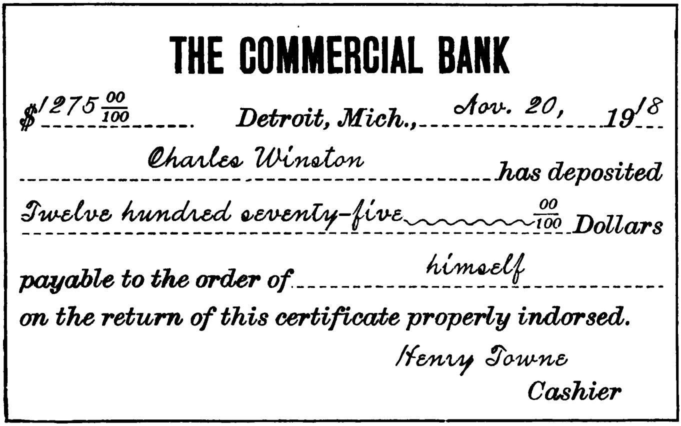
What is the rate for a money order for:

1. 75¢?
2. $1.65?
3. $4.98?
4. $11.32?
5. $9.45?
6. $22.63?
7. $2.78?
8. $6.32?
9. $35.69?
10. $67.84?
11. $43.26?
12. $72.58?

What will be the charge for a traveler's check for:

13. $200?
14. $350?
15. $575?
16. $1500?
17. $2500?
18. $1850?
19. $675?
20. $3200?
21. $1250?
22. $225?
23. $385?
24. $750?

229. A **Certificate of Deposit** also may be used in transferring credit.



THE COMMERCIAL BANK

$1275 $\frac{00}{100}$ *Detroit, Mich.,* *Nov. 20,* *19 18*

Charles Winston *has deposited*

Twelve hundred seventy-five $\frac{00}{100}$ *Dollars*

payable to the order of *himself*

on the return of this certificate properly indorsed.

Henry Towne
Cashier

Such certificates of deposit serve as receipts for funds left on deposit with a bank. Banks pay interest on such funds.

In one community, 2½ % interest is allowed on funds remaining on deposit three months, 3 % on money on deposit six months, and 4 % on money on deposit one year.

By properly indorsing the certificate, the owner of it may transfer his credit to some one else.

The certificate may be made payable to some one else at the start.

EXERCISE 139

1. *a.* Write a certificate of deposit, dated to-day from a fictitious bank in your town, stating that you have deposited $ 350 payable to yourself.

b. Assume that you leave the money on deposit three months. What interest will you draw?

c. Assume that you wish to transfer the money at the close of the three months to Henry Randolph. Write the proper indorsement.

2. What interest at the rates mentioned above will be due on a certificate of deposit:

a. For $ 1800 left on deposit six months?

b. For $ 785 left on deposit three months?

c. For $ 1235 left on deposit for one year?

Drafts and Acceptances

230. A more popular means of sending money to a distance is a special form of check called a **Draft**.

A **Draft** is a written order from one individual, firm, or bank to another, directing the latter to pay a specified sum of money to a third party.

The **Drawer** is the person signing the draft; the **Drawee** is the person, firm, or bank to whom the draft is directed; the **Payee** is the one to whom payment is to be made.

Example. — Assume that Mr. Sampson of Philadelphia, Pa., owes Mr. Bristol of Peoria, Ill., $ 600. He may go to a bank in Philadelphia and purchase a draft, payable to Mr. Bristol. Since the Philadelphia bank does not have a correspondent in Peoria, it will probably give him a draft on a Chicago bank.

To obtain this draft, Mr. Sampson must pay the Philadelphia bank $ 600. Also, a charge of $\frac{1}{10}$% of the face of the draft is often made. This charge is very commonly omitted when the purchaser of the draft (Mr. Sampson) is a regular depositor in the bank.

Mr. Sampson will send the draft to Mr. Bristol. The latter, to obtain the money, will proceed exactly as if he had received a check (§ 221). The bank cashing the draft may make a charge for the cost of collecting the face of the draft.

NOTE. — 1. Mr. Sampson may have the draft made payable to him. In this case, he will indorse it over to Mr. Bristol, using the full indorsement (§ 210) and then send it to Mr. Bristol.

NOTE. — 2. The words "value received" and "charge to the account of" are not necessary.

When both the drawee and drawer are banks, the draft is called a **Bank Draft**. A bank draft is particularly desirable as a means of sending money to a distance, because it has behind it the business reputation of a bank.

Usually, a bank draft is a sight or demand draft. Payment is specified by such phrases as "pay on demand" or "pay at sight." It is payable as soon as presented and will be cashed or accepted for collection by any bank.

EXERCISE 140

Write the following drafts, and determine the charge if the rate in each case is $\frac{1}{10}$ %; date each *to-day.*

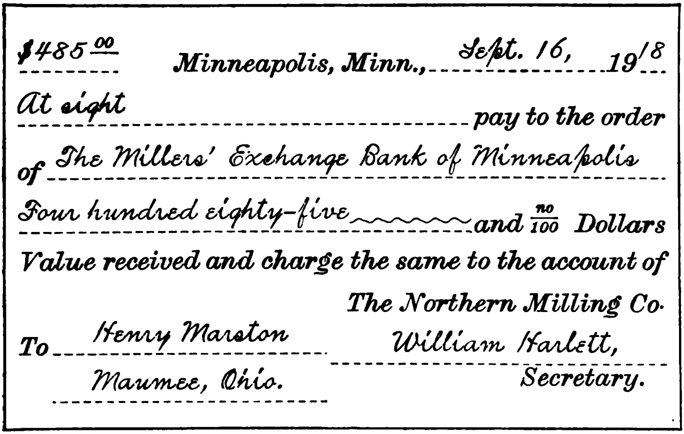
Ex. No.	Drawer	Drawee	Payee	Amount
1.	Farmer's Bank, Canton, Ohio	First National Bank, Cleveland, Ohio	Charles Hancock	$ 325
2.	A fictitious bank in your town	Exchange Bank, Detroit, Mich.	Sarles and Davis	674.45
3.	A fictitious bank in your town	Continental Bank, Chicago, Ill.	Marshall Field and Co.	835.60

4. Charles Johnson, Ames, Pa., owed William Jordan of Pittsburgh, Pa., $ 175. He applied at his bank, the Ames Merchants Bank, for a sight draft payable to William Jordan. The Merchants Bank through their cashier, Arthur Giles, drew upon the Second National Bank of Pittsburgh on Feb. 14, 1919. Write the draft.

5. Using the same data as in Ex. 4, make the draft payable to Charles Johnson and then write a full indorsement of it by Johnson over to William Jordan.

231. Drafts are used also **as a Means of Collecting Money** in commercial transactions. Such drafts are called **Commercial Drafts.** They may be either *sight drafts* or *time drafts.*

Commercial Sight Draft. Suppose that the Northern Milling Company of Minneapolis, Minn., sells to Henry Marston of Maumee, Ohio, a carload of bran. They receive from the railroad company a **Bill of Lading.** Mr. Marston must present this bill of lading to the railroad delivering the car before he can get possession of the car. If the Milling Company wants to insure payment for the bran, they make out the following sight draft upon Mr. Marston.



$\$485^{00}$ Minneapolis, Minn., Sept. 16, 1918

At sight pay to the order

of The Millers' Exchange Bank of Minneapolis

Four hundred eighty-five ~~~~~~~~ and $\frac{no}{100}$ Dollars

Value received and charge the same to the account of

The Northern Milling Co.

To Henry Marston William Harlett,

Maumee, Ohio. Secretary.

The Milling Company deposits this draft, attached to the bill of lading, at its bank in Minneapolis. The bank sends it to a bank in Maumee, Ohio. That bank notifies Mr. Marston that it has the bill of lading and draft. Mr. Marston, after paying the face of the draft, receives the bill of lading, and can then get his carload of bran.

The Maumee bank *may charge* a small fee for its services. This charge may be a small fee (25 ¢) or a small per cent ($\frac{1}{10}\%$) of the face of the draft. It sends the net proceeds to the Minneapolis bank, and the latter credits the account of the Milling Company. The Minneapolis bank may, but probably does not, make any charge for its services to the Milling Company, because the latter are regular customers of the bank. The minimum charge is usually 25 ¢.

232. A **Commercial Time Draft** is payable after the expiration of a specified length of time. Payment is specified by such phrases as "15 days after date," or "15 days after sight."

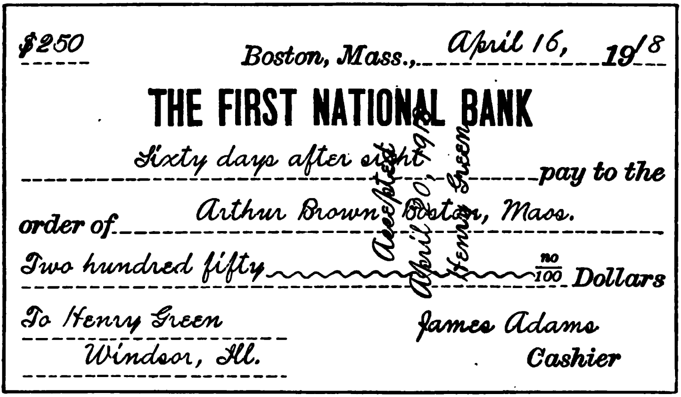
In such cases, the draft is presented to the drawee to determine whether he intends to pay the money. If the drawee decides to pay the draft, he signifies his intention by writing across the face of the draft the word "Accepted" with the date and his name.

After the draft is accepted, it is called an **Acceptance.**

An acceptance is a form of promissory note. The drawee, having accepted it, thereby promises to make payment at a certain time.

The date of maturity is reckoned as the specified number of days from the date of the draft, if it is payable a number of days after date, or as the specified number of days from the date of acceptance if it is payable a number of days after sight.

Example. — Assume that Henry Green of Windsor, Ill., has owed Arthur Brown of Boston, Mass., $250 for some time, and that Brown wishes to compel Green to pay off the debt. Brown may have his bank draw up the following draft.



$250 Boston, Mass., April 16, 1918

THE FIRST NATIONAL BANK

Sixty days after sight pay to the order of Arthur Brown, Boston, Mass.

Two hundred fifty no/100 Dollars

To Henry Green
Windsor, Ill.

James Adams
Cashier

Accepted
April 20, 1918
Henry Green

Mr. Brown deposits the draft in his bank, indorsing it as at the right.

Pay to
the order of the
First National Bank
for collection only.
Arthur Brown.

The First National Bank sends the draft to the Windsor State Bank of Windsor, Ill., for collection, indorsing it as at the right.

Pay to the order of
The Windsor State Bank
Windsor, Ill.
for collection only.
The First National Bank
Boston, Mass.

The Windsor State Bank informs Mr. Green that it has a draft drawn upon him. He, wishing to retain a good business reputation with the bank, decides to pay the draft; he writes across the face of it his "acceptance," as indicated on the draft, by writing across its face the word "Accepted," the date, and his name.

Having accepted it on April 20, 1918, he must pay it on or before June 19, 1918.

When Green pays the draft, his bank marks it as paid and delivers it to him; also, after deducting, possibly, a small fee such as $\frac{1}{10}$ % of the face for collection, his bank remits the net proceeds to the First National Bank. The latter places the proceeds to the credit of Mr. Brown.

EXERCISE 141

1. *a.* Write a thirty-day time draft of which you are the payee, and also the drawer, and of which Samuel Hughes, Pittston, Pa., is the drawee. Date it to-day. Assume that Mr. Hughes accepts the draft five days from to-day. *b.* Write across the face of the draft his acceptance. *c.* What is the date of maturity?

2. *a.* Write a sixty-day time draft, dated to-day, at your town, of which you are the payee and also the drawer, drawn upon the Paducah, Tenn., Milling Company. *b.* Assume that you deposit it for collection at a fictitious bank in your town. Properly indorse it. *c.* Assume that your bank sends it for collection to a fictitious bank in Paducah.

Properly indorse it for the bank. *d.* Assume that the bank presents it to the Milling Company and that the company accepts it ten days from to-day. Write the acceptance. *e.* What is the date of maturity? *f.* Suppose that the Paducah bank charges $\frac{1}{10}$% for collection. What will be the proceeds?

3. Ernest Waters, Williamstown, Ky., bought $850 worth of merchandise from Dickson, Hughes, and Company, of Toledo, Ohio. The latter, after waiting some time for their money, made out on July 25, 1918, a thirty-day time draft, payable to themselves, naming Mr. Waters as drawee. They deposited the draft for collection with the Republic Bank of Toledo, Ohio. The latter sent it to the Center Co. National Bank at Williamstown, Ky., for presentation. On July 30, Mr. Waters accepted it.

a. Write the draft, the indorsements, and the acceptance.

b. When is the draft due?

c. If the Williamstown Bank makes a charge of $\frac{1}{10}$% for collection, what amount will they remit to the Toledo Bank?

233. In case the payments from one community to another are much greater than the collections for it from the other, then it becomes necessary to send actual money from the first community to the second to pay the indebtedness. Persons in the first community asking for drafts to make payments in the second community will then be asked to pay a fee for exchange, amounting possibly to 1% of the face of the draft. In this case, **Exchange** on the second community is **at a Premium** or **above Par.**

On the other hand, if the credits of one community are greater than its debits, then the person offering to purchase a draft payable in the second community may be offered a discount. **Exchange** is then **at a Discount** or **Below Par.**

The Rate of Exchange is the rate per cent of the face of a draft charged for drawing the draft when business conditions make it necessary to control the flow of money from or into a community. This charge is in addition to the fee for issuing the draft.

EXERCISE 142

1. Is the flow of money encouraged or discouraged when exchange is above par? Below par?

Find the cost of a sight draft for:

2. $500 when exchange is at a premium of $\frac{1}{8}$%, — no charges for issuing the draft.

3. $285 when exchange is at a discount of $\frac{3}{4}$%, if the charge for issuing the draft is $\frac{1}{10}$%.

4. $1975 when exchange is at a discount of $\frac{1}{4}$%, — charges $\frac{1}{10}$% for issuing the draft.

5. $2500 when exchange is at a premium of $\frac{7}{8}$%, — no charge for issuing the draft.

6. $650 when exchange is at a discount of $\frac{1}{2}$%, — $\frac{1}{10}$% for issuing the draft.

234. Discounting Acceptances. Since an acceptance is a form of promissory note, it will be discounted on the same conditions as a promissory note. It will be assumed that the drafts are payable a number of days after sight, unless something is said to the contrary.

Example. — The Hartford Manufacturing Co. had a ninety-day draft for $1250 drawn upon James Markley, dated May 12, and accepted by Markley on May 15. Needing the money, they asked their bank to discount the acceptance on May 25. What were the proceeds if the rate of discount was 6%?

Solution. — 1. The date of acceptance was May 15.

2. ∴ the date of maturity was 90 days after May 15.

3. The acceptance was presented 10 days after May 15, hence the term of discount (§ 218) was 90 – 10 or 80 days.

4. ∴ the discount was the interest on $1250 at 6 % for 80 days. This is $16⅔ or $16.67.

5. ∴ the proceeds were $1250 – $16.67 or $1233.33.

EXERCISE 143

Find the proceeds of the following acceptances if discounted at 6 %:

Ex. No.	Face	Date	Time	Accepted	Discounted	
					a	*b*
1.	$ 300	7/14/18	30 da.	7/15/18	7/15/18	7/20/18
2.	2850	10/3/18	90 da.	10/10/18	10/10/18	11/1/18
3.	625	9/24/18	60 da.	9/30/18	9/30/18	10/25/18
4.	1340	11/8/18	30 da.	11/10/18	11/10/18	11/15/18
5.	932	12/17/18	20 da.	12/20/18	12/20/18	1/1/19

6. Assume that the Brown Hardware Co. of Middlesex, Delaware, received a statement dated Jan. 5, 1919, from the Lamson Manufacturing Co. of New York City, of an overdue account amounting to $ 626.50, with the request that it be paid at once or that the Brown Hardware Co. accept a 30-day time draft, payable to the Lamson Manufacturing Co.

a. Write the draft, making the Lamson Co. both payee and drawer.

b. Indicate upon it the Brown Hardware Co.'s acceptance under date of Jan. 7, 1919.

c. Assume that the Lamson Manufacturing Co. discounted it on Jan. 10, 1919, at the First National Bank of New York. Properly indorse the acceptance, and determine the proceeds if the rate of discount was 6 %.

7. Dr. Theodore Daniels, Athens, So. Carolina, demanding payment of Mr. Sommers on an overdue bill amounting to $ 135, addressed to him on Aug. 19, 1918, a 60-day time draft for that amount payable to the doctor.

a. Write the draft.

b. Indicate Mr. Sommers' acceptance on Aug. 25, 1918.

c. On Oct. 1, 1918, Dr. Daniels discounted the acceptance with Stephen Andrews at 6 %, making out a full indorsement of it to the latter. Write the indorsement, and determine the proceeds.

8. On Nov. 25, 1918, the Commoners Bank of Watertown, New Jersey, discounted for customers the following commercial paper:

1. A 3-mo. note for $ 1725 at 6 % dated Nov. 10.
2. A 30-day time acceptance for $ 7235 dated Nov. 12, and accepted Nov. 15.
3. A 60-day non-interest-bearing note for $ 800 dated Nov. 25.
4. A 90-day note for $ 2375 at 6 % dated Oct. 18.
5. A 30-day acceptance for $ 10,000 dated Nov. 18, and accepted Nov. 19.

a. Find the amount the bank paid for each of these securities.

b. On Nov. 30 the bank re-discounted all these notes at the National Bank of New York. Find the amount credited to their account by the latter bank.

Foreign Exchange

235. A draft used in **Foreign Exchange** is always called a **Bill of Exchange.** Formerly, when ocean transportation was slow, and less certain, bills of exchange were always issued in *triplicate.* These were sent by different routes and the first to arrive was paid, the others becoming void thereby. Now, however, they are issued *at most* only in duplicate, one copy being placed on file as a matter of record in case the other is lost. When issued in duplicate, they take the following form:

£250 *Boston, Mass.,* Dec. 15, *19*18

At sight of this First of Exchange (Second, of the same date and tenor, unpaid) pay to the order of
George Lewis,

Two hundred fifty Pounds Sterling.

Value received and charge to the account of

To Messrs. Baring Bros.
London, Eng. *Kidder, Peabody and Co.*

Exchange may be transmitted also by cable.

236. Rates of Foreign Exchange depend upon the balance in trade between this country and the foreign country to which it is desired to send money. Exchange on foreign countries is quoted daily in the newspapers of large cities.

In exchange between the United States and England, £100 is theoretically equivalent to $486.65, and $100 is theoretically equivalent to 518.1 fr. in exchange between the United States and France. In the table following are given the rates of exchange on July 23, 1918.

London	Cables 476.425 Checks 475.275	France	Cables 570 Checks 571.5

This means that £100 was equal to $476.42½ if exchanged by cable, and to $475.275 if exchanged by draft; that $100 was equal to 570 francs if exchanged by cable, and to 571.5 if exchanged by draft.

Comparison with the theoretical equivalents shows that exchange with England and France was below par on July 23, 1918, indicating that the balance of trade was in favor of the United States, since bankers deemed it necessary to encourage shipment of funds abroad by giving more than the theoretical equivalent.

Example. — What was the cost of cable exchange on London for £160 8*s.* on July 23, 1918?

Solution. — 1. £160 8*s.* = £160.4.

2. Since each 1£ cost \$4.76425, then £160.4 cost 160.4 × \$4.76425, or \$764.19.

Example 2. — Find the cost of a bill of exchange on France for 632 fr. on July 23, 1918.

Solution. — 1. \$100 was considered the equivalent of 570 fr., or \$1 of 5.7 fr.

2. ∴ the cost of 632 fr. was as many dollars as 5.7 is contained times in 632, or \$110.87.

EXERCISE 144

What will be the cost of a bill of exchange:

1. On London for £257 when exchange is quoted at 485?

2. On Paris for 1750 fr. when exchange is quoted at 524?

3. On London for £3525 when exchange is quoted at 484.75?

4. On Paris for 4275 fr. when exchange is quoted at 516.25?

5. On London for £8500 when exchange is quoted at 485.875?

6. How large a bill of exchange on London can be bought for \$2500 when exchange is quoted at 486?

7. A family wishes to send $ 1000 to a member in Paris. If exchange is quoted at 518, how many francs can be obtained?

8. Find the cost of a bill of exchange for £ 750 10*s.* when exchange is quoted at 485.5.

9. An importer wants to send his purchasing agent in London $ 5000. What will be the face of the bill of exchange if exchange is quoted at 484.5?

10. The United States Government loaned the French Government $ 100,000,000 at one time during the month of November in 1918. If exchange was quoted at 571.5, what was the equivalent in francs?

XIII. PARTNERSHIPS AND CORPORATIONS

237. A **Partnership** is an association of two or more persons, called **Partners**, formed by a written contract, to engage in business in common for the purpose of making profits.

NOTE. — Students will find an interesting article discussing the subject of Partnership in the Encyclopædia Britannica.

Partnerships are organized under special laws of the various states.

The total money and property contributed by the partners is called the **Capital**.

The **Assets** of a company comprise all its money and property at any particular time, and the **Liabilities** of a company are its debts.

238. A **General Partner** may be a real partner, one who actually shares in the management and profits of the business; a silent partner, one who shares in the profits but not in the management of the business; an ostensible partner, one who appears to be but is not actually a partner.

The general partners of a business are individually responsible for all liabilities in excess of the assets.

In some states, *special partnerships* may be formed. In such, there must be at least one general partner. The special partners contribute to the capital, and are liable for debts of the company only to the extent of their contribution to the capital.

A partnership is sometimes for a fixed term of years and sometimes for an indefinite period. A partner cannot withdraw from a firm before the close of the term of agreement unless the partnership is dissolved.

The death of a partner, the bankruptcy, or the insanity of a partner automatically bring about the dissolution of the firm. A partner cannot dispose of his interests without the consent of the other partners.

The manner of distributing the profits is specified in the contract between the partners. In the following examples some methods of distribution are described.

EXERCISE 145

1. Andrews, Baldwin, and Davis form a partnership, each contributing $ 3000, and each contributing services of equal worth. They agree to divide the profits equally. If the net profits are $ 3780, what should each receive?

2. A and B formed a partnership, A contributing $ 2000 and B $ 4000. They gained $ 2460. They agreed to divide the profits in proportion to their respective contributions to the capital.

Suggestions. — 1. What was the total capital? 2. What part of the capital did A contribute? 3. What part of the profits should he receive?

3. A, B, and C form a partnership to which each contributes equal services; A contributes $ 2000, B $ 3200, and C $ 4800. They agree to divide the profits in proportion to each partner's contribution to the capital. What should each partner receive out of $ 4350 profits?

4. A, B, and C form a partnership. A contributes $ 8000, B $ 7500, and C $ 6500. They agree to divide the profits in proportion to their shares of the capital. How much should each receive out of the $ 6275 profits?

5. Suppose that, in the preceding example, the partners also agree to divide any losses as they divide the profits. If the assets of the company at the time of dissolution are $ 22,750, and the liabilities are $ 28,430, how much should each contribute to pay off the indebtedness?

6. Assume that partner A in Example 5 is bankrupt at the time of dissolution. How much must B and C each pay if they agree: *a.* To pay their own shares and each to pay one half of A's share? *b.* To pay the debts in proportion to their contributions to the capital?

7. Assume that partners A and B in Example 5 are both bankrupt at the time of dissolution and that C has great wealth. How much must C pay to the creditors to close up the affairs of the firm?

8. A and B form a partnership. A contributes $15,000 and B $5000 to the capital. They agree that A is to receive from the business a salary of $150 per month and B $100 per month, and that the net profits at the end of the year shall be divided in proportion to their contributions to the capital. The average monthly income is $450, from which the salaries are deducted. What should each receive at the end of the year from the accumulated net profits?

9. A and B form a partnership as follows: A contributes $7300 and B, $4700. They agree that each shall receive at the end of the year 6 % interest on his share of the capital, and that they will divide equally the surplus profits. What should each receive out of $2350 gross profits?

10. Suppose that A and B in Example 9 agree that the surplus profits, after paying the interest on invested capital, shall be divided in proportion to the amount of capital invested. What should each receive in this case?

11. The following plan is a very common one.

A and B invest $6000 and $10,000 respectively in a business. They agree that A is to receive from the business annually $1500 and B, $1800; that each is to receive 6 % interest on his invested capital; and that the surplus profits are to be divided equally. What does each receive out of gross profits of $5200?

12. Assume that A, B, and C contribute \$ 6300, \$ 5200, and \$ 7500, respectively, to the capital of a company; that they agree to allow each a salary of \$ 125 per month; that each is to receive 6 % interest on his investment; and that surplus profits at the end of the year are to be divided equally. What should each receive during the year out of average monthly profits of \$ 925?

13. A, B, and C formed a partnership, agreeing that each should receive from the profits 6 % interest on his invested capital, and that surplus profits should be divided equally. Each invested \$ 2500 on Jan. 2. A invested \$ 500 on May 1, and \$ 400 on Aug. 15. B invested \$ 200 on March 10, \$ 100 on July 20, and \$ 150 on Sept. 1.

The gross profits at the end of the year were \$ 2350. How much should each partner receive?

14. A, B, and C contributed to the capital of their firm \$ 1200, \$ 3300, and \$ 2700, respectively. They agree that each is to receive 6 % interest on his invested capital and that the net profits shall be divided equally. At the close of the year (Dec. 31), they found that they had liabilities as follows: bills payable, \$ 736.45; a note for \$ 500, dated Nov. 20, with interest at 6 %, which they must pay on or before Jan. 30.

They have the following assets: cash on hand, \$ 1638.75; bills receivable, \$ 1482.31; a 90-day note for \$ 375, with interest at 6 %, dated Dec. 1; a 30-day acceptance for \$ 250, dated Dec. 15; a 60-day acceptance for \$ 425, dated Nov. 25.

a. Assume that they discount at their bank at 6 % on Jan. 2 the note payable to them and the acceptances, adding the proceeds to the cash on hand. What is the total amount of cash on hand then?

b. What is the amount of the note which they owe (principal + interest to date)? Assume that they pay this amount and also all their outstanding bills. How much cash on hand remains?

c. Out of their remaining cash assets they pay to each partner the interest on his investment. What does each receive?

d. What is the balance of their book profits, including cash on hand and bills receivable?

e. What is each partner's share of these book assets?

Corporations — Stocks and Bonds

239. Men wishing to engage in a large business undertaking usually organize a **Joint Stock Corporation.** To do so, they must comply with the laws of the state in which they organize. A copy of the statutes relating to private corporations can be obtained by applying to the secretary of state in any state.

240. The ownership of a corporation is divided into **Shares of Stock.** Stock is either **Common Stock** or **Preferred Stock.** The owner of each share of common stock is entitled to one vote in the management of the corporation and to dividends when and only when all other obligations of the company are paid. The owner of preferred stock is not entitled to any votes in the management of the business, unless it be the matter of issuing additional stock; he is entitled, however, to **Dividends,** usually at a specified rate per cent, before any dividends are paid on common stock.

The preferred stock may be **Cumulative Preferred,** in which case the owner is guaranteed income at a fixed rate per cent per year; if the dividend for any year or years is unpaid when due, the owner is guaranteed that it will be paid ultimately if the corporation is solvent.

Each share of stock has a nominal value, called the **Par Value** as a rule. Recently, corporations have been issuing common stock having *no par value.* Often but not always, the par value is $ 100.

The shares of stock are sold in order to obtain the **Capital** of the corporation.

Each purchaser of stock is given a **Stock Certificate** stating the number and kind of shares purchased, and other information about them.

The owner of shares of stock may sell them to some one else, by indorsing his stock certificate. The new purchaser then sends the indorsed certificate to the transfer agent of the corporation in order to have a record of the transfer made upon the books of the corporation.

241. The **Market Value of Stock** depends upon various factors. Among these are the present net earnings of the corporation, the future prospects of it, and the current rate of interest paid upon sound investments.

When a share of stock is purchased, the buyer acquires ownership of the next dividend payable on the share of stock, — unless the stock is sold *ex dividend;* in this latter event, the seller retains the rights to the dividend.

Stocks that sell for more than their par value are said to be **Above Par**, and those that sell for less, **Below Par**. The price at which a share sells is called the **Market Price**.

EXERCISE 146

What is the par value and what is the market value of:

Ex. No.	No. of Shares	Par Value	Market Value
1.	40	$100	$300
2.	15	100	101
3.	20	100	118
4.	50	10	14½
5.	25	100	58
6.	50	No par value	63½
7.	75	15	46½
8.	10	100	69¾
9.	5	100	460
10.	20	100	162

242. The actual rate of income to the owner of a share of stock depends upon the price he paid for the stock and the present annual dividends received from it. To find the present rate of income on a share of stock, find what per cent the current annual dividends are of the current market price. Notice that the rate of income has nothing whatever to do with the par value.

Example. — What was the annual rate of income on a share of Anaconda Copper when the market price was $ 91.75 and the dividends were $ 8 per year?

Solution. — 1. What per cent of $ 91.75 is $ 8?
2. $ 8 is 8.71 % of $ 91.75.

EXERCISE 147

Find the annual rate of income when one share of:

Ex. No.	Name of Company	Was Selling For	And Annual Dividends Were
1.	American Tobacco Common	$ 172	$ 20
2.	Buckeye Pipe Line Common	83	14
3.	Delaware and Hudson R. R. Common	97½	9
4.	Southern Pacific Common	80	6
5.	Eureka Pipe Line Common	245	24
6.	Federal Sugar Refining Pfd.	95	6
7.	B. F. Goodrich Common	72	4
8.	Norfolk and Western Pfd.	89	4
9.	United Cigar Stores Common	101	7
10.	Vacuum Oil	355	6
11.	American Beet Sugar Common	87	8
12.	Colorado Fuel and Iron Co.	46	3
13.	New York Central Common	80	5
14.	National Transit Co.	14	2
15.	Savage Arms Co.	55	6

243. Buying and Selling Stocks. Stocks are bought and sold through stock **Brokers**, located in large cities. They, in turn, or their clerks, meet at certain hours on business days at the **Stock Exchange** of their city. The procedure involved in buying and selling follows:

Mr. Jones wants to sell 100 shares of U. S. Steel Common Stock. He gives his broker an order to that effect, telling him to sell "at the market" or at some specified price. If the market price happens to be $116\frac{3}{8}$, Mr. Jones may ask $116\frac{1}{2}$ for his stock. Mr. Smith may have decided to buy 75 shares, and Mr. Cummings 25 shares of Steel Common, and ask *their* brokers to get it for them. At the stock exchange, the clerk of Mr. Jones' broker offers 100 shares for sale, announcing the price; the clerks of the other brokers *bid* some other price. If a sale is arranged, Mr. Jones indorses his stock certificates over to the purchaser, who arranges for the transfer of the ownership on the books of the corporation. Mr. Jones' certificate is canceled and new ones are made out for the purchasers and sent to them through their brokers.

The brokers charge for their services $12\frac{1}{2}$¢ per share of stock involved in the transaction, with a minimum charge of $2 or $2.50 for each transaction. The fee of $12\frac{1}{2}$¢ amounts to $\frac{1}{8}$% when the par value is $100. This fee is called **Brokerage.** The buyer and seller both pay brokerage.

Example 1. — What are the proceeds to the seller of 100 shares of U. S. Steel Common Stock at $108\frac{3}{4}$?

Solution. — 1. Each share sells for $108.75.

2. Hence the gross receipts are $10,875.

3. Since the brokerage is $12\frac{1}{2}$¢ per share, the total brokerage is $100 \times .12\frac{1}{2}$ or $12.50.

4. ∴ the net proceeds to the seller are $10,875 − $12.50 or $10,862.50.

Example 2. — What is the cost to the buyer in the preceding example?

Solution. — 1. The cost = $10,875 + $12.50 = $10,887.50.

EXERCISE 148

Below are the quotations for certain stocks on the N. Y. Stock Exchange on a certain day in 1918.

BID	ASKED	NAME OF STOCK	HIGH	LOW	CLOSE
33½	34	Allis Chalmers	34	33½	33½
66¼	66¾	American Locomotive	66⅞	66½	66½
105½	106½	American Smelting Co. Pfd.	106¼	106¼	106¼
91½	91¾	Baldwin Locomotive Co.	92⅝	91⅝	91⅝
104½	105	Bethlehem Steel Pfd.	105¼	105¼	105¼
143½	145	General Electric Co.	145	145	145
90½	91	Great Northern Pfd.	90⅞	90⅞	90⅞
103⅛	103¼	Norfolk and Western	103½	103¼	103¼
111⅜	112	Pullman Co.	111½	111½	111½
91⅞	92¼	Republic Iron and Steel	92¾	91¾	91⅞
44⅝	44¾	Studebaker	44⅝	44	44⅝
104	105¼	U. S. Rubber 1st Pfd.	104⅜	104⅜	104⅜
109⅛	109¼	U. S. Steel	110	108¾	109¼
41⅜	41⅝	Westinghouse Electric	42	41¾	41½

Find the cost to the buyer and the proceeds to the seller in the sale of:

EX. NO.	NO. OF SHARES	NAME OF STOCK	AT PRICE
1.	100	American Locomotive Co.	Low
2.	50	Bethlehem Steel Pfd.	High
3.	20	Allis Chalmers	Close
4.	40	Great Northern Pfd.	High
5.	15	Republic Iron and Steel	Low
6.	200	Studebaker	High
7.	75	U. S. Rubber 1st Pfd.	Low
8.	35	U. S. Steel	Close
9.	60	Pullman Co.	Bid
10.	80	Westinghouse Electric	Asked

Find the gain or loss by a stock speculator who

Ex. No.	Bought			And Sold at
	No. of Shares	Name of Stock	At	
11.	100	U. S. Steel	Low	High
12.	200	Westinghouse Electric	Low	High
13.	1500	American Smelting Co. Pfd.	105	106⅛
14.	500	Baldwin Locomotive	High	Low
15.	1000	Great Northern Pfd.	87½	90¾

16. A man having $ 2500 to invest decides to buy as many shares as he can of U. S. Steel Common. How many shares can he safely order through his broker? (Use the quoted "high" price.)

17. A man having $ 3750 to invest decides to put ⅓ into Bethlehem Steel Pfd., ⅓ into American Smelting Co. Pfd., and ⅓ into General Electric. How many shares of each can he order?

18. How many shares of Norfolk and Western can a man buy who has $ 1500 to invest?

19. A man having $ 5000 to invest decides to place ⅖ in U. S. Rubber Pfd., ⅕ in Studebaker Common, and ⅖ in Westinghouse Electric. How many shares of each can he order?

20. The trustees of an estate wish to invest $ 35,000, distributing the investment so that ⅗ is invested in preferred stocks and ⅖ in standard common stock. They decide to buy U. S. Rubber Pfd., Great Northern Pfd., and Bethlehem Steel Pfd., investing approximately equal amounts in each. For common stock, they select Baldwin Locomotive, General Electric, and Republic Iron and Steel, investing about equal amounts in each. Find the number of shares of each stock that they can order.

Bonds

244. A **Bond** is a certificate indicating ownership of part of an indebtedness of a corporation; it is a promise to pay a fixed sum of money at a definite future time, with interest at a fixed rate, payable periodically. Often there is some sort of security pledged to secure payment of the bonds.

Thus, the owners of a piece of property, wanting a large sum of money, borrow it from some banking institution, and, as security, mortgage their property to the institution as trustees.

The bankers then issue bonds whose total face value is the amount of the sum loaned the owners of the property.

On the other hand, governments of nations, states, and municipalities issue bonds, the only security for the payment of which is the honor of the issuing government.

The rate of interest and the time when the loan is repayable are specified in the bond. The interest is generally payable semi-annually. It is a rate per cent of the face of the bond.

A **Registered Bond** provides a place in which the owner's name may be written. The borrower then mails the interest to the owner of the bond when it falls due.

A **Coupon Bond** has coupons for each of the interest payments due upon it attached to it. As these become due, the owner of the bond cuts them off and deposits them in his bank for collection.

A complete description of the various kinds of bonds is beyond the scope of this book.

NOTE. — In the section of a magazine, like *World's Work*, devoted to financial advertisements, investment houses often advertise their willingness to send without charge a copy of a pamphlet describing Stocks and Bonds. One such copy for a school is enough.

Some banker in the town can probably give to the school a copy of a recent number of the *Bond Record*, published by Halsey and Hudnut of New York City.

245. Bonds are described in such manner as to indicate such items as the name of the debtor, the year of maturity, the rate of interest, and often the character of the security.

Thus, Western Pacific R. R. Co. 1st Mortgage 5's, Series A, is a 5 % 1st mortgage bond of the Western Pacific R. R. Co.

246. The great advantage of bonds, in general, over stocks is that they represent a definite sum of money that must be paid at a certain time; that they bear interest at a definite rate that must be paid at specified times, taking precedence over all dividend payments of the debtor.

247. Cost of Bonds. The current market price of bonds may be above or below the face value of the bond, — *above or below par.* Whatever the market price, the owner of a bond is entitled to the face value of the bond at maturity.

Bonds sell at a certain market price plus accrued interest.

Thus, if a man buys on March 1 a bond on which the interest is payable on January 1 and July 1, he must pay the market price plus interest on the face of the bond from January 1 to March 1. On July 1 he receives the interest on the face from January 1 to July 1, thus getting the interest he paid the previous owner of the bond as well as the interest due on the bond since he purchased it.

When bonds are purchased through a broker, **Brokerage** amounting to $\frac{1}{8}$ % of the face of the bond is charged.

Hence the purchase price equals market price plus accrued interest plus brokerage.

248. The **Rate of Income** from a bond is the rate per cent which the annual interest is of the purchase price.

Example. — Find the purchase price of a $1000 5 % bond at 95 on March 12, if the interest is payable Feb. and Aug. 1. Also find the annual rate of income.

Solution.—1. The market price = 95 % of $1000 = $950.

2. The accrued interest on $1000 from Feb. 1 to March 12—that is, for 39 days—is $5.42.

3. The brokerage is $\frac{1}{8}$ % of $1000 or $1.25.

4. Hence the purchase price = $950 + $5.42 + $1.25 = $956.67.

5. The annual income is 5 % of $1000 or $50.

6. The annual rate of income is 5.2^+ %,
for $50 is 5.2^+ % of $956.67.

249. The **Yield of a Bond** is the rate of income realized from the bond if the bond is held until maturity.

Thus, suppose a man buys a $100 bond for $98; that the rate of interest is 5 % and that the bond matures in 1 year. During the year, the man receives $5 interest (5 % of $100), and at the end of the year $2 more than he paid for the bond, for the bond is worth $100 at maturity. All together he receives $7 on an investment of $98 for 1 year. His rate of income is about 7.1 %.

If this same bond matures in two years, he will have received a total of $10 interest and $2 profit when the bond matures. All together he will have received $12 for the use of $98 for two years, or at the rate of $6 per year. This is about 6.1 %. This result is the approximate yield of income on this bond, as compound interest on the interest paid semi-annually has been neglected.

NOTE.—The computation of yields on bonds is quite difficult. Investment houses have tables showing the rate yielded by bonds at various rates of interest for bonds maturing at the ends of various periods of years.

EXERCISE 149

The following table contains the names of some bonds, the rate of interest they pay, the interest dates, and the price bid for the bonds on July 30, 1918. Determine the necessary items to fill in the two blank columns of the table.

Ex. No.	Title of Bond	Rate	Interest Payable on 1st of	Price	Rate of Income at Bid Price	Cost of $1000 Bond on July 30
1.	Chicago and Alton 1st Mortgage	3½ %	Jan. July	39⅛		
2.	Chicago and Great Western 1st Mortgage	4 %	Mar. Sept.	57½		
3.	Chicago and N. Western General Mortgage	4 %	May Nov.	82½		
4.	Great Northern 1st Refunding	4¼ %	Jan. July	86		
5.	Louisville and Nashville General Mortgage	6 %	June Dec.	106¾		
6.	N. Y. Central & Hudson River Mortgage	3½ %	Jan. July	72		
7.	Oregon Short Line 1st Mortgage.	6 %	Feb. Aug.	100⅞		
8.	United States 3d Liberty Loan	4¼ %	Mar. Sept.	95$\frac{1}{10}$		
9.	United States 2d Liberty Loan	4¼ %	Mar. Sept.	93.36		
10.	Detroit City Gas Co.	5 %	Jan. July	94⅛		
11.	Baldwin Locomotive Co. 1st Mortgage	5 %	May Nov.	99½		
12.	Chic. Union Station 1st Mortgage	4½ %	Jan. July	87		
13.	United States Steel Sinking Fund	5 %	May Nov.	98½		
14.	Western Electric 1st Mortgage	5 %	Jan. July	96		
15.	Brooklyn Rapid Transit	5 %	Apr. Oct.	81⅛		

250. United States Liberty Bonds were purchased most widely during 1917–1919. They were sold in denominations of $ 50, $ 100, $ 500, $ 1000, etc. These bonds, even more so than ordinary bonds, are a *good* investment, for they have exceptional security behind them, — the honor of the United

States of America. These bonds are Coupon bonds, — that is, attached to each are coupons for the semi-annual interest. When the interest becomes payable, the coupon should be cut from the bond and taken *to a bank*, where the full cash value of it (less a small collection charge possibly) can be obtained for it. Coupons for the interest for two years' time were attached to the bonds. At the close of that time, information about the manner of obtaining the semi-annual interest may be obtained from any banker.

If the holder of a bond is compelled by circumstances to sell it, he should go to a bank where he will be able to obtain the current market price of the bond.

EXERCISE 150

1. What is the semi-annual interest on a $ 50 bond bearing 4½ % interest? On a $ 100 bond? On a $ 500 bond?

2. What should be obtained for a $ 50 U. S. Liberty Bond on an interest payment date when the market price is 98½? 98¼? 99⅛? 97¾?

XIV. EQUATION OF PAYMENTS

251. Equation of Payments is the process of finding at what time several payments which are due at different times may be paid at one time without injustice to either debtor or creditor.

The **Date of Payment** so determined is called the **Equated Date**, — and the time is called the **Equated Time.**

Example 1. — A owes B $ 25 due in 3 mo., $ 50 due in 4 mo., $ 35 due in 5 mo., and $ 40 due in 7 mo. What is the equated date of payment?

Solution. — 1. A at present is entitled to the use of the $ 25 for 3 mo. This is the same as the use of $ 1 for 25 × 3 mo. or 75 mo.

2. Thus,

A is entitled to the use of $ 25 for 3 mo. or of $ 1 for 25 × 3 or 75 mo.

A is entitled to the use of $ 50 for 4 mo. or of $ 1 for 50 × 4 or 200 mo.

A is entitled to the use of $ 35 for 5 mo. or of $ 1 for 35 × 5 or 175 mo.

A is entitled to the use of $ 40 for 7 mo. or of $ 1 for 40 × 7 or 280 mo.

3. All together, A is entitled to the use of $ 1 for 730 mo.

4. The total sum owed is $ 150. He is therefore entitled to the use of $ 150 as many months as 150 is contained times in 730.

This is $4\frac{13}{15}$ mo. or 4 mo. 26 da.

5. ∴ A can make the payment 4 mo. 26 da. from now.

Note 1. — To check the solution, find the interest up to the equated date on all payments due before that date, and the discount on all payments due after it at any rate per cent. These two should be equal approximately.

NOTE 2. — If in a result a fraction of a day is ½, or more, then the length of time is increased by one day; otherwise, the fractional part of a day is omitted.

Example 2. — What is the equated date of payment of $ 200 due May 4, $ 300 due June 12, and $ 400 due July 24?

Solution. — 1. Assume that all payments are made on May 4.

2. Then the June 12 payment is made 39 days ahead of time, and therefore the debtor loses the use of $ 300 for 39 days or of $ 1 for 11,700 da.

3. The July 24 payment is made 81 days ahead of time, and therefore the debtor loses the use of $ 400 for 81 days or of $ 1 for 32,400 days.

4. All together, the debtor loses the equivalent of the use of $ 1 for 44,100 days.

5. Hence, the date of payment is as many days after May 4 as 900 is contained times in 44,100, or 49 days.

6. ∴ the equated date = May 4 + 49 days = June 22.

Example 3. — Find the equated time and the balance to be paid on the following account.

Dr. *James Johnson* *Cr.*

1918			1918		
Aug. 27	To Mdse. 30 da.	$ 350	Sept. 8	By Draft, 30 da.	$ 275
Sept. 12	To Mdse. 30 da.	125	Sept. 28	By Cash	250
Oct. 7	To Mdse. 60 da.	275	Oct. 30	By Cash	125

Solution. — 1. The Aug. 27 item is actually due on Sept. 26; and the Sept. 8 credit matures on Oct. 8. These and the other due dates are in the third column below.

2. Assume that the settlement is made on the earliest of the dates in column 3.

3. The first debit item is paid 14 days before it is due. Hence Mr. Johnson loses the use of $ 350 for 14 days or of $ 1 for 4900 days (350 × 14). Similarly for items 2 and 3 of the debits. All together he loses the use of $ 1 for 28,275 days. (See Cols. 4 and 5.)

4. The first credit item is paid 26 days after the settlement date. Hence Mr. Johnson gains the use of $275 for 26 days or of $1 for 7150 days. Similarly, for items 2 and 3 of the credits. All together, Mr. Johnson gains the use of $1 for 17,150 days.

5. By paying on Sept. 12, Mr. Johnson loses the use of $1 for 11,125 days. This is equal to the use of the balance of $100 for 111.25 days.

6. Hence the real date of settlement should be 111¼ days after Sept. 12. This is Jan. 1, 1918. Legally this makes the date of settlement Jan. 2, 1918.

DEBITS	DATE	AMOUNT	DUE	9/12 DIFFERENCES	IF PAID ON 9/12 MR. JOHNSON SUFFERS
	Aug. 27	$350	Sept. 26	14 da.	4900 da. loss of use of $1
	Sept. 12	125	Sept. 12	0 da.	0 da. loss of use of $1
	Oct. 7	275	Dec. 6	85 da.	23,375 da. loss of use of $1
Total		$750			28,275 da. loss of use of $1
Credits	Sept. 8	$275	Oct. 8	26 da.	7150 da. gain in use of $1
	Sept. 28	250	Sept. 28	16 da.	4000 da. gain in use of $1
	Oct. 30	125	Oct. 30	48 da.	6000 da. gain in use of $1
Total		$650			17,150 da. gain in use of $1
Balance		$100			11,125 da. loss in use of $1

Example 4. — Another method of averaging accounts is known as the **Interest Method.**

Dr. *Charles Stuart* *Cr.*

1918			1918		
5/28	To Mdse. 30 da.	$350	6/8	By Draft, 30 da.	$275
6/16	To Mdse.	125	7/15	By Cash	250
7/7	To Mdse. 2 mo.	275	8/21	By Cash	125

Solution. — 1. Take as the focal date Sept. 7.

2. If the first payment (due June 27) is not made until Sept. 7, Mr. Stuart should pay interest on $350 for 72 da., or $4.20. Similarly for the second debit item. (See Col. 5 below.)

3. The payment of June 8, maturing on July 8, being made 61 days in advance of the focal date should draw interest for 61 da. This is $2.7958. Similarly for credit items 2 and 3.

4. All together, Mr. Stuart owes interest to the amount of $5.9292 – $5.40 or $.5292, if he settles the account on Sept. 7.

5. He owes $100. The interest on it for one day is $.0167. Hence he should pay the debt as many days before Sept. 7 as $.0167 is contained times in $.5292, or 31.6 da.

6. 32 days before Sept. 7 is Aug. 6.

Debit	Date	Amount	Due	Focal Date Diff.	Interest on Amt. at 6 %	
	5/28	$350	6/27	72 da.	$4.20	interest due
	6/16	125	6/16	83 da.	1.7292	
	7/7	275	9/7	0	0	
Total		$750			$5.9292	
Credit	6/8	$275	7/8	61 da.	$2.7958	interest earned
	7/15	250	7/15	54 da.	2.25	
	8/21	125	8/21	17 da.	.3542	
Total		$650			$5.4000	

EXERCISE 151

1. What is the equated date of payment of $55 due in 3 mo., $170 due in 9 mo., and $135 due in 7 mo.?

2. A owes B $325, of which $50 is due in 6 mo., $95 is due in 8 mo., $80 in 9 mo., and $100 in 11 mo. What is the equated date of payment?

3. Find the equated date for paying $425 due in 30 days, $725 due in 60 days, $125 due in 90 days, and $540 due in 120 days.

4. What is the equated date for paying $25 due March 15, $135 due April 6, and $100 due May 25?

5. Find the equated date for paying $275 due Aug. 29, $325 due Sept. 23, $250 due Oct. 5, and $410 due Nov. 13.

6. On May 23, I bought a piece of land for $ 2500, on four months' credit. If I pay $ 1000 on July 23, when should I pay the balance?

7. If goods are bought on three months' credit as follows: Sept. 19, 1918, $ 1250; Oct. 3, $ 900; Nov. 12, $ 750, what is the equated date of payment?

8. Four sixty-day notes bear the following dates: Jan. 4, 1918, $ 500; Feb. 27, 1918, $ 350; June 18, 1918, $ 550; July 30, 1918, $ 275. What is the average date of payment?

9. On Oct. 21, Henry Williams owed $ 160 due in 40 days, $ 225 due in 60 days, and $ 175 due in 90 days. What is the equated date of payment?

10. A bill of $ 1200 is due in 5 months from Jan. 13, 1919. If payments are made as follows: $ 260, March 20, 1919; $ 225, May 5, 1919, when is the balance due?

11. A tradesman owes $ 300 due in 6 months, and $ 750 due in 9 months. If at the end of 7 months he pays $ 450, when should the balance be paid?

12. Hooker and Ingalls bought merchandise on 60 days' credit as follows: June 28, $ 75; Aug. 1, $ 150; Sept. 4, $ 100. What is the average date of payment?

13. A merchant owes $ 2350 due in 10 months. If he pays $ 400 in 2 months, $ 350 in 4 months, and $ 550 in 8 months, when should he pay the balance?

14. Four ninety-day notes bear date as follows: March 9, 1918, $ 375; May 24, 1918, $ 250; Aug. 13, 1918, $ 525; Oct. 30, 1918, $ 400. What is the average date of payment?

15. Find the equated time for paying the balance of the following accounts:

Dr. *George Adams* *Cr.*

1918			1918		
Sept. 16	To Mdse. 30 da.	$550	Oct. 4	By Cash	$500
Oct. 28	To Mdse.	375	Oct. 29	By Cash	325

TABLES OF MEASURES

Measures of Length or Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards or 16½ feet = 1 rod (rd.)
320 rods
1760 yards = 1 mile (mi.)
5280 feet
1 knot or nautical mile = 6080 ft. (U. S.)

Surveyors, in measuring land, use a chain (ch.) the length of which is 4 rods, divided into 100 links (li.) of 7.92 inches each. 80 chains = 1 mile.

Measures of Surface or Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods
43560 square feet = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
10 square chains = 1 acre
36 square miles = 1 township

Measures of Volume or Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet (a pile 4 ft. high, 4 ft. wide, and 8 ft. long) = 1 cord

Liquid Measures

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

It has been customary to consider 31½ gallons the contents of a barrel (bbl.) and 63 gallons the contents of a hogshead. In practice, however, barrels are of various sizes; thus the standard gasoline barrel contains 56 gallons. A liquid quart contains 57¾ cubic inches; a liquid gallon 231 cubic inches.

Dry Measures

(Used in measuring grain, vegetables, fruits, etc.)

2 pints (pt.) = **1 quart** (qt.)
8 quarts = **1 peck** (pk.)
4 pecks = **1 bushel** (bu.)

A dry quart contains 67⅕ cubic inches; a bushel contains 2150.42 cubic inches.

Avoirdupois Weight

16 ounces (oz.) = **1 pound** (lb.)
100 pounds = **1 hundredweight** (cwt.)
20 hundredweight
2000 pounds } = **1 ton** (T.)

At United States Custom Offices, and at iron and coal mines, the following weights are used:

112 pounds = 1 long hundredweight
2240 pounds = 1 long ton

Troy Weight

(Used in weighing gold, silver, and jewels)

24 grains (gr.) = **1 pennyweight** (pwt.)
20 pennyweights = **1 ounce** (oz.)
12 ounces = **1 pound** (lb.)

Apothecaries' Weight

20 grains (gr.) = **1 scruple** (℈)
3 scruples = **1 dram** (ʒ)
8 drams = **1 ounce** (℥)
12 ounces = **1 pound** (℔)

Apothecaries' Liquid Measures

60 minims (m.) = **1 fluid dram** (f ʒ)
8 fluid drams = **1 fluid ounce** (f ℥)
16 fluid ounces = **1 pint** (O)

Measures of Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
$\left.\begin{array}{l}\text{52 weeks}\\ \text{365 days}\end{array}\right\}$ = 1 year (yr.)
366 days = 1 leap year

If the number of a year is divisible by 4 but not by 100, the year is a leap year; also when the number of the year is divisible by 400 the year is a leap year. In leap years, February has 29 days.

The Solar Year has nearly $365\frac{1}{4}$ days.

Angle and Arc Measures

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)

A quadrant contains 90 arc degrees: a circle contains 360 arc degrees. A right angle contains 90 angular degrees.

The length of a degree ($\frac{1}{360}$) of the earth's equator is about $69\frac{1}{6}$ miles.

English Money

4 farthings (far.) = 1 penny (d.)
12 pence = 1 shilling (sh.)
20 shillings = 1 pound or sovereign (£)
= $4.8665

French Money

10 centimes (ct.) = 1 decime (dc.)
10 decimes = 1 franc (fr.)
1 franc = 19.3 cents

German Money

100 pfennige = 1 mark
1 mark = 23.8 cents

Miscellaneous Measures

Numbers

12 units = 1 dozen (doz.)
20 units = 1 score
12 dozen = 1 gross (gr.)
12 gross = 1 great gross

Paper

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

Some Standard Weights

(Used in most of the States)

Wheat	60 lb. = 1 bushel
Rye	56 lb. = 1 bushel
Oats	32 lb. = 1 bushel
Barley	48 lb. = 1 bushel
Corn, on cob	70 lb. = 1 bushel
Corn, shelled	56 lb. = 1 bushel
Potatoes	60 lb. = 1 bushel
Water	1 cu. ft. = $62\frac{1}{2}$ lb.

Carat. (1) A term used to express the purity of gold. Pure gold = 24 carats fine. If 2, 4, etc., parts of alloy are present, the gold is 22, 20, etc., carats fine. Thus, 14 carat gold, used for good chains, contains 14 parts of gold to 10 parts of alloy.

(2) Also used as a unit for measuring precious stones.

1 carat = 4 grains = about $3\frac{1}{6}$ Troy grains.

Since 1905, 1 carat = 200 milligrams, among international jewelers.

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ANSWERS TO WELLS AND HART'S NEW HIGH SCHOOL ARITHMETIC

EXERCISE 3. Page 4.

1. *a.* 8; *b.* 18; *c.* 19; *d.* 27; *e.* 40.
2. *a.* 90; *b.* 110; *c.* 1600; *d.* 1150; *e.* 609.
3. XI; XII; XIII; XIV; XV; XVI; XVII; XVIII; XIX.
4. XXII; XXXII; XLIII; XLVIII.
5. L; LII; LV; LIX.
6. LX; LXXII; LXXXVI; LXXXIX.
7. XC; XCIII; XCV; XCVIII.
8. CC; CDL; DCXXXV; DCCCLXXII.
9. M; MDC; MDCCCLXX; MCMXXIII.
10. *a.* 1722; *b.* 1776; *c.* 1492; *d.* 1819; *e.* 1861; *f.* 1914.

EXERCISE 8. Pages 9–12.

1. 18,532.
2. 33,082.
3. 27,879.
4. 34,044.
5. 43,775.
6. 268,665.
7. 286,721.
8. 334,723.
9. 404,080.
10. 465,536.
11. 265 lb.
12. $728.18.
13. Texas, 265,896.
    California, 158,297.
    Montana, 146,997.
    New Mexico, 122,634.
    Arizona, 113,956.
    Nevada, 110,690.
    Colorado, 103,948.
    Wyoming, 97,914.
    Oregon, 96,699.
    Utah, 84,990.
    Land, total, 1,289,428.
    Water, total, 12,593.
14. 298,817.
15. 91,972,266.
16. 1919, $13,504,357,934.
    1918, $18,688,960,431.
17. $7104.15.
18. 6,361,502.
19. 268,491.

EXERCISE 9. Page 12.

1. 105.
2. 65.
3. 93.
4. 138.
5. 129.
6. 162.
7. 93.
8. 97.
9. 129.
10. 95.
11. 118.
12. 124.
13. 115.
14. 151.
15. 111.
16. 104.
17. 121.
18. 118.

EXERCISE 10. Pages 13–16.

1. 168.
2. 162.
3. 273.
4. 1176.
5. 1328.
6. 1677.

7.

England,	177.
Germany,	87.
France	50.
Japan,	43.
Russia,	22.
Italy,	26.
Austria-H.,	16.

8.

Ewing,	330.
Bomer,	142.
Smith,	222.
Swenson,	365.
Crane,	275.
Sullivan,	284.
Anderson,	280.
Brown,	442.
Derby,	171.
Ritz,	231.
Total,	2742.

Mon.	423.
Tues.	456.
Wed.	450.
Thurs.	477.
Fri.	477.
Sat.	459.
Total,	2742.

9.

A.	876	Mon.	2908.
B.	6152	Tues.	2888.
C.	1620	Wed.	2897.
D.	1670	Thurs.	2910.
E.	1869	Fri.	2899.
F.	8113	Sat.	2904.
Total,	20,300	Sun.	2894.
		Total,	20,300.

10.

A.	$1262.	Mon.	$1283.
B.	1777.	Tues.	1172.
C.	562.	Wed.	1051.
D.	2979.	Thurs.	1162.
E.	927.	Fri.	1487.
Total,	7507.	Sat.	1352.
		Total,	7507.

11.

1910,	523,210.
1911,	533,606.
1912,	547,289.
1913,	564,460.
1914,	580,058.

12.
270,082.
7,707,751.
$18,490,749,000.
$18,526,436,000.
$20,767,546,000.

13.

Jan.,	$361.92.	Coal,	$141.00.
Feb.,	189.51.	Ins.,	212.23.
Mar.,	131.04.	Taxes,	63.45.
Apr.,	113.20.	Doctor,	113.07.
May,	136.32.	House,	1107.79.
June,	116.45.	Clubs,	57.70.
July,	115.47.	Recreation,	144.80.
Aug.,	148.79.	Total,	1840.04.
Sept.,	185.17.		
Oct.,	108.73.		
Nov.,	109.10.		
Dec.,	124.28.		
Total,	1840.04.		

14.

Jan.,	$13,170.	Coal,	$130,685.
Feb.,	15,543.	Wood,	2,645.
Mar.,	12,705.	Ice,	41,648.
Apr.,	12,952.	Lime,	6,915.
May,	11,158.	Cement,	7,558.
June,	11,932.	Total,	189,451.
July,	18,109.		
Aug.,	20,873.		
Sept.,	21,797.		
Oct.,	15,645.		
Nov.,	16,689.		
Dec.,	18,878.		
Total,	189,451.		

EXERCISE 13. Pages 19–22.

5. *a.* $664.75. *b.* $174.75.

c. *A.* $25.75. *C.* $97.00. *E.* $47.50. *G.* $91.10.
B. $14.75. *D.* $73.75. *F.* $62.65. *H.* $77.50.

d. The sum of the amounts in part *c* equals the difference between the results of parts *b* and *a*.

6.

1911.	Earned,	$230,896.92.
1912.	Lost,	1,781,434.20.
1913.	Earned,	4,551,985.32.
1914.	Earned,	4,376,463.05.
1915.	Lost,	11,297,861.15.

7.

1912.	$10,876,523.40.
1913.	$13,542,090.25.
1914.	$21,490,562.29.
1915.	$14,987,923.80.

8. Total, 1914; $1,136,099.16.
1915; $1,213,971.66.

Gas lights, decrease,	$28,761.08.
Gasoline lights, decrease,	$17,777.44.
Rented electric lights, increase,	$20,544.34.
Municipal electric lights, increase,	$103,866.68.

9. Total, 1900; $20,439,901,164.
1910; $40,991,449,090.
North, $13,025,814,580.
South, $4,702,272,170.
West, $2,823,461,176.

10. 25; $1,894,746,000; $277,589,000.

	TOTAL	ON HAND	TOTAL	ON HAND	TOTAL	ON HAND
11.	3830	2158	4812	1942	783	458
	9746	5389	3980	1693	468	216
	5352	2087	3797	2152	1531	1533
	4885	2452	1458	605		

	TOTALS	BALANCES	TOTALS	BALANCES	TOTALS	BALANCES
12.	950,388	808,418	1,086	855	350	180
	2,208	1,880	663	582	802	589

13. Increases:

1900,	15,977,691.	1860,	7,115,050.	1820,	3,227,567.
1890,	13,046,861.	1850,	8,251,445.	1810,	2,398,572.
1880,	12,791,931.	1840,	6,122,423.	1800,	1,931,398.
1870,	11,597,412.	1830,	4,203,433.	1790,	1,379,269.

14.	TOTAL SUPPLY	ON HAND	TOTAL SUPPLY	ON HAND
	185	38	1498	336
	357	47	821	242
	269	54		

EXERCISE 14. PAGES 23-24.

COINS AND BILLS

Ex.	1¢	5¢	10¢	25¢	50¢	$1.00	$2.00	Total value
1	2	1	1		1		2	$4.67
2	3			1			2	4.28
3	1	1					2	4.06
4	4	1	1	1	1	1	1	3.94
5	3	1		1			2	4.33
6	2			1	1		2	4.77
7		1	1				2	4.15
8			1	1	1	1	1	3.85
9		1			1			.55
10	3		1			1		1.13
11	2		2				1	2.22
12	3			1	1		2	4.78
13	1	1		1		1	1	3.31
14		1	1		1	1	1	3.65
15	1	1				1	1	3.06
16	3		1	1		1		1.38

COINS AND BILLS

Ex.	1¢	5¢	10¢	25¢	50¢	$1.00	$2.00	$5.00	$10.00	Total
17	4	1	1	1			1	1		$7.44
18				1				1		5.25
19	1		1			1	1			3.11
20	3		2		1					.73
21	3	1	1		1	1				1.68
22	2	1			1	1		1		6.57
23	4			1			1	1		7.29
24	1		2		1		2			4.71
25	2	1	1	1				1		5.42
26	4		1				1			2.14
27			2	1		1	1			3.45
28					1					.50
29		1			1		1			2.55
30	2	1			1					.57
31	1	1			1			1		5.56
32	1			1		1		1		6.26
33		1	1			1	1			3.15
34		1		1	1					.80
35	4		1	1	1					.89
36				1		1				1.25

EXERCISE 17. Pages 28–29.

1. 834,588.
2. 10,270,000.
3. 3,900,383.
4. 4,054,799.
5. 2,172,500.
6. 4,116,328.
7. 228,958,488.
8. 148,732,062.
9. 433,403,622.
10. 37,709,424.
11. 15,716,910.
12. 602,025,849.
13. 229,480,867.
14. 248,644,665.
15. $770.
16. $123.25.
17. $556.80.
18. $361.00.
19. $351.00.
20. $3,152,500.
21. $9,187,500.
22. $5491.20.
23. $207,390.
24. $3937.50.
25. $1658.

EXERCISE 18. Pages 32–33.

1. *a.* 243. *g.* 4464.
b. 414. *h.* 4923.
c. 702. *i.* 5616.
d. 846. *j.* 8442.
e. 2367. *k.* 4167.
f. 3375. *l.* 6822.

2. *a.* 2673. *g.* 49,104.
b. 4554. *h.* 54,153.
c. 7722. *i.* 61,776.
d. 9306. *j.* 92,862.
e. 26,037. *k.* 45,837.
f. 37,125. *l.* 75,042.

3. *a.* 26,973. *g.* 495,504.
b. 45,954. *h.* 546,453.
c. 77,922. *i.* 623,376.
d. 93,906. *j.* 937,062.
e. 262,737. *k.* 462,537.
f. 374,625. *l.* 757,242.

4. *a.* 1350. *g.* 24,800.
b. 2300. *h.* 27,350.
c. 3900. *i.* 31,200.
d. 4700. *j.* 46,900.
e. 13,150. *k.* 23,150.
f. 18,750. *l.* 37,900.

5. *a.* 675. *g.* 12,400.
b. 1150. *h.* 13,675.
c. 1950. *i.* 15,600.
d. 2350. *j.* 23,450.
e. 6575. *k.* 11,575.
f. 9375. *l.* 18,950.

6. *a.* 405. *g.* 7440.
b. 690. *h.* 8205.
c. 1170. *i.* 9360.
d. 1410. *j.* 14,070.
e. 3945. *k.* 6945.
f. 5625. *l.* 11,370.

7. *a.* 2025. *g.* 37,200.
b. 3450. *h.* 41,025.
c. 5850. *i.* 46,800.
d. 7050. *j.* 70,350.
e. 19,725. *k.* 34,725.
f. 28,125. *l.* 56,850.

8. *a.* 2619.
b. 4462.
c. 7566.
d. 9118.
e. 25,511.
f. 36,375.

9. *a.* 2754.
b. 4692.
c. 7956.
d. 9588.
e. 26,826.
f. 38,250.

10. *a.* 2646. *d.* 9212.
b. 4508. *e.* 25,774.
c. 7644. *f.* 36,750.

11. *a.* 2835. *d.* 9870.
b. 4830. *e.* 27,615.
c. 8190. *f.* 39,375.

12. $74.25.
13. $338.00.
14. $127.41.
15. 1900 in.
16. 6500 lb.
17. $42.00.
18. $5.75.
19. $5.25.
20. $54.00.
21. $27.00.
22. $812.50.
23. $819.50.
24. $336.25.
25. $19.50.
26. *a.* $27.00; *b.* $13.50; *c.* $40.50; *d.* $8.10.
27. $35.45.
28. $1905.75.
29. $12.22.
30. $77.70.
31. 10,560.

EXERCISE 19. Pages 35–37.

1. $97.82.
2. $107.65.
3. $107.88.
4. $187.59.
5. $126.70.
6. $466.03.
7. $168.58.
8. $609.98.
9. $984.25.
10. $198.10.

EXERCISE 21. Page 39.

1. *a.* 199. *b.* 262. *c.* 284. *d.* 343. *e.* 498.
2. *a.* 190. *b.* 297$\frac{21}{34}$. *c.* 408. *d.* 587. *e.* 719.
3. *a.* 142. *b.* 239. *c.* 368. *d.* 487. *e.* 738.
4. *a.* 285. *b.* 454. *c.* 564. *d.* 692. *e.* 856.
5. *a.* 300$\frac{6}{93}$. *b.* 482. *c.* 624$\frac{25}{93}$. *d.* 746$\frac{10}{93}$. *e.* 876$\frac{22}{93}$.

EXERCISE 22. Pages 41–43.

1. 36.
2. 840.
3. $85.75.
4. 17.
5. $4.67.
6. 4186.
7. 12.
8. 18.
9. 10.
10. 17.
11. 12 bu., 4¢.
12. 42.
13. 6.
14. 48.
15. $3933.
$11,321.
16. 1900 : 146 A. ; $3562.
1910 : 138 A. ; $6443.
17. $.95 : $1.28 ; $1.24.
18. Silage, 57+ T. ; Beets, 19+ T.
Hay, 19+ T. ; Feed, 10 T.
19. $1926.
20. 80 m. per hr.

EXERCISE 23. Page 45.

1. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 87, 89, 97.

2. 2, 13.
3. 2, 19.
4. 2, 3, 7.
5. 2, 3, 3, 3.
6. 3, 3, 7.
7. 2, 2, 17.
8. 3, 5, 5.
9. 2, 2, 3, 7.
10. 2, 2, 2, 2, 3.
11. 2, 5, 11.
12. 2, 2, 2, 2, 2, 2, 2.
13. 5, 29.
14. 2, 2, 53.
15. 3, 79.
16. 2, 2, 2, 2, 2, 2, 2, 2.
17. 2, 2, 2, 3, 11.
18. 5, 5, 11.
19. 2, 3, 7, 7.
20. 2, 5, 31.
21. 5, 3, 23.
22. 2, 191.
23. 2, 2, 111.
24. 2, 263.
25. 2, 2, 153.
26. 2, 3, 123.
27. 2, 2, 207.
28. 5, 3, 59.
29. 2, 13, 37.
30. 2, 2, 2, 5, 5, 5.
31. 2, 2, 2, 157.

EXERCISE 24. Page 46.

1. 36.
2. 12.
3. 9.
4. 15.
5. 72.
6. 64.
7. 225.
8. 12.
9. 15.
10. 18.
11. 9.
12. 13.
13. 75.
14. 4.
15. 21.
16. 96.

EXERCISE 25. Page 49.

1. 43.
2. 23.
3. 31.
4. 19.
5. 41.
6. 43.
7. 118.
8. 161.
9. 37.
10. 295.
11. 73.
12. 83.
13. 79.
14. 89.

EXERCISE 26. Page 50.

1. 36.
2. 30.
3. 168.
4. 120.
5. 84.
6. 40.
7. 30.
8. 72.
9. 36.
10. 1026.
11. 936.
12. 660.
13. 432.
14. 900.
15. 3872.
16. 588.
17. 1155.
18. 4212.
19. 8820.
20. 690.

EXERCISE 29. Pages 55–56.

1. $\frac{5}{9}$.
2. $\frac{5}{8}$.
3. $\frac{3}{8}$.
4. $\frac{3}{7}$.
5. $\frac{11}{24}$.
6. $\frac{15}{32}$.
7. $\frac{3}{8}$.
8. $\frac{23}{6}$.
9. $\frac{19}{8}$.
10. $\frac{7}{16}$.
11. $\frac{19}{8}$.
12. $\frac{3}{2}$.
13. $\frac{2}{3}$.
14. $\frac{1}{6}$.
15. 7.
16. $\frac{7}{2}$.
17. $\frac{19}{30}$.
8. $\frac{1}{8}$.
19. $\frac{7}{4}$.
20. $\frac{21}{10}$.
21. $\frac{7}{10}$.
22. $\frac{3}{5}$.
23. $\frac{5}{42}$.
24. 10.
25. $\frac{21}{8}$.
26. $\frac{1}{3}$.
27. $\frac{4}{35}$.
28. $\frac{49}{6}$.
29. $\frac{1}{9}$.
30. $\frac{27}{35}$.

EXERCISE 32. Page 59.

1. $\frac{4}{6}$; $\frac{3}{6}$.
2. $\frac{4}{12}$; $\frac{9}{12}$.
3. $\frac{10}{15}$; $\frac{9}{15}$.
4. $\frac{15}{20}$; $\frac{8}{20}$.
5. $\frac{5}{6}$, $\frac{3}{6}$.
6. $\frac{12}{20}$; $\frac{5}{20}$.
7. $\frac{9}{12}$; $\frac{2}{12}$.
8. $\frac{21}{24}$; $\frac{8}{24}$.
9. $\frac{15}{40}$; $\frac{16}{40}$.
10. $\frac{4}{14}$; $\frac{7}{14}$.
11. $\frac{15}{24}$; $\frac{14}{24}$.
12. $\frac{18}{42}$; $\frac{35}{42}$.
13. $\frac{8}{10}$; $\frac{8}{10}$.
14. $\frac{20}{24}$; $\frac{9}{24}$.
15. $\frac{15}{10}$; $\frac{8}{10}$.
16. $\frac{20}{8}$; $\frac{7}{8}$.
17. $\frac{20}{30}$; $\frac{18}{30}$; $\frac{25}{30}$.
18. $\frac{36}{48}$; $\frac{10}{48}$; $\frac{60}{48}$.
19. $\frac{15}{18}$; $\frac{7}{18}$; $\frac{16}{18}$.
20. $\frac{124}{96}$; $\frac{68}{96}$; $\frac{50}{96}$.
21. $\frac{21}{70}$; $\frac{65}{70}$; $\frac{44}{70}$.
22. $\frac{35}{60}$; $\frac{27}{60}$; $\frac{38}{60}$.
23. $\frac{77}{84}$; $\frac{45}{84}$: $\frac{60}{84}$.
24. $\frac{25}{120}$; $\frac{92}{120}$; $\frac{51}{120}$.
25. $\frac{6}{32}$; $\frac{5}{32}$; $\frac{28}{32}$.
26. $\frac{3}{24}$; $\frac{18}{24}$; $\frac{21}{24}$.
27. $\frac{5}{64}$; $\frac{24}{64}$; $\frac{28}{64}$.
28. $\frac{6}{96}$; $\frac{80}{96}$; $\frac{56}{96}$.
29. $\frac{14}{30}$; $\frac{18}{30}$; $\frac{27}{30}$.
30. $\frac{10}{36}$; $\frac{42}{36}$; $\frac{28}{36}$.

EXERCISE 33. Pages 60–62.

1. $\frac{5}{4}$.
2. $\frac{5}{6}$.
3. $\frac{7}{12}$.
4. $\frac{11}{16}$.
5. $\frac{17}{12}$.
6. $\frac{5}{8}$.
7. $1\frac{1}{2}$.
8. $1\frac{5}{8}$.
9. $\frac{17}{24}$.
10. $1\frac{2}{3}$.
11. $1\frac{1}{6}$.
12. $1\frac{3}{16}$.
13. $1\frac{14}{15}$.
14. $2\frac{7}{8}$.
15. $1\frac{11}{20}$.
16. 1[illegible].
17. $\frac{29}{30}$.
18. $1\frac{11}{40}$.
19. $1\frac{11}{24}$.
20. $\frac{31}{36}$.
21. $1\frac{7}{32}$.
22. $1\frac{9}{20}$.
23. $2\frac{23}{104}$.
24. $\frac{103}{105}$.
25. $1\frac{7}{16}$.
26. $\frac{8}{15}$.
27. $2\frac{92}{385}$.
28. $1\frac{23}{28}$.
29. $\frac{8}{9}$.
30. $1\frac{41}{70}$.
31. $5\frac{7}{18}$.
32. $7\frac{5}{6}$.
33. $13\frac{1}{4}$.
34. 9.
35. $16\frac{19}{120}$.
36. $19\frac{19}{24}$.
37. $9\frac{5}{6}$.
38. $17\frac{1}{2}$.
39. $11\frac{7}{20}$.
40. $14\frac{23}{24}$.
41. $34\frac{5}{8}$ lb.
42. *A*. $46\frac{5}{6}$ hr.
B. $46\frac{1}{4}$ hr.
C. $47\frac{1}{12}$ hr.
D. 48 hr.
E. 46 hr.
43. $17\frac{3}{4}$ in.
44. $\frac{27}{32}$ in.
45. $1\frac{9}{32}$ in.
46. 48 in.
47. $5\frac{1}{2}$ in.
48. $7\frac{1}{2}$ in.

EXERCISE 34. Page 63.

1. $\frac{17}{5}$.
2. $\frac{23}{4}$.
3. $\frac{37}{8}$.
4. $\frac{20}{3}$.
5. $\frac{30}{7}$.
6. $\frac{35}{6}$.
7. $\frac{38}{5}$.
8. $\frac{75}{8}$.
9. $\frac{31}{3}$.
10. $\frac{38}{3}$.
11. $\frac{98}{11}$.
12. $\frac{97}{9}$.
13. $\frac{27}{4}$.
14. $\frac{39}{7}$.
15. $\frac{81}{8}$.
16. $\frac{77}{6}$.
17. $\frac{56}{3}$.
18. $\frac{108}{7}$.
19. $\frac{87}{4}$.
20. $\frac{34}{7}$.
21. $\frac{50}{11}$.
22. $\frac{53}{6}$.
23. $\frac{49}{12}$.
24. $\frac{51}{16}$.
25. $\frac{117}{10}$.
26. $\frac{44}{3}$.
27. $\frac{37}{4}$.
28. $\frac{97}{11}$.
29. $\frac{65}{6}$.
30. $\frac{133}{10}$.
31. $\frac{77}{8}$.
32. $\frac{101}{8}$.
33. $\frac{89}{5}$.
34. $\frac{5403}{100}$.
35. $\frac{645}{16}$.
36. $\frac{351}{64}$.
37. $\frac{55}{2}$.
38. $\frac{200}{3}$.
39. $\frac{500}{7}$.
40. $\frac{432}{5}$.

EXERCISE 35. Pages 64–65.

1. $\frac{1}{6}$.
2. $\frac{1}{6}$.
3. $\frac{5}{12}$.
4. $\frac{7}{20}$.
5. $\frac{1}{6}$.
6. $\frac{19}{40}$.
7. $\frac{1}{12}$.
8. $\frac{1}{24}$.
9. $\frac{3}{20}$.
10. $\frac{5}{24}$.
11. $\frac{1}{3}$.
12. $\frac{3}{16}$.
13. $\frac{1}{16}$.
14. $\frac{1}{40}$.
15. $\frac{13}{80}$.
16. $\frac{5}{32}$.
17. $\frac{19}{64}$.
18. $\frac{1}{24}$.
19. $\frac{13}{48}$.
20. $\frac{1}{32}$.
21. $4\frac{5}{8}$.
22. $6\frac{1}{3}$.
23. $9\frac{3}{8}$.
24. $5\frac{1}{3}$.
25. $7\frac{4}{9}$.
26. $11\frac{4}{7}$.
27. $5\frac{5}{12}$.
28. $5\frac{8}{9}$.
29. $5\frac{5}{16}$.
30. $13\frac{1}{16}$.
31. $14\frac{15}{32}$.
32. $21\frac{1}{8}$.
33. $96\frac{5}{8}$.
34. $28\frac{7}{8}$.
35. $14\frac{9}{10}$.
36. $17\frac{1}{35}$.

37. $3\frac{1}{4}$.
38. $1\frac{3}{8}$.
39. $\frac{11}{12}$.
40. $4\frac{3}{8}$.
41. \$$28\frac{1}{3}$.
\$$30\frac{1}{3}$.
\$$13\frac{1}{3}$.
\$$60\frac{7}{8}$.
\$$49\frac{3}{4}$.
\$$88\frac{3}{8}$.
\$$15\frac{7}{8}$.
\$$34\frac{5}{8}$.
42. $\frac{3}{8}$.
43. 14″ wide.
26″ high.

EXERCISE 36. Pages 66–67.

1. *a.* $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{3}{4}$, $\frac{8}{5}$, $\frac{7}{5}$, $\frac{8}{9}$, $\frac{7}{4}$, $\frac{11}{6}$.
b. 2, $\frac{9}{4}$, $\frac{5}{2}$, $\frac{9}{8}$, $\frac{12}{5}$, $\frac{21}{10}$, $\frac{4}{3}$, $\frac{21}{8}$, $\frac{11}{4}$.
c. $\frac{8}{3}$, 3, $\frac{10}{3}$, $\frac{3}{2}$, $\frac{16}{5}$, $\frac{14}{5}$, $\frac{16}{9}$, $\frac{7}{2}$, $\frac{11}{3}$.
d. $\frac{10}{3}$, $\frac{15}{4}$, $\frac{25}{6}$, $\frac{15}{8}$, 4, $\frac{7}{2}$, $\frac{20}{9}$, $\frac{35}{8}$, $\frac{55}{12}$.
e. $\frac{16}{3}$, 6, $\frac{20}{3}$, 3, $\frac{32}{5}$, $\frac{28}{5}$, $\frac{32}{9}$, 7, $\frac{22}{3}$.
f. $\frac{40}{3}$, 15, $\frac{50}{3}$, $\frac{15}{2}$, 16, 14, $\frac{80}{9}$, $\frac{35}{2}$, $\frac{55}{3}$.
2. *a.* $\frac{3}{10}$; *b.* $\frac{3}{4}$; *c.* $\frac{21}{20}$; *d.* $\frac{3}{2}$; *e.* $\frac{39}{20}$.
3. *a.* $\frac{7}{9}$; *b.* $\frac{7}{6}$; *c.* $\frac{77}{36}$; *d.* $\frac{91}{36}$; *e.* $\frac{7}{3}$.
4. *a.* $\frac{9}{4}$; *b.* $\frac{45}{28}$; *c.* $\frac{9}{2}$; *d.* $\frac{99}{28}$; *e.* 9.

5. $1\frac{1}{3}$.
6. 30.
7. 6.
8. 76.
9. $25\frac{5}{7}$.
10. $16\frac{4}{5}$.
11. 96.
12. $671\frac{1}{4}$.
13. 60.
14. $204\frac{3}{4}$.
15. $15\frac{3}{4}$.
16. $267\frac{1}{2}$.
17. $723\frac{3}{4}$.
18. 1315.
19. 511.
20. $245\frac{7}{10}$.
21. 33.
22. $57\frac{1}{2}$.
23. $79\frac{1}{2}$.
24. $109\frac{2}{3}$.
25. 69.
26. $346\frac{2}{3}$.
27. 805.
28. 300.
29. 750.
30. $1133\frac{1}{3}$.
31. 3200.
32. $7088\frac{3}{9}$.
33. $1784\frac{1}{2}$.
34. 2500.
35. $6533\frac{1}{3}$.
36. $941\frac{4}{5}$.
37. $980\frac{1}{2}$.
38. 220.
39. 560.
40. 201.

EXERCISE 37. Pages 68–71.

1. $\frac{3}{4}$.
2. $\frac{21}{40}$.
3. $\frac{11}{64}$.
4. $\frac{35}{44}$.
5. $\frac{3}{16}$.
6. $\frac{8}{21}$.
7. $5\frac{3}{5}$.
8. $\frac{3}{8}$.
9. $1\frac{2}{3}$.
10. $\frac{5}{294}$.
11. $2\frac{3}{4}$.
12. $\frac{8}{27}$.
13. $306\frac{3}{7}$.
14. $\frac{18}{65}$.
15. $19\frac{5}{7}$.
16. 54.
17. 155.
18. 3.
19. 245.
20. $2\frac{2}{3}$.
21. 1.
22. \$1.17.
23. \$162.40.
24. \$.55.
25. \$1.17.
26. \$6.94.
27. \$3.02.
28. \$76.29.
29. \$347.53.
30. \$1326.75.
31. \$1288.13.
32. \$3634.38.
33. \$468.75.
34. \$235.63.
35. \$85.31.
36. \$427.63.

37. \$450.66.
38. \$65.31.
39. \$116.62.
40. \$3012.19.
41. \$1771.88.
42. \$158.13.
43. \$51.87.
44. \$223.65.
45. \$333.

46. Cornmeal, $1\frac{1}{8}$ c.; sugar, $2\frac{1}{4}$ c.; baking powder, 6 t.; salt, $\frac{3}{4}$ t.; milk, $1\frac{1}{2}$ c.; egg, $1\frac{1}{2}$; butter, $1\frac{1}{2}$ T.

47. Milk, $\frac{3}{4}$ c.; ground coffee, $\frac{1}{4}$ c.; sugar, $\frac{1}{6}$ c.; arrowroot, $\frac{3}{8}$ T.

48. Gelatine, $\frac{1}{3}$ T.; cold water, $\frac{1}{12}$ c.; boiling water, $\frac{1}{4}$ c.; sugar $\frac{1}{6}$ c.; orange juice, $\frac{1}{4}$ c.; lemon juice, $\frac{2}{3}$ T.; white of egg, $\frac{1}{3}$.

Gelatine, $\frac{3}{2}$ T.; cold water, $\frac{3}{8}$ c.; boiling water, $1\frac{1}{8}$ c.; sugar, $\frac{3}{4}$ c.; orange juice, $1\frac{1}{8}$ c.; lemon juice, 3 T.; white of egg, $\frac{3}{2}$.

49. #1, 8 lb.; #2, $8\frac{4}{7}$ lb.; #3, $7\frac{1}{7}$ lb.; #4, $9\frac{3}{7}$ lb.

50. 14 lb. fruit: $10\frac{1}{2}$ lb. sugar, and $10\frac{1}{2}$ c. water.
9 lb. fruit: $6\frac{3}{4}$ lb. sugar, and $6\frac{3}{4}$ c. water.
17 lb. fruit: $12\frac{3}{4}$ lb. sugar, and $12\frac{3}{4}$ c. water.

51. *a.* \$160; *b.* \$233.33; *c.* \$300.

52. \$55.58.
53. \$.25.
54. \$24.15.
55. \$4089.
56. \$17.56.
57. \$23.31.
58. \$2.50.
59. \$72,276.71.

EXERCISE 38. Pages 72–74.

1. $\frac{5}{6}$.
2. 3.
3. $1\frac{1}{4}$.
4. $1\frac{5}{9}$.
5. $\frac{2}{3}$.
6. $\frac{5}{16}$.
7. $1\frac{3}{5}$.
8. $\frac{13}{14}$.
9. $1\frac{1}{2}$.
10. $\frac{3}{4}$.
11. $6\frac{2}{9}$.
12. 20.
13. $24\frac{3}{4}$.
14. 49.
15. 27.
16. $42\frac{2}{3}$.
17. $14\frac{2}{5}$.
18. $11\frac{3}{7}$.
19. $22\frac{1}{2}$.
20. $\frac{5}{32}$.
21. $\frac{7}{36}$.
22. $\frac{9}{80}$.
23. $\frac{5}{18}$.
24. $\frac{3}{16}$.
25. $\frac{7}{32}$.
26. $\frac{11}{3}$.
27. $\frac{4}{5}$.
28. $\frac{11}{16}$.
29. $\frac{3}{88}$.
30. $2\frac{1}{2}$.
31. $12\frac{3}{5}$.
32. $1\frac{2}{3}$.
33. $1\frac{8}{27}$.
34. $2\frac{2}{3}$.
35. $\frac{11}{64}$.
36. $\frac{3}{25}$.
37. $1\frac{1}{4}$.
38. $\frac{77}{153}$.
39. $1\frac{3}{4}$.
40. $7\frac{1}{3}$.
41. $\frac{5}{84}$.
42. $\frac{7}{180}$.
43. $3\frac{19}{24}$
44. $\frac{6}{65}$.
45. $\frac{75}{152}$.
46. $2\frac{13}{28}$.
47. $42\frac{1}{2}$.
48. $\frac{3}{10}$.
49. $\frac{7}{40}$.
50. $3\frac{3}{7}$.
51. 11 towels.
52. 5 garments
53. $1\frac{57}{64}$ in.
54. $\frac{3}{4}$ in.
55. $1\frac{3}{4}$ in.
56. 12 yd.
57. \$9.21.

EXERCISE 39. Pages 74–75.

1. $\frac{8}{15}$.
2. $\frac{11}{16}$.
3. $7\frac{1}{2}$.
4. 1.
5. $3\frac{3}{4}$.
6. $5\frac{1}{5}$.
7. $8\frac{5}{8}$.
8. 12.
9. $2\frac{4}{5}$.
10. $2\frac{3}{20}$.
11. $3\frac{7}{18}$.
12. $4\frac{4}{5}$.
13. $\frac{48}{55}$.
14. $\frac{3}{32}$.
15. $14\frac{2}{3}$.
16. $\frac{10}{21}$.
17. $16\frac{1}{2}$.
18. $7\frac{1}{2}$.
19. $1\frac{6}{7}$.
20. $\frac{44}{45}$.
21. $\frac{7}{10}$.
22. $\frac{14}{15}$.
23. $1\frac{4}{5}$.
24. $\frac{4}{25}$.
25. $\frac{10}{21}$.
26. $4\frac{2}{3}$.
27. $\frac{3}{2}$.
28. $2\frac{4}{35}$.

EXERCISE 40. Pages 75–77

1. *a.* $\frac{1}{6}$; *b.* $\frac{1}{4}$; *c.* $\frac{1}{3}$; *d.* $\frac{5}{12}$; *e.* $\frac{7}{12}$; *f.* $\frac{3}{4}$.
2. *a.* $\frac{1}{18}$; *b.* $\frac{1}{6}$; *c.* $\frac{13}{54}$; *d.* $\frac{7}{18}$; *e.* $\frac{7}{12}$; *f.* $\frac{7}{9}$.
3. *a.* $\frac{1}{3}$; *b.* $\frac{3}{20}$; *c.* $\frac{3}{16}$; *d.* $\frac{1}{9}$; *e.* $\frac{1}{14}$; *f.* $\frac{3}{70}$.
4. *a.* $\frac{1}{2}$; *b.* $\frac{7}{18}$; *c.* $\frac{14}{45}$; *d.* $\frac{4}{19}$; *e.* $\frac{14}{75}$; *f.* $\frac{1}{6}$.
5. *a.* $\frac{3}{7}$; *b.* $\frac{6}{5}$; *c.* $\frac{12}{7}$; *d.* $\frac{6}{7}$.
6. *a.* $\frac{3}{28}$; *b.* $\frac{25}{192}$; *c.* $\frac{1}{5}$; *d.* $\frac{11}{45}$.
7. *a.* $\frac{3}{4}$; *b.* $\frac{10}{7}$; *c.* $\frac{7}{36}$.
8. *a.* $\frac{16}{45}$; *b.* $\frac{20}{9}$; *c.* $\frac{9}{4}$.
9. $\frac{6}{5}$.
10. $\frac{15}{28}$.
11. $\frac{71}{440}$.
12. $\frac{11}{7}$.
13. $\frac{27}{28}$.
14. $\frac{558}{1135}$.
15. $\frac{1}{6}$.
16. $\frac{25}{28}$.
17. $\frac{25}{32}$.
18. $\frac{3}{10}$.
19. $\frac{4}{5}$.
20. $\frac{112}{125}$.
21. \$3.21.
22. \$4.25.
23. With 4 qt. — $\frac{1}{2}$ qt. vinegar.
    3 lb. sugar.
    $\frac{1}{8}$ c. cloves and cinnamon.
    With 10 qt. — $1\frac{1}{4}$ qt. vinegar.
    $7\frac{1}{2}$ lb. sugar.
    $\frac{5}{16}$ c. cloves and cinnamon.
24. With 3 lb. — 4 qt. raspberries, and 3 lb. sugar.
    With 5 lb. — $6\frac{2}{3}$ qt. raspberries, and 5 lb. sugar.
    With $3\frac{1}{2}$ lb. — $4\frac{2}{3}$ qt. raspberries, and $3\frac{1}{2}$ lb. sugar.
25. With 5 lb. — $3\frac{4}{7}$ lb. sugar; $2\frac{1}{7}$ T. cinnamon;
    $2\frac{1}{7}$ T. cloves; $\frac{5}{7}$ pt. vinegar.
    With 1 lb. — $\frac{5}{7}$ lb. sugar; $\frac{3}{7}$ T. cinnamon;
    $\frac{3}{7}$ T. cloves; $\frac{1}{7}$ pt. vinegar.
    With 13 lb. — $9\frac{2}{7}$ lb. sugar; $5\frac{4}{7}$ T. cinnamon;
    $5\frac{4}{7}$ T. cloves; $1\frac{6}{7}$ pt. vinegar.

EXERCISE 41. Page 78.

1. 32.
2. 42.
3. *a.* 72; *b.* 64; *c.* 54; *d.* 88.
4. 48.
5. 3750.
6. 85.
7. 48 da.
8. $1242.
9. 875 eggs.
10. 1063 ft.

EXERCISE 42. Page 79.

1. $28\frac{4}{7}$.
2. $71\frac{3}{7}$.
3. $22\frac{2}{3}$.
4. $55\frac{5}{9}$.
5. $77\frac{7}{9}$.
6. 30.
7. $63\frac{7}{11}$.
8. $41\frac{2}{3}$.
9. $13\frac{1}{8}$.
10. $36\frac{4}{11}$.
11. $18\frac{3}{4}$.
12. 35.
13. 16.
14. $27\frac{1}{2}$.
15. 6.
16. $31\frac{1}{4}$.
17. 28.
18. $22\frac{1}{2}$.
19. 22.
20. $43\frac{3}{4}$.
21. $58\frac{1}{8}$.
22. $44\frac{4}{9}$.
23. $45\frac{5}{11}$.
24. $16\frac{2}{3}$.
25. $54\frac{6}{11}$.
26. $26\frac{2}{3}$.
27. $53\frac{1}{3}$.
28. $91\frac{2}{3}$.
29. $46\frac{2}{3}$.
30. $72\frac{8}{11}$.
31. $56\frac{1}{4}$.
32. 52.
33. $37\frac{1}{2}$.
34. $81\frac{9}{11}$.
35. $73\frac{1}{8}$.
36. $90\frac{10}{11}$.
37. 68.
38. $68\frac{3}{4}$.
39. $32\frac{1}{2}$.
40. $93\frac{3}{4}$.
41. $35\frac{2}{3}$.
42. 34.
43. $\frac{2}{5}$.
44. $\frac{7}{10}$.
45. $\frac{1}{25}$.
46. $\frac{3}{4}$.
47. $\frac{3}{5}$.
48. $\frac{1}{8}$.
49. $\frac{1}{6}$.
50. $\frac{1}{3}$.
51. $\frac{1}{12}$.
52. $\frac{1}{16}$.
53. $\frac{3}{8}$.
54. $\frac{1}{15}$.
55. $\frac{5}{8}$.
56. $\frac{3}{16}$.
57. $\frac{7}{8}$.
58. $\frac{1}{30}$.
59. $\frac{1}{40}$.
60. $\frac{3}{40}$.

EXERCISE 43. Pages 80–82.

1. *a.* $5; *b.* $8; *c.* $4; *d.* $6; *e.* $6.67; *f.* $3.33; *g.* $2.67; *h.* $3.67; *i.* $1.67; *j.* $5.33.

2. *a.* $1.88; *b.* $3; *c.* $1.50; *d.* $2.25; *e.* $2.50; *f.* $1.25; *g.* $1; *h.* $1.38; *i.* $.63. *j.* $2.

3. *a.* $.94; *b.* $1.50; *c.* $.75; *d.* $1.13; *e.* $1.25; *f.* $.63; *g.* $.50; *h.* $.69; *i.* $.31; *j.* $1.00.

4. *a.* $1.25; *b.* $2; *c.* $1; *d.* $1.50; *e.* $1.67; *f.* $.83; *g.* $.67; *h.* $.92; *i.* $.42; *j.* $1.33.

5. *a.* $2.50; *b.* $4.00; *c.* $2.00; *d.* $3.00; *e.* $3.33; *f.* $1.67; *g.* $1.33; *h.* $1.83; *i.* $.83; *j.* $2.67.

6. *a.* $5.63; *b.* $9.00; *c.* $4.50; *d.* $6.75; *e.* $7.50; *f.* $3.75; *g.* $3.00; *h.* $4.13; *i.* $1.88; *j.* $6.00.

7. *a.* $11.25; *b.* $18.00; *c.* $9.00; *d.* $13.50; *e.* $15.00; *f.* $7.50; *g.* $6.00; *h.* $8.25; *i.* $3.75; *j.* $12.00.

8. *a.* $9.00; *b.* $14.40; *c.* $7.20; *d.* $10.80; *e.* $12.00; *f.* $6.00; *g.* $4.80; *h.* $6.60; *i.* $3.00; *j.* $9.60.

9. *a.* $13.13; *b.* $21.00; *c.* $10.50; *d.* $15.75; *e.* $17.50; *f.* $8.75; *g.* $7.00; *h.* $9.63; *i.* $4.38; *j.* $14.00.

10. *a.* $20.00; *b.* $32.00; *c.* $16.00; *d.* $24.00; *e.* $26.67; *f.* $13.33; *g.* $10.67; *h.* $14.67; *i.* $6.67; *j.* $21.33.

11. *a.* $21.00; *b.* $33.60; *c.* $16.80; *d.* $25.20; *e.* $28.00; *f.* $14.00; *g.* $11.20; *h.* $15.40; *i.* $7.00; *j.* $22.40.

12. *a.* $20.13; *b.* $33.00; *c.* $16.50; *d.* $24.75; *e.* $27.50; *f.* $13.75; *g.* $11.00; *h.* $15.13; *i.* $6.88; *j.* $22.00.

13. *a.* $10.00; *b.* $12.00; *c.* $3.75; *d.* $50.00; *e.* $5.00; *f.* $11.25.

14. *a.* $160.00; *b.* $180.00; *c.* $16.00; *d.* $300.00; *e.* $20.00; *f.* $150.00.

15. *a.* $106.75; *b.* $128.00; *c.* $40.00; *d.* $533.33; *e.* $53.33; *f.* $120.00.

16. *a.* $800.00; *b.* $900.00; *c.* $80.00; *d.* $1500.00; *e.* $100.00; *f.* $750.00.

17. *a.* $21.00; *b.* $18.25; *c.* $3.00; *d.* $4.75.

18. *a.* $4.50; *b.* $8.50; *c.* $14.50; *d.* $17.00; *e.* $23.50: *f.* $34.50.

19. *a.* $2.25; *b.* $3.00; *c.* $3.38; *d.* $5.63; *e.* $8.63; *f.* $12.25.

20. *a.* $3.00; *b.* $6.25; *c.* $7.00; *d.* $8.33; *e.* $10.00; *f.* $12.50.

21. $6.75. 23. $61.26. 25. $1001.25.
22. $1.71. 24. $265.83.

EXERCISE 44. Pages 82-83.

1. $1176. 2. $37.84. 3. A, $5.80; B, $4.35; C, $7.25.
4. The second is greater by $.62. 5. 23⅓ yd.
6. A, $5; B, $3.75; C, $2.50.
7. Wife, $874.80; son, $546.75; daughter, $328.05.
8. $171.60. 9. 14½ lb. 10. A, $51; B, $99; C, $66.
11. Wife, $24,650; elder son, $18,487.50; younger, $14,790; daughter, $12,325.
12. Real estate $15,000; railway shares, $2700; city bonds, $6300.
13. Older son, $132; younger, $88.
14. *a.* $\frac{15}{121}$; *b.* $\frac{20}{121}$; *c.* $7260; *d.* $6352.50. 15. 25½ min.

EXERCISE 45. Pages 86–87.

20. *a.* Divides it by 10; *b.* Divides it by 100; *c.* Divides it by 1000.
21. 52.43. 22. .158. 23. 9.0013. 24. .059398. 25. 26.805303.

EXERCISE 46. Page 88.

1. $\frac{14}{5}$.
2. $\frac{1}{200}$.
3. $\frac{171}{250}$.
4. $\frac{87}{20}$.
5. $\frac{1064}{125}$.
6. $\frac{1886}{25}$.
7. $\frac{123}{4}$.
8. $\frac{79}{400}$.
9. $\frac{68461}{1000}$.
10. $\frac{1}{40}$.
11. $\frac{47}{1250}$.
12. $\frac{6}{625}$.
13. $\frac{247}{80}$.
14. $\frac{8809}{1000000}$.
15. $\frac{1}{800}$.
16. $\frac{1}{16}$.
17. $\frac{35}{4}$.
18. $\frac{101}{4}$.
19. $\frac{3}{80}$.
20. $\frac{347}{200}$.
21. $\frac{41}{4}$.
22. $\frac{8825}{4}$.
23. $\frac{25}{4}$.
24. $\frac{6521}{500}$.
25. $\frac{43}{8}$.

EXERCISE 47. Pages 89–90.

1. 32.3768.
2. 38.66775.
3. 205.18578.
4. 18.28524.
5. 7.732546.
6. 899.58245.
7. 101.03096.
8. 4716.08012.
9. 82.85578.
10. 44.16584.
11. 4.8598.
12. .24693.
13. 1.683.
14. .01086.
15. 9.2807.
16. 2.3632.
17. 9.93603.
18. 3.43918.
19. 782.8523.
20. .05910.
21. 8298.6070.
22. 10669.7956.
23. 11079.8125.
24. 9841.3280.
25. 9683.2631.

EXERCISE 48. Pages 91–92

1. 225.771.
2. .53856.
3. .0258420.
4. 6.33216.
5. .463136.
6. 96.6735.
7. .947232.
8. .15561902.
9. 1.0912.
10. 24.13468.
11. 13.539.
12. 104.9688.
13. 4.93051.
14. .07599176.
15. .428424.
16. 142.981616.
17. .063297.
18. 30.2567496.
19. 2.19912.
20. .000418.
21. 210.1896.
22. 3.8608812.
23. 6.9957125.
24. 109.375.

25. Copper, 528.125 lb.; cork, 15 lb.; gold, 1203.75 lb.; ice, 57.50 lb.; lead, 709.375 lb.; steel, 489.375 lb.; air, .0808125 lb.

26. *a*. 144.72 ft.; *b*. 257.28 ft.; *c*. 402 ft.; 4th second, 112.56 ft.; 5th second, 144.72 ft.

27. $2435.06.

28. $1705.16.

29. *a*. .029878 in.;
b. .061364 in.

EXERCISE 49. Pages 93–94.

7. 852.
8. 3.77.
9. 6.5.
10. 2.08.
11. .1261.
12. .05839.
13. .14170.
14. .3688.
15. 9.37628.
16. 6023.
17. $5.50.
18. $12.50.

19. $937.50.
20. $13.25.
21. $5325.60
22. $5044.60.
23. $571.40.
24. $787.50.
25. $37.17.
26. $7.25.
27. $332.50.
28. $5574.00.
29. $418.75.
30. $156.38.
31. $78.20; $700.10.
32. $10,763; $35.90.
33. $1831.73.
34. $376.
35. $13,730.
36. $1255.
37. $26,570.
38. $3717.

EXERCISE 50. Pages 97–99.

1. 50.
2. 700.
3. 6.
4. .6.
5. 180.
6. .22.
7. 7.
8. 30.
9. 310.
10. 3200.
11. 2000.
12. 62,800.
13. 309.
14. 400.
15. 7480.
16. 1.63.
17. .896+.
18. 62,510.
19. 2.35.
20. 45.6.
21. 508.3.
22. 9.643.
23. 49,387.593.
24. 6812.5.
25. 24,902+.
26. *a.* 45 m. per hr.
    b. 29.2 m. per hr.
    c. 34.9 m. per hr.
    d. 51.4 m. per hr.
27. 32.3 m. per hr.
28. 18.22 tons.
29. 11.2 m.
30. 3.1¢.
31. 20.6 A.
32. 30.9 people.
33. $.63\frac{7}{10}$¢.
34. $28.98.
35. *a.* 3026.6+.
    b. $293.047.
36. 36.1 times.
    2.9 times.
    23.6 times.
    3.6 times.
    18.2 times.

EXERCISE 51. Pages 100–102.

6. 1.26.
7. .325.
8. 2.531.
9. .000463.
10. .0039075.
11. .132.
12. 1.2892.
13. 9.288+.
14. 34.436.
15. 641.123.
16. 2.670.
17. 86.4212+.
18. 3.84; $33.60.
19. 15.35; $552.60.
20. 121.06; $277.74.
21. 27.6.
22. 2.65; $34.58.
23. 141.4; $1948.49.
24. $91.68.
25. $312.58.
26. $4563.95.
27. $213.63.
28. $425.
29. $37.38.
30. $24.03.
31. $32.25.
32. $14.63.
33. $168.75.
34. $96.44.
35. $311.38.
36. *a.* $20.21;
    b. $24.19;
    c. $13.98.
37. *a.* $123.75;
    b. $103.13;
    c. $233.06.

38. *a.* $22.69;
b. $13.94;
c. $60.64.

39. *a.* $234;
b. $138.38;
c. $177.

40. *a.* $2462.64;
b. $1638.45;
c. $1546.10.

EXERCISE 52. Page 103.

1. .777+.
2. .2.
3. .3125.
4. .333+.
5. .238+.
6. .111+.
7. .285+.
8. .230+.
9. .511+.
10. .478+.
11. .222+.
12. .468+.
13. .518+.
14. .272+.
15. .357+.
16. 1.888+.
17. .653+.
18. .281+.
19. .916+.
20. 1.437+.
21. 4.575+.
22. 3.506+.
23. .0682.
24. 7.022+.
25. .338+.
26. .067+.
27. .100+.
28. 21.397+.
29. .031+.
30. .072+.
31. .550+.
32. .851+.
33. 24.381+.
34. .087+.
35. .198+.
37. .25.
38. .5.
39. .333.
40. .666.
41. .2.
42. .6.
43. .166.
44. .125.
45. .375.
46. .083.
47. .416.
48. .875.
49. .833.
50. .583.
51. .0625.
52. .066.
53. .625.
54. .8.
55. .28.
56. .225.

EXERCISE 53. Pages 104–107.

1. *a.* .260 lb.
b. .282 lb.
c. .411 lb.

2. 452.3904 cu. in.
127.57 lb.

3. $2200.54.

4. $557.89.

5. $2,150,465,600.

6. 102.5 a.

7. Gt. Britain, 2808.3 t.;
Germany, 2871.7 t.;
United States, 1254 t.

8. *a.* 7.48 gal.;
b. 8.35 lb.

11. $95.85.

12. *a.* 426.83+;
b. 96.054+;
c. 1.406+;
d. 15.526+;
e. 283.498+;
f. 1386.2+;
g. 288.55+;
h. .0838+.

13. *a.* 63+;
b. 46+.

14. 341.5 in.

15. 12.3 h. p.

16. 20 lb.

17. Income, $1,883,017.17;
Total Assets and Income, $6,762,888.67;
Disbursements, $1,011,152.20;
Balance, $5,751,736.47;
Ledger Assets, $5,751,736.47.

EXERCISE 54. Pages 109–110.

1. 1600 rd. ; 26,400 ft.
2. 48 qt. ; 96 pt.
3. 135 sq. ft. ; 19,440 sq. in.
4. 168 hr. ; 10,080 min.
5. 60 cwt.; 6000 lb.
6. 256 qt. ; 512 pt.
7. 900′ ; 54,000″.
8. 81 cu. ft. ; 139,968 cu. in.
9. 121 sq. yd. ; 1089 sq. ft.
10. 132 yd. ; 396 ft.
11. 18 cu. ft. ; 31,104 cu. in.
12. 448 rd. ; 7392 ft.
13. 192.5 yd. ; 577.5 ft.
14. 27′ ; 1620″.
15. 3.75 pk. ; 22 qt. ; 44 pt.
16. 16 sq. rd. ; 4356 sq. ft.
17. 12 cwt. ; 1200 lb.
18. 38 pk. ; 608 pt.
19. 94.08 hr. ; 5644.8 min.
20. 2341.35 ft.
21. 14.4 qt. ; 28.8 pt.
22. 102 pt.
23. 3581 lb.
24. 14,671.25 sq. yd.
25. 8765 hr.
26. 14,734.5 ft.
27. 304 qt.
28. 137 in.
29. 39 pt.
30. 52 sq. ft.
31. 413 cu. ft.
32. 3 qt. 1 pt.
33. 2 pk. 2 qt.
34. 2 hr. 30 min. 36 sec.
35. 1 rd. 3 yd. 2 ft. 6 in.
36. 2 lb. 13 oz.
37. 5 da. 6 hr.
38. 4 yd. 1 ft. 2.4 in.
39. 2 pk. 4 qt.
40. 13 cwt. 33 lb. 5¼ oz.
41. 4 mo.
42. 53 rd. 1 yd. 2 ft. 6 in.
43. 1 yd. 1 ft. 1.5 in.

EXERCISE 55. Pages 111–112.

1. $\frac{3}{4}$; .75.
2. $\frac{7}{8}$; .875.
3. $\frac{7}{12}$; .58$\frac{1}{3}$.
4. $\frac{25}{32}$; .78125.
5. $\frac{5}{8}$; .625.
6. $\frac{5}{9}$; .55$\frac{5}{9}$.
7. $\frac{5}{8}$; .625.
8. $\frac{25}{44}$; .56$\frac{9}{11}$.
9. $\frac{250}{363}$; .68+.
10. $\frac{5}{9}$; .55+.
11. $\frac{175}{264}$; .66+.
12. $\frac{27}{40}$; .675.
13. $\frac{15}{64}$; .23+.
14. $\frac{50}{73}$; .68+.
15. $\frac{15}{26}$; .57+.
16. $\frac{7}{18}$; .38+.
17. $\frac{35}{64}$; .54+.
18. $\frac{3}{4}$; .75.
19. $\frac{5}{6}$; .83$\frac{1}{3}$.
20. $\frac{7}{10}$; .7.
21. $3\frac{299}{320}$; 3.93+.
22. $3\frac{7248}{7845}$; 3.92+.
23. $2\frac{2613}{2640}$; 2.98+.
24. $69\frac{1}{2}$; 69.5.
25. $1\frac{757}{1089}$; 1.69+.
26. $1\frac{59}{66}$; 1.89+.
27. $1\frac{87}{88}$; 1.98+.
28. $390\frac{5}{6}$; 390.83+.
29. $977\frac{11}{32}$; 977.34+.
30. $351\frac{11}{24}$; 351.45+.

31. 15.
32. $346\frac{23}{27}$; 346.85+.
33. $2\frac{111}{132}$; 2.84+.
34. $4\frac{23}{88}$; 4.26+.
35. $833\frac{1}{3}$; 833.33+.
36. $17\frac{39}{73}$; 17.53+.
37. 15.
38. $3\frac{161}{363}$; 3.44+.
39. $\frac{753}{1000}$; .753+.
40. $\frac{9}{16}$; .5625.
41. $16\frac{5}{12}$; 16.41+.
42. $\frac{5}{8}$; .625.
43. $8\frac{11}{12}$; 8.91+.
44. $4\frac{2}{5}$; 4.4.
45. $\frac{287}{400}$; .7175.
46. $\frac{25}{32}$; .78+.
47. $\frac{7}{8}$; .875.
48. $\frac{29}{42}$; .69+.
49. 15 bu. 7 qt. 1 pt.
50. 1 cwt. 47 lb. 5 oz.
51. 326 gal. 1 pt.
52. 229 rd. 2 yd. 1 ft. 6 in.
53. 2 A. 96 sq. rd. 4 sq. yd.
54. 22 hr. 13 min. 20 sec.
55. 2 A. 42 sq. rd. 3 sq. yd. 72 sq. in.
56. 1 T. 17 cwt. 58 lb.
57. 2 mi. 274 rd. 5 yd.
58. 160 rd. 4 yd. 2 ft.
59. 2 mi. 269 rd. 1 ft. 6 in.
60. 1 mi. 195 rd. 2 ft. 6 in.
61. 13 T. 10 cwt.
62. 8 mi. 167 rd. 1 yd. 1 ft. 6 in.

EXERCISE 56. Pages 113–114.

1. 44 gal. 3 qt.
2. 18 cwt. 99 lb.
3. 164 bu. 3 pk. 3 qt.
4. 218 da. 12 hr. 1 min.
5. 313° 27′ 20″.
6. 26 mi. 118 rd. 3 yd.
7. 40 T. 7 cwt. 37 lb.
8. 226 cu. ft. 645 cu. in.
9. 5 sq. mi. 236 A. 61 sq. rd.
10. 80 mi. 234 rd. 1 yd.
11. 10 bu. 2 pk. 6 qt.
12. 37° 46′ 44″.
13. 81° 5′ 15″.
14. 9 gal. 3 qt. 1 pt.
15. 9 lb. 12 oz.
16. 11 cu. yd. 9 cu. ft. 976 cu. in.
17. 8 mi. 22 rd.
18. 8 T. 15 cwt.
19. 5 A. 13 sq. rd.
20. 3 da. 19 hr. 48 min.
21. 74 A. 1 sq. rd.
22. 39 rd. 5 yd. 2 ft.
23. 5 yd. 1 ft. $3\frac{1}{2}$ in.
24. 28 rd. 2 yd.
25. 24 gal. 3 qt. 1 pt.

EXERCISE 57. Pages 119–121.

1. 263 da.
2. 281 da.
3. 116 da.
4. 322 da.
5. 260 da.
6. 252 da.
7. 255 da.
8. 173 da.
9. 205 da.
10. 299 da.
11. 4 yr. 10 mo. 15 da.
12. 5 yr. 3 mo. 22 da.
13. 1 yr. 10 mo. 26 da.
14. 3 yr. 8 mo. 24 da.
15. 1 yr. 11 mo. 21 da.
16. 5 yr. 4 mo. 18 da.
17. 6 yr. 2 mo. 23 da.
18. 10 yr. 3 mo. 20 da.
19. 3 yr. 3 mo. 9 da.
20. 3 yr. 6 mo. 8 da.
21. 4 yr. 10 mo. 15 da.
22. 5 yr. 3 mo. 21 da.
23. 1 yr. 10 mo. 27 da.
24. 3 yr. 8 mo. 24 da.
25. 1 yr. 11 mo. 21 da.
26. 5 yr. 4 mo. 20 da.
27. 6 yr. 2 mo. 22 da.
28. 10 yr. 3 mo. 19 da.
29. 3 yr. 3 mo. 9 da.
30. 3 yr. 6 mo. 8 da.

EXERCISE 58. Page 117.

1. 41 bu. 1 pk.
2. 42 ft. 11 in.
3. 14 gal. 1 qt.
4. 57 sq. ft. 112 sq. in.
5. 22 rd. 5 yd.
6. 15 bu. 3 pk. 2 qt.
7. 79° 54′.
8. 95 gal. 1 qt.
9. 75 mi. 143 rd. 3½ yd.
10. 58 da. 4 hr. 10 min.
11. 82 T. 4 cwt. 25 lb.
12. 410 cu. ft. 976 cu. in.
13. 310 lb. 15 oz.
14. 146 rd. 4 yd.
15. 55 A. 70 sq. yd.
16. 86′ 8″.
17. 308′ 4″.
18. $7970.63.
19. $314.05.
20. 75.9 cu. ft.
21. About 24½ yd.

EXERCISE 59. Page 118.

1. 11 bu. 2 pk. 5⅓ qt.
2. 10 gal. 3 qt.
3. 2′ 11½″.
4. 5° 37′ 30″.
5. 67 A. 120 sq. rd.
6. 8 lb. 4 oz.
7. 5 T. 15 cwt.
8. 4 sq. yd. 2 sq. ft.
9. 9 rd. 4 yd.
10. 3 A. 132 sq. rd.
11. 23 T. 9 cwt. 71 lb.
12. 11 bu. 3 pk. 7 qt.
13. 3′ 7″.
14. 1 rd. 1 yd.
15. 6 da. 13 hr.
16. 45 lb. 6⅔ oz.
17. 4 T. 18 cwt.
18. 30 mi. 203 rd. 3½ yd.
19. 6 cu. ft. 1495$\frac{3}{7}$ cu. in.
20. 58° 27′ 24″.
21. 2′ 11¼″ wide; 4′ 3″ long.
22. 8″.

EXERCISE 60. Pages 119–121.

1. 20.
2. 12.
3. 8.
4. 55.
5. 4.
6. 121.
7. 15.
8. $\frac{5}{16}$.
9. 68 pcs.
10. 1350.
11. 51.
12. 12.
13. 76A. 106⅔ sq. rd.
14. 11.7 bbl.
15. 3344 ft. per min.
16. $545.75.
17. 191^+ lb. per da.
18. 12.1 sec.
19. *a.* 1′ 30″; *b.* 40 mi.
20. $.03^+$.
21. 110.88 T.
22. 105.1 cu. in.
23. 1781.2 oz.
24. 44 rails.
25. $3284.
26. $499.49.
27. $1967.68.
28. $1274.91.
29. $554.95.
30. $14.20.

EXERCISE 61. Page 122.

1. About 25.9 sec.
2. 2.08 mi.
3. 23.1 mi. per min.
4. $143.61.
5. 371.2 sq. rd.
6. *a.* 3.4 mi.; *b.* 3 mi. 130 rd. 5 yd.
7. *a.* 1.11 gal.; *b.* $.42.
8. $360.36.

10. 115.9 gal.

EXERCISE 62. Page 126.

1. 76.5 mm; 32 mm.
2. $1\frac{25}{32}$ in.; $3\frac{21}{32}$ in.
3. 19 mm.
4. $21\frac{1}{2}$ mm.

EXERCISE 63. Pages 127–128.

1. 472 cm.; 4720 mm.; .00472 Km.
2. 73 l.; 7300 cl.
3. 8325 g.; 83,250 dg.; 8,325,000 mg.
4. 73,850 Km.; 7385 Dm.
5. 548 sq. m.
6. 354 cm.; 3.54 m.
7. 7065 g.
8. 5.406 l.
9. 7856 cm.; .7856 Hm.
10. 735,000 cu. dm.
11. 3931.26 mg.
12. 1328.1 m.
13. 323.92 sq. m.
14. 1919 g.
15. 449,118 mm.
16. 424.66 l.
17. .12575 Hm.
18. .289065 sq. m.
19. 2.862 cg.
20. .037.
21. 31.956 g.
22. .568.
23. 93.6 cu. mm.
24. 244.326 dm.
25. 7.533^{+}.

EXERCISE 64. Pages 129–130.

1. 9.14 dm.
2. .8808 Dl.
3. 9.29088 sq. dm.
4. 35.316 cu. ft.
5. 10.76 sq. ft.
6. 4.7315 dl.
7. 616.3^{+} rd.
8. 26.7552 sq. m.
9. 5.179^{+} m.
10. $.566^{+}$ cu. m.
11. 11.7369 Kg.
12. 44.931 Dl.
13. 5.833^{+} Km.
14. 7716.1 lb.
15. 1718.75 mi.
16. 37.065 A.
17. 16.188 Ha.
18. 27.216 Kg.
19. $.119^{+}$ Kl.
20. $9\frac{3}{8}$ mi.
21. $147.25.
22. $133.08.
23. 457 m.
24. About 1056.
25. $.529^{+}$ oz.
26. $.2^{+}$ l.
27. 367^{+} bu.
28. 1722^{+} bu.
29. 378,500 l.
30. 22,680 Kg.
31. $28\frac{1}{8}$ mi.

EXERCISE 65. Page 131.

1. 240*d.*
2. £$\frac{5}{12}$.
3. £3 15*s.*
4. 100 fr.
5. $14.48.
6. 777 fr. 20 cen.

7. $364.99.
8. 12 fr. 50 cen.
9. 180*d.*
10. 9*s.*; 108*d.*
11. 78 cen.
12. 60 cen.
13. £1 2*s.* 2*d.*
14. £205 9*s.* 7*d.*
15. £13 17*s.* 6*d.*
16. *a.* 3.86¢;
b. $14.48.

EXERCISE 66. Pages 132–133.

1. 288 ℈.
2. 69 ʒ.
3. 208 ʒ.
4. $1\frac{1}{4}$ ʒ.
5. $1\frac{1}{24}$ ℥.
6. 4 ℥; 32 ʒ.
7. 128 f ʒ.
8. 192 f ℥.
9. $3\frac{1}{4}$ f ℥.
10. $\frac{53}{96}$ O.
11. 1 ℔, 4 ℥ 5 ʒ 2 ℈ 7 gr.
12. 4 O 10 f ℥ 4 f ʒ 38 m.
13. 9906 ℔ 3 ℥.
14. 2 ℔ 3 ℥ 2 ʒ 2 ℈.
15. 2 ℔ 8 ℥.
16. 5 O 7 f ℥.
17. 2 O 4 f ℥.
18. 32.
19. 4 O 7 f ℥.
20. 1 ℥ 1 ʒ.
21. nux vomica, 2 ʒ 1 ℈ 4 gr.;
caffeine citrate, 1 ℥ 1 ʒ 1 ℈ 16 gr.;
sodium salicylate, 1 ℥ 4 ʒ;
sodium phosphate, 3 ℥.

EXERCISE 67. Pages 134–135.

1. 25.
2. 27.
3. 36.
4. 16.
5. 64.
6. 1000.
7. 81.
8. 625.
9. 216.
10. 400.
11. 10,000.
12. 343.
13. 81.
14. 625.
15. 27,000.
16. 121.
17. 256.
18. 243.
19. 64.
20. 1296.
21. $\frac{1}{4}$.
22. $\frac{8}{27}$.
23. $\frac{1}{9}$.
24. $\frac{8}{125}$.
25. $\frac{9}{16}$.
26. $\frac{25}{36}$.
27. $\frac{1}{81}$.
28. $\frac{16}{121}$.
29. 1728.
30. 7225.
31. 6.25.
32. 1.21.
33. 12.25.
34. 18.49.
35. 2.25.
36. 1.69.
37. .0625.
38. .0016.
39. .0441.
40. 4.1209.
41. .000009.
42. 9.7344.
43. 405.2169.
44. .046225.

EXERCISE 68. Page 136.

1. 9.
2. 4.
3. 12.
4. 3.
5. 11.
6. 10.
7. 14.
8. 5.
9. 16.
10. 8.
11. 20.
12. 7.
13. 18.
14. 9.
15. 19.
16. 6.
19. $\frac{2}{3}$.
20. $\frac{5}{7}$.
21. $\frac{3}{10}$.
22. $\frac{4}{9}$.
23. $\frac{3}{7}$.
24. 5.
25. $\frac{13}{10}$.
26. $\frac{7}{6}$.
27. $\frac{7}{10}$.
28. $\frac{2}{3}$.
29. $\frac{3}{4}$.
30. $\frac{1}{2}$.
31. $\frac{1}{10}$.
32. $\frac{1}{8}$.
33. $\frac{1}{4}$.
34. $\frac{3}{5}$.
35. $\frac{9}{13}$.
36. $\frac{3}{5}$.
37. $\frac{1}{16}$.
38. $\frac{5}{2}$.
39. $\frac{1}{3}$.
40. $\frac{4}{5}$.
41. $\frac{13}{5}$.
42. $\frac{6}{7}$.
43. $\frac{3}{16}$.
44. $\frac{5}{8}$.
45. $\frac{19}{20}$.
46. $\frac{11}{16}$.
47. $\frac{4}{3}$.
48. $\frac{13}{15}$.

EXERCISE 69. Pages 139–140.

1. 43.
2. 62.
3. 56.
4. 74.
5. 91.
6. 103.
7. 235.
8. 514.
9. 323.
10. 3.79.
11. 52.3.
12. 5.76.
13. 7.22.
14. 82.53.
15. 211.2.
16. 4.97.
17. 57.83.
18. 76.41.
19. 473.8.
20. .506.
21. 2.645.
22. 4.795.
23. 5.916.
24. 6.855.
25. 4.159.
26. 1.688.
27. 6.099.
28. 12.067.
29. 8.488.
30. 25.362.
31. 7.674.
32. 19.381.
33. 11.201.
34. 58.401.
35. 81.944.
36. 137.681.

EXERCISE 70. Pages 140–141.

1. 1521.
2. 438,976.
3. 3364.
4. 2209.
5. 24,389.
6. 6.164.
7. 4.344.
8. 9.539.
9. 3.825.
10. 8.366.
11. 7396.
12. 4.626.
13. 12,167.
14. 3.893.
15. 8.062.
16. 2,116.
17. 4.140.
18. 6.082.
19. 3.036.
20. 4.272.
21. .215+.
22. .034+.
23. 9.817+.
24. .096+.
25. 1.531+.
26. 1.179+.
27. 1.392+.
28. 1.924+.
29. 1.755+.
30. .994+.
31. 1.388+.
32. 1.784+.
33. 1.489+.
34. 1.218+.
35. .894+.
36. .732+.
37. .719+.
38. .821+.
39. 2.731+.
40. 1.469+.

EXERCISE 71. Page 144.

1. 31.
2. 27.
3. 53.
4. 39.
5. 8.8.
6. 12.3.
7. 37.2.
8. 80.3.
9. 3.59.
10. 5.72.
11. 76.3.
12. 9.24.

EXERCISE 72. Pages 146–147.

6. Rt. angle ; 90°.
7. *a.* Acute angle ; 60°. *b.* Obtuse angle ; 120°.
   c. Obtuse angle ; 150°.
8. By stretching the string, chalking it, and snapping it.
11. Parallel lines.
12. Right angles.
13. Merely fold it.

EXERCISE 73. Page 149.

2. Yes. 3. The altitudes meet at a point.
4. The two forming the right angle.

EXERCISE 74. Pages 151–152.

1. *a.* 81 sq. ft.; *b.* 144 sq. yd.; *c.* 400 sq. rd.; *d.* 14.0625 sq. yd.
2. *a.* 528 sq. ft.; *b.* 83$\frac{1}{3}$ sq. ft.; *c.* 59.6 sq. ft.
3. 3.29^{+} ft.
4. *a.* 14 yd.; *b.* 20 ft.; *c.* 25 rd.
5. *a.* 12.2^{+} ft.; *b.* 14.1^{+} yd.; *c.* 31.6^{+} ft.
6. 8 ft.
7. 3.6 A.
8. $.68^{+}$ A.
9. $.16^{+}$ A.
10. $236.25.
11. $15.00.
12. $971.25.
13. 40,000.
14. $129.00.
15. $32.63.
16. $119.17.
17. 6188.
18. $648.
19. 845 M.
20. $10.00.
21. $88.00.
22. $5.56.
23. 15 gal.

EXERCISE 75. Page 153.

1. $42.49. 2. $187.50. 3. $48.89. 4. $21.20. 5. $283.16.

EXERCISE 76. Pages 155–156.

1. 22 rolls (11 double rolls). 2. $14.25.
3. For walls, 9 rolls @ $1.10 is $9.90.
For ceiling, 5 rolls @ $1.10 is $5.50.
4. For walls, 13 rolls, $12.35.
For ceiling, 9 rolls, $4.50.
5. Walls, 8 rolls, $5.60.
Ceiling, 6 rolls, $2.70.
6. Walls, 15 rolls.
Ceiling, 10 rolls.
7. Walls, 12 rolls, $14.29.
Ceiling, 8 rolls, $9.52.
8. First method, 12 rolls.
Second method, 10 rolls.

EXERCISE 77. Pages 157–158.

1. $18.43.
2. 42 yd.
3. 55$\frac{5}{9}$ yd.
4. $21.96.
5. $61.06.
6. 34$\frac{1}{8}$ yd.
7. Run strips crosswise.
8. 28$\frac{1}{2}$ sq. yd.
9. The 12′ wide linoleum. Buy 7′ of it, 9$\frac{1}{3}$ sq. yd. $17.27.
10. $138.
11. $429.20.
12. Buy a piece 6′ wide and 9′ long. $10.50.

EXERCISE 78. Page 159.

1. *a.* 130 sq. ft.
   b. $77\frac{1}{12}$ sq. ft.
   c. 40 sq. ft.
   d. 15.525 sq. ft.
   e. 495 sq. yd.

EXERCISE 79. Page 160.

1. 40 sq. in.; 20 sq. in.
2. 96 sq. in.; 48 sq. in.
3. 150 sq. in.; 75 sq. in.
4. *a.* 48 sq. ft.
   b. 10 sq. yd.
   c. $4\frac{1}{8}$ sq. ft.
   d. 8.225 sq. ft.
   e. 314.5 sq. ft.
5. 8 ft.
6. 10 ft.

EXERCISE 80. Page 161.

1. *a.* 36 sq. in.
   b. 78 sq. ft.
   c. 25 sq. ft.
   d. 8 sq. yd.
   e. $58\frac{1}{8}$ sq. ft.
2. 198 sq. yd.
3. $60.47.
4. 786 sq. in.

EXERCISE 81. Page 163.

1. *a.* 10 in.
   b. 25 in.
   c. 23.8 ft.
2. *a.* 28 ft.
   b. 29.4^+ ft.
   c. 14.1^+ yd.
3. 11.2^+ in.
4. 44.9^+ in.
5. 36.4^+ ft.
6. 141.5^+ ft.
7. 13 ft.
8. AG, 12.8^+ ft.
   GC, 12.8^+ ft.
   $BH = HD = CI = 15.6^+$ ft.
   $GH = 10.7^+$ ft.

EXERCISE 82. Page 164.

1. 67.3^+ sq. yd.
2. 44.8^+ sq. ft.
3. 60 sq. ft.
4. 12.6 A.
5. 31.89 A.

EXERCISE 83. Pages 167–168.

1. *a.* 106.814 in.;
   907 92 sq. in.
   b. 23.248 in.;
   43.0084 sq. in.
   c. 78.540 in.;
   490.874 sq. in.
   d. 13.352 in.;
   14.1863 sq. in.
2. 47.12^+ yd.
3. 44.3^+ ft.
4. 20.3^+ in.
5. 314.15 sq. yd.
6. $24,856.3^+$ mi.
7. About 593 times.
   (593.2^+).
8. 7.854 in.
9. 47.124 in.
10. 12.5^+ T.
11. 11.04^+ sq. in.
12. 117.7^+ ft.
    739.8^+ ft.
13. 160.221 sq. in.
14. *a.* 4947.9 ft. per min.
    b. 471.2 ft. per min.
15. About 1273 times.
    (1273^+).
16. 5497.8 ft.$^+$ per min.
17. 2094. 3 ft.$^+$ per min.

EXERCISE 84. Pages 170–171.

1. *a.* 240 cu. ft.
   b. 945 cu. in.
   c. 864 cu. ft.
2. 4968 cu. ft.
3. *a.* 12,096 cu. in.
   b. 406 lb. (About).
4. About 124 lb.
5. About 1631 tons.
6. 142.4 lb.
7. $55.66.
8. 761.9 bbl.
9. *a.* 225 gal.
   b. 1940 lb.
10. 38.8 cu. yd.
11. 18.9 tons.
12. 26.4 tons.
13. 13+ ft.
14. 63,360 cu. ft.
15. About 403 bu.
16. 6¼ cu. ft.
17. 160 bu.
18. 133⅓ cu. yd.; $33.33.
19. 249.6 cu. ft.
20. 17.3 tons.

EXERCISE 85. Pages 172–173.

1. 51.8 or 52 sacks.
2. Cement, $92.63; sand, $18.88; stone, $37.46.
3. Gravel, 10⅔ or 11 yd.; $13.75. Cement, 11 bbl.; $20.35.
4. 2053+ or 2054 cu. yd.
   Cement, 2383 bbl.; $4527.70.
   Sand, 1068 cu. yd.; $ 961.20.
   Gravel, 1767 cu. yd.; $2297.10.
   Total, $7786.00.
5. *a.* Cement, 75 bbl.;
   Stone, 37½ cu. yd.;
   Sand, 22½ cu. yd.;
   Cinders, 82½ cu yd.
   b. 75 bbl. cement, $142.50.
   38 cu. yd. stone, $ 39.40.
   23 cu. yd. sand, $ 20.70.
   83 cu. yd cinders, $ 63.25.
   Total, $274.85.
6. About 30 cu. yd.
   Gravel, $33.00.
   Cement, $58.50.
   Total, $91.50.

EXERCISE 86. Pages 174–175.

1. 12 B. F.
2. 8⅓ B. F.
3. 13⅓ B. F.
4. 37⅓ B. F.
5. 48 B. F.
6. 128 B. F.
7. $69.30.
8. $65.52.
9. $6.03.
10. $12.99.
11. $3.17.
12. $5.95.
13. $37.50.
14. $17.60.
15. $100.80.
16. Oak, 17⅓ B. F.; $ 1.30.
    Pine, 193⅓ B. F.; $ 9.18.
    Total, $10.48.

EXERCISE 87. Pages 176–178.

1. *a.* 188.50 sq. in.; 282.743 cu. in.
   b. 1005.3 sq. ft.; 4021.24 cu. in.
   c. 1256.6 sq. ft.; 5026.55 cu. ft.
2. 31 linear feet.
3. 10 linear feet.
4. 98.1 gal.
5. 137.3 gal.
6. 2.33 ft.
7. 98.175 bbl.
8. 13.2 ft.
9. 113.09 tons.
10. 1095 lb.
11. 52.3 sq. ft.
12. 256.5 lb.
13. 216.6 lb.
14. 461.3 lb.
15. 360.8 lb.
16. 564.6 lb.
17. 282.4 lb.
18. 40 in.
19. 65.4 gal.
20. 145.7 gal.

EXERCISE 88. Page 179.

1. *a.* 1437.3 cu. in.
   b. 4190.4 cu. in.
   c. 7241.3 cu. in.
2. *a.* 616 sq. in.
   b. 1257.1 sq. in.
   c. 1810.2 sq. in.
3. *a.* Vol., 2145.5 cu. in.
   Surface, 804.5 sq. in.
   b. Vol., 1767.8 cu. in.
   Surface, 707.1 sq. in.
   c. Vol., 5577.5 cu. in.
   Surface, 1521.1 sq. in.
4. 56.588 lb.
5. 91.6 lb.
6. $184.15.
7. Vol., 240,256,116,638 cu. mi.
   (Or better, 240,256,000,000 cu. mi.).
   Surface, 186,921,252 sq. mi.

EXERCISE 89. Pages 180–182.

1. 2 pcs. $2'' \times 2'' \times 12'$, 24 lin. ft.
   8 pcs. $2'' \times 2'' \times 9'$, 72 lin. ft.
   Cost, $2.24.
2. $2.16.
3. 8 pcs. $2'' \times 4'' \times 8'$, $42\frac{2}{3}$ B. F.; $1.47.
   9 pcs. $1'' \times 10'' \times 12'$, 90 B. F.; $3.11.
   Total cost, $4.58.
4. 41 pcs. $4'' \times 4'' \times 10'$, 547 B. F.; $ 18.05.
   120 pcs. $2'' \times 4'' \times 10'$, 800 B. F.; $ 26.40.
   240 pcs. $1'' \times 8'' \times 14'$, 2240 B. F.; $ 85.12.
   Total cost, $129.57.

EXERCISE 90. Pages 182–183.

1. 654.5 B. F.
2. 16 pcs. 2″ × 6″ × 14′, 224 B. F.; $ 8.29.
   2 pcs. 2″ × 6″ × 20′, 40 B. F.; $ 1.48.
   726 l. ft. 1″ × 6″ flooring, 363 B. F.; $ 16.34.
   Total, $ 26.01.
3. 5 pcs. 4″ × 4″ × 16′, 106⅔ B. F.
   7 pcs. 2″ × 4″ × 16′, 74⅔ B. F.
   4 pcs. 2″ × 4″ × 12′, 32 B. F.
   Total, 213⅔ B. F.
   Cost $ 7.47.
   Also 385 l. ft. 1″ × 6″ flooring
   192½ B. F. cost $ 8.66.
   Total cost, $ 16.13.

EXERCISE 91. Pages 184–188.

1. *a*. 6 M shingles; *b*. 4⅘ M, or 5 M shingles.

2. 5.37 M, or 5½ M shingles. (22 bundles.)

3. *a*. 14′ in length; *b*. 7 rafters; 65⅓ B. F.; *c*. 400 l. ft.; *d*. 8 bundles; *e*. 10 bundles; *f*. 388 B. F.

4. *a*. 9 16′ rafters; $ 3.17; *b*. 29 strips, 24′ long; $ 7.66; *c*. 2¾ M shingles; $ 17.88. Total, $ 28.71.

5. *a*. Rafters, $ 3.17; *b*. 33 strips, 396 B. F.; $ 16.63; *c*. 5 strips, 120 l. ft., 4 rolls; $ 12.00. Total, $ 31.80.

6. *a′*. 10′; 7½′; 6′; *b′*. ¼; ⅖; ½; *c′*. 8.7′; *d′*. 10′ rafters.

	Purpose	No.	Dimensions		Quantity
e′.	Sills	2	2″ × 4″	12′	16 B. F.
		2	2″ × 4″	18′	24 B. F.
	Studding	25	2″ × 4″	10′	166⅔ B. F.
	Plate	4	2″ × 4″	12′	32 B. F.
		4	2″ × 4″	18′	48 B. F.
	Rafters	20	2″ × 4″	10′	133⅓ B. F.
	Shingle strips	36	1″ × 4″	22′	264 B. F.
	Ridge board	1	1″ × 4″	22′	7⅓ B. F.
	Horizontal braces	22	2″ × 4″	3′	44 B. F.
	Headers	2	2″ × 4″	8′	10⅔ B. F.
	Diagonal braces	8	2″ × 4″	4½′	24 B. F.

f′. 655 B. F. of sheathing; one roll of building paper; 1053 linear feet siding.

7. *a.* 15 M shingles; $93.75; *b.* 156 pcs. 1″ × 12″ 14′; 2184 B.F.; 40 pcs. 1″ × 12″ 16′; 640 B. F. Total, 2824 B.F.; $169.44.

8. *a* and *b.* Living room, 1575 l. ft.; halls, 656 l. ft.; dining room, 875 l. ft.; total oak, 854 B.F.; $72.59; kitchen, 732 l. ft.; 183 B.F.; $13.18. *c.* $98.40.
9. $31.34.
10. $88.96.

EXERCISE 92. Pages 193–194.

1. Lat. area, 420 sq. in.; vol., 420 cu. in.
2. 130 sq. in.
3. 480 cu. in.
4. 360 sq. ft.
5. 11 in.
6. 595 cu in.
7. 28 in.
8. 19½ in.
9. 53⅓ cu. ft.
10. 15 in.
11. Vol., 1568 cu. in.;
Lat. area, 700 sq. in.
12. Side, 8 ft.;
Lat. area, 448 sq. ft.
13. Height, 3.7 ft.;
Surface, 115.3 sq. ft.
14. 2400 sq. in.
15. 5780 lb.

EXERCISE 93. Pages 196–197.

1. Lat. area, 357.7 sq. in.; vol., 678.582 cu. in.
2. 104.72 sq. yd.
3. 33.63 cu. ft.
4. 19.02 cu. ft.
5. 1075.47 cu. in.
6. 376.99 sq. ft.
7. Slant ht., 10 in.;
alt., 8 in.
vol., 301.592 cu. in.
8. 6 ft.
9. 7.84 in.
10. 6.1 ft.
11. 13.8 in.
12. 2.11 in.
13. Diam., 10 in.;
vol., 523.6 cu. in.

QUESTIONS. Page 198.

1. 1895, 7.8¢; 1900, 8¢; 1905, 8¢; 1910, 9.2¢; 1915, 12.5¢.
2. 1915.
3. One half as long.
4. Twice as long.

QUESTIONS, § 142. Page 200.

1. $26.50.
2. $23.00.

QUESTIONS, § 143. Page 200.

1. About one half. Almost double.
2. About one fourth. About equal.
3. Population of Africa equals that of North America and about equals that of South America and Oceania.

QUESTIONS, § 144. Page 202.

1. *a.* $30; *b.* $40; *c.* $80; *d.* $105.
2. *a.* $90; *b.* $120; *c.* $180; *d.* $220.

QUESTIONS.

1. *a.* $90; *b.* $110; *c.* $150; *d.* $170; *e.* $35; *f.* $70.
2. *a.* $200; *b.* $300; *c.* $370; *d.* $155; *e.* $102; *f.* $55.

EXERCISE 95. Page 207.

1. 640 A.
2. I. 40 A. II. 160 A. III. 80 A. IV. 320 A. V. 40 A.
3. N. W. ¼ of the S. W. ¼ of the section.
4. I is the W. ½ of Section 22 of Township 3 N., Range 23 E. of the Fourth Principal Meridian.

II is the N. ½ of the N. E. ¼ of Section 22, etc.
III is the S. ½ of the N. E. ¼ of Section 22, etc.
IV is the N. W. ¼ of the S. E. ¼ of Section 22, etc.
V is the N. E. ¼ of the S. E. ¼ of Section 22, etc.
VI is the S. ½ of the S. E. ¼ of Section 22, etc.

7. For that of Example 5, 480 rd. of fencing.
For that of Example 6, 320 rd. of fencing.

8. $10,800.
9. $7000.

EXERCISE 96. Pages 208–209.

1. .06.
2. .10.
3. .12.
4. .15.
5. .20.
6. .35
7. .16.
8. .14.
9. .92.
10. .83.
11. One.
12. One and one half.
13. Two.
14. One and three quarters.
15. Three.
16. Two and one half.
17. 3.10.
18. 2.60.
19. 5.40.
20. 3.75.
21. 8.26.
22. 3.50.
23. 1.30.
24. 2.25.
25. 1.40.
26. 10.0.
27. 400 %.
28. 600 %.
29. 325 %.
30. 180 %.
31. .025.
32. .0425.
33. .0325.
34. .05125.
35. .875.
36. .0566⅔.
37. .0016⅔.
38. .0007.
39. .085.
40. .0006.
41. .005.
42. .05⅔.
43. .035.
44. .055.
45. .045.
46. .05 and .0005.

47. .75 and .0075.
48. .02 and .002.
49. .25 and .0025.
50. .015 and .00015.
51. .50 and .005.

EXERCISE 97. PAGE 210.

1. $\frac{1}{2} = 50\,\%$; $\frac{1}{5} = 20\,\%$; $\frac{5}{6} = 83\frac{1}{3}\,\%$; $\frac{1}{10} = 10\,\%$;
$\frac{1}{3} = 33\frac{1}{3}\,\%$; $\frac{2}{5} = 40\,\%$; $\frac{1}{8} = 12\frac{1}{2}\,\%$; $\frac{3}{10} = 30\,\%$;
$\frac{2}{3} = 66\frac{2}{3}\,\%$; $\frac{3}{5} = 60\,\%$; $\frac{3}{8} = 37\frac{1}{2}\,\%$; $\frac{7}{10} = 70\,\%$;
$\frac{1}{4} = 25\,\%$; $\frac{4}{5} = 80\,\%$; $\frac{5}{8} = 62\frac{1}{2}\,\%$; $\frac{9}{10} = 90\,\%$;
$\frac{3}{4} = 75\,\%$; $\frac{1}{6} = 16\frac{2}{3}\,\%$; $\frac{7}{8} = 87\frac{1}{2}\,\%$; $\frac{1}{12} = 8\frac{1}{3}\,\%$.
2. *a.* 150 % ; *b.* 250 % ; *c.* 350 % ; *d.* 125 % ; *e.* 450 %.
3. *a.* 50 % ; *b.* 75 % ; *c.* 50 % ; *d.* 80 % ; *e.* 80 %.
4. *a.* $137\frac{1}{2}$ % ; *b.* 120 % ; *c.* 130 % ; *d.* $116\frac{2}{3}$ % ; *e.* 280 %.
5. $57\frac{1}{7}$ %.
6. $9\frac{1}{11}$ %.
7. $58\frac{1}{3}$ %.
8. 85 %.
9. 24 %.
10. $17\frac{1}{2}$ %.
11. $93\frac{3}{4}$ %.
12. 60 %.
13. $58\frac{2}{11}$ %.
14. 160 %.
15. $39\frac{13}{17}$ %.
16. $90\frac{5}{8}$ %.
17. $59\frac{1}{11}$ %.
18. $76\frac{11}{14}$ %.
19. $80\frac{15}{17}$ %.
20. $77\frac{9}{11}$ %.
21. $48\frac{16}{33}$ %.
22. $237\frac{1}{2}$ %.
23. $71\frac{5}{9}$ %.
24. $40\frac{5}{13}$ %.

EXERCISE 98. PAGES 211-214

1. .6, .75, 1.26, 84.09, 275.36.
2. 1.2, 1.50, 2.52, 168.18, 550.72.
3. 6, 7.5, 12.6, 840.9, 2753.6.
4. 15, 18.75, 31.5, 2102.25, 6884.
5. 4.2, 5.25, 8.82, 588.63, 1927.52.
6. $4.20, $13.80, $100.80, $351.60, $821.40.
7. $ 3.50, $11.50, $ 84, $ 293, $ 684.50,
$10.50, $34.50, $252, $ 879, $2053.50,
$17.50, $57.50, $420, $1465, $3422.50,
$24.50, $80.50, $588, $2051, $4791.50.
8. $100 ; $168 ; $96 ; $50.
9. $87.50 ; $106.25 ; $112.50.
10. $150 ; $200 ; $2400.
11. $308.
12. 18.75 bu.
13. 77 ; 91.
14. $2000 ; $1250.
15. 39,375 sq. ft.
16. $2981.25 ;
$2385.
17. *a.* $93.50 ;
b. $137.50 ;
c. $154 ;
d. $104.50 ;
e. $192.50.
18. 1125.
19. $412.50.
20. *a.* 69¢ ;
b. 37¢ ;
c. 75¢ ;
d. $1.61 ;
e. 23¢.

21. A, $274.75;
B, $82.08;
C, $192.07.

22. $5, $10, $10, $10, $15.
$350; $700; $700; $700; $1050.
$40; $80; $80; $80; $120.

23. $1148.93.

24. $2440.

25. 5.2 T. lead;
.08 T. silver.

26. Nitrogen, 80 lb.;
Phosp. Acid, 200 lb.;
Potash, 160 lb.

27. (1) 351.12 lb.; $105.34.
(2) 261.20 lb.; $783.59.
(3) 251.22 lb.; $753.66.

28. Copper, 1300 lb.;
Zinc, 700 lb.

29. 560 lb.

30. *a.* $2.01;
b. $3.74;
c. $7.36.

31. 3420 board feet.

32. 304.5 ft.

33. $501.34.

34. $378.42.

35. 2.85 H. P.

36. Tin, 1200 lb.;
Copper, 150 lb.;
Antimony, 150 lb.

37. 27.5 A.

38. 11.775 yd.

39. Copper, 18.8 lb.;
4.7 lb.;
23.5 lb.

40. $618.75 per day.

EXERCISE 99. PAGES 215–220.

1. 50 %; 25 %; $33\frac{1}{3}$ %; $16\frac{2}{3}$ %; $8\frac{1}{3}$ %; $12\frac{1}{2}$ %.
2. 10 %; 20 %; $16\frac{2}{3}$ %; 50 %; $33\frac{1}{3}$ %.
3. $33\frac{1}{3}$ %; $12\frac{1}{2}$ %; 50 %; $66\frac{2}{3}$ %.
4. 10 %; 40 %; 60 %; 50 %; 20.
5. $12\frac{1}{2}$ %; 25 %; $66\frac{2}{3}$ %; 50 %; 200 %.
6. $66\frac{2}{3}$ %; $133\frac{1}{3}$ %; 200 %; 250 %; 1000 %.
7. 25 %; $33\frac{1}{3}$ %; $12\frac{1}{2}$ %; 10 %.
8. $33\frac{1}{3}$ %; $16\frac{2}{3}$ %; 25 %; 20 %.
9. 40 %; $33\frac{1}{3}$ %; 50 %; $16\frac{2}{3}$ %.
10. $66\frac{2}{3}$ %; 50 %; 10 %; 20 %; 200 %.
11. 20 %; 100 %; 200 %; 400 %.
12. $66\frac{2}{3}$ %; 200 %; 80 %; $44\frac{4}{9}$ %.
13. 200 %; 50 %; $66\frac{2}{3}$ %.
14. 25 %; 100 %; 200 %; 150 %.
15. 140 %; 10 %; 175 %; 700 %.
16. 25 %; 70 %, 125 %; $87\frac{1}{2}$ %.

17. 66 %, silver ; 24 %, copper.

18. $\frac{12}{50}$, or 24 %.

19. *a.* $1\frac{1}{4}$ % ; *b.* 30.85 gal.

20. $12\frac{1}{2}$ %.

21. $\frac{3}{4}$ %.

22. 4 %.

23. 220 %.

24. *a.* $33\frac{1}{3}$ % ; *b.* $133\frac{1}{3}$ %.

25. *a.* 60 % ; *b.* 160 %.

26. $127\frac{7}{9}$ %.

27. 100 %.

28. No ; 9% were in short lengths.

29. A. 84 % ; B. 80 % ; C, 88 %.
Average, 84 %. No.

30. $28\frac{4}{7}$ %.

31. Bricklayer, $8\frac{1}{3}$ % increase.
Carpenter, 30.1^{+} % increase.
Cement finisher, 24% increase.
Gas fitter, $33\frac{1}{3}$ % increase.
Painter, 55.1^{+} % increase.

32. $8\frac{1}{3}$ %.

33. 79.1^{+} %.

34. Wheat bran, $18\frac{2}{11}$ %.
Ground oats, $18\frac{2}{11}$ %.
Corn meal, $27\frac{3}{11}$ %.
Gluten meal, $13\frac{7}{11}$ %.
Brewers' grain, $13\frac{7}{11}$ %.
Cottonseed meal, $9\frac{1}{11}$ %.

35. Deaths, 47.3 %.
Loss of fingers, 71^{+} %.
Injured, 47.4 %.

36. 75 %.

37. American, 37.8 %.
British, 43.8 %.

38. 94.6 %.

39. $22\frac{2}{9}$ %.

40. 13.1 %.

41. $11\frac{1}{9}$ %.

42. 80.1 %.

43. 12.5 %.

44. Anthracite,

1905, 35.2 %.
1910, 8.7 %.
1914, 7.4 %.

Bituminous,

1905, 48.3 %.
1910, 32.3 %.
1914, 1.3 %.

45. 10.6 %.

46. Urban, 38.4 %.
Rural, 9.1 %.

47. White, increase 22.3 %.
Negro, increase 11.2 %.
Indian, increase 12 %.
Chinese, decrease 20.4 %.
Japanese, increase 196.6 %.

48. 42.4 %.

49. Entente, 78.5 %.
Neutrals, 13 %.
Central Powers, 8.3 %.

EXERCISE 100. Pages 221–223.

1. *a.* 1000 ; *b.* 400 ; *c.* 250 ; *d.* 50 ; *e.* 40.

2. *a.* \$800 ; *b.* \$$933\frac{1}{3}$; *c.* \$560 ; *d.* \$200 ; *e.* \$$133\frac{1}{3}$.

3. *a.* 650 mi. ; *b.* \$750 ; *c.* 3850 T.

4. *a.* 4200 ; *b.* \$610 ; *c.* \$2930 ; *d.* 5000.

5. *a.* 300 rd.; *b.* $4000; *c.* 24 T.
6. $656.25.
7. About $1630.
8. 2,500,000.
9. $7500.
10. $35,000.
11. 588 eggs.
12. 18.34 yd.
13. About 68,642.
14. 11,896.
15. 29,412 tons.

EXERCISE 101. Pages 223–225.

1. $B = 2900$.
   $R = 19\%$.
   $P = 356.27$.
   $R = 25.8\%$.
   $B = 7199\frac{5}{9}$.
   $P = 572.83$.
2. 3780 sq. ft.
3. $295.33.
4. 58+ %.
5. 51.8 %; 17.3 %.
6. About 9 %.
7. 76 %.
8. Native whites, 3.09 %.
   Foreign whites, 12.7 %.
   Negroes, 30.4 %.
9. $50,206.50.
10. New York City, 11.6 %.
    Chicago, 13.4 %.
    Philadelphia, 16.2 %.
    St. Louis, 12.7 %.
    Boston, 10.3 %.
11. 2625 T.
12. Idaho, 101.2 %.
    Oklahoma, 109.6 %.
    Iowa, .3 %.
    Washington, 120.4 %.
13. $15,266.25.
14. Proprietors, 3.5 %.
    Salaried employees, 10.2 %.
    Wage earners, 86.1 %.
15. City, increase, 34.8 %;
    Rural, increase, 11.1 %.
    Total, increase, 21 %.

EXERCISE 102. Pages 228–234.

1. *a.* $13.50; *b.* $12; *c.* $7.50; *d.* $11.25; *e.* $10; *f.* $14.25; *g.* $13.13; *h.* $12.50.

2. *a.* $22.50; *b.* $20; *c.* $12.50; *d.* $18.75; *e.* $16.67; *f.* $23.75; *g.* $21.87; *h.* $28.83.

3. *a.* $2.11; *b.* $1.88; *c.* $1.18; *d.* $1.76; *e.* $1.57; *f.* $2.23; *g.* $2.06; *h.* $1.96.

4. *a.* $7.07; *b.* $6.09; *c.* $3.93; *d.* $5.89; *e.* $5.24; *f.* $7.47; *g.* $6.89; *h.* $6.55.

5. *a.* $302.40; *b.* $268.80; *c.* $168; *d.* $254; *e.* $224; *f.* $319.20; *g.* $294; *h.* $280.

6. *a.* $112.50; *b.* $100; *c.* $62.50; *d.* $93.75; *e.* $83.33; *f.* $118.75; *g.* $109.37; *h.* $104.17.

7. *a.* $42.84; *b.* $38.08; *c.* $23.80; *d.* $35.70; *e.* $31.73; *f.* $45.22; *g.* $41.65; *h.* $39.67.

8. *a.* $24.75; *b.* $22; *c.* $13.75; *d.* $20.62; *e.* $18.33; *f.* $26.12; *g.* $24.06; *h.* $22.92.

9. *a.* $3.73; *b.* $3.32; *c.* $2.08; *d.* $3.11; *e.* $2.77; *f.* $3.94; *g.* $3.63; *h.* $3.46.

10. *a.* $7.85; *b.* $6.98; *c.* $4.36; *d.* $6.54; *e.* $5.81; *f.* $8.28; *g.* $7.63; *h.* $6.27.

11. $36.
12. $48.
13. $3.92.
14. $10.92.
15. $21.01.
16. $3.
17. $196.87.
18. $418.88.
19. $18.30.
20. $30.09.
21. $49.41.
22. $158.
23. $12.73.
24. $79.70.
25. $2500.87.
26. $4.94.
27. $25.24.
28. $15.44.
29. $15.59.
30. $11.08.
31. $.29.
32. $702.35.
33. $124.70.
34. $91.14.
35. $840.33.
36. $12.91.
37. $13.58.
38. $131.60.
39. $28.71.
40. $155.22.
41. $92.32.
42. $2641.66.

EXERCISE 103. Pages 235–236.

1. $2420.
2. $1980.
3. $787.50.
4. $1484.
5. $2800.
6. $500.
7. $910.
8. $3200.
9. $2902.50.
10. $3375.
11. 20 %.
12. 10 %.
13. 15 %.
14. 25 %.
15. 12½ %.
16. 20 %.
17. 75 %.
18. 33⅓ %.
19. 60 %.
20. $9\frac{1}{11}$ %.
21. 20 % gain.
22. 12½ % gain.
23. 10 % loss.
24. $27\frac{3}{11}$ % loss.
25. $14\frac{2}{7}$ % loss.
26. 10 % gain.
27. 16⅔ % loss.
28. 37½ % gain.
29. 17.6^{+} % gain.
30. 5.6^{+} % loss.
31. $81.67.
32. $498.27.
33. $12.272 per bbl.
34. $356.
35. $434.78.
36. $1.133 per bu.
37. 34.5 %.
38. 113⅓ % gain.
39. 17.7^{+} % gain.
40. 7.54^{+} % profit.
41. 37.7^{+} % profit.

EXERCISE 104. Page 238.

1. Cost *ic*;
S. P. *a.* *pz*; *b.* *mtn.*

2. Cost *mzn*;
S. P. *a.* *mnc*; *b.* *mar.*

3. Cost *rin*;
S. P. *a.* *prp*; *b.* *mtmi.*

4. Cost *ci*;
S. P. *a.* *pa*; *b.* *is.*

5. Cost *mna*;
S. P. *a.* *mip*; *b.* *mrt.*

6. Cost *sns*;
S. P. *a.* *sip*; *b.* *cta.*

7. Cost *maut*;
S. P. *a.* *muzr*; *b.* *muac.*

8. Cost *acas*;
S. P. *a.* *apsn*; *b.* *aizz.*

9. Cost *nams*;
S. P. *a* *nisi*; *b.* *utmp.*

10. Cost *csns*;
S. P. *a.* *pmup*; *b.* *ipic.*

11. *a.* Net cost $51.22.
b. S. P. of # 200, *nst*;
S. P. of # 400, *uti*;
S. P. of # 173, *irs.*

12. *a.* Net cost $118.50.
b. S. P. of # 3721, *sai*;
S. P. of # 3945, *ciu*;
S. P. of # 592, *nms.*

EXERCISE 105. Pages 240–244.

1. $2526.71.
2. $7304.25.
3. $3512.25.
4. $835.50.
5. $744.14.
6. 2.5 % (about).
7. $263.25.
8. $31.50.
9. 5 %.
10. 6.89+ %.
11. $1280.31.
12. $1684.35.
13. $590.69.
14. $290.69.
15. $1295.06.
16. $11.88.
17. $5764.
18. $3312.06.
19. $2444.98;
i.e. $2445.
Commission, $55.
20. 224 bbl.
21. $8815.
22. $357.08.
23. $2517.69.
24. 1½ ¢ per bu.
25. 3¾ %.

EXERCISE 106. Page 247.

1. $375.
2. $968.33.
3. $351.50.
4. $121.50.
5. $33.64.
6. $8.85.
7. $57.60.
8. $18.50.
9. $29.17.
10. $63.60.
11. $111.68.
12. $141.37.
13. $246.89.
14. $44.08.
15. $144.30.
16. $343.33.

EXERCISE 107. Pages 248–249.

1. *a.* $14.40; *b.* $18.00; *c.* $3.60; *d.* $9.00; *e.* $12.60; *f.* $5.40.
2. *a.* $5.60; *b.* $7.00; *c.* $1.40; *d.* $3.50; *e.* $4.90; *f.* $2.10.

3. *a.* $10.72; *b.* $13.40; *c.* $2.68; *d.* $6.70; *e.* $9.38; *f.* $4.02.

4. *a.* $5.44; *b.* $6.80; *c.* $1.36; *d.* $3.40; *e.* $4.76; *f.* $2.04.

5. *a.* $7.60; *b.* $9.25; *c.* $1.85; *d.* $4.63; *e.* $6.48; *f.* $2.78.

6. *a.* $37.60; *b.* $47.00; *c.* $9.40; *d.* $23.50; *e.* $32.90; *f.* $14.10.

7. *a.* $17.02; *b.* $21.28; *c.* $4.26; *d.* $10.64; *e.* $14.90; *f.* $6.38.

8. *a.* $22.52; *b.* $28.19; *c.* $5.63; *d.* $14.10; *e.* $19.73; *f.* $8.46.

9. *a.* $5.91; *b.* $7.38; *c.* $11.81; *d.* $29.53; *e.* $41.34; *f.* $17.72.

10. *a.* $65.30; *b.* $81.63; *c.* $16.33; *d.* $40.81; *e.* $57.14; *f.* $24.49.

11. *a.* $85.74; *b.* $107.18; *c.* $21.44; *d.* $53.59; *e.* $75.02; *f.* $32.15.

12. *a.* $202.55; *b.* $253.19; *c.* $50.64; *d.* $126.59; *e.* $177.23; *f.* $75.96.

13. *a.* $.72; *b.* $1.08; *c.* $2.52; *d.* $3.24; *e.* $.84; *f.* $1.26; *g.* $.90; *h.* $.54.

14. *a.* $.28; *b.* $.42; *c.* $.98; *d.* $1.26;
e. $.33; *f.* $.49; *g.* $.35; *h.* $.21.

15. *a.* $.54; *b.* $.80; *c.* $1.88; *d.* $2.41;
e. $.63; *f.* $.94; *g.* $.67; *h.* $.40.

16. *a.* $.27; *b.* $.41; *c.* $.95; *d.* $1.22;
e. $.32; *f.* $.48; *g.* $.34; *h.* $.20.

17. *a.* $.37; *b.* $.56; *c.* $1.30; *d.* $1.67;
e. $.44; *f.* $.65; *g.* $.46; *h.* $.27.

18. *a.* $1.88; *b.* $2.82; *c.* $6.58; *d.* $8.46;
e. $2.19; *f.* $3.29; *g.* $2.35; *h.* $1.41.

19. *a.* $.85; *b.* $2.98; *c.* $.99; *d.* $1.06;
e. $1.28; *f.* $3.83; *g.* $1.49; *h.* $.64.

20. *a.* $1.13; *b.* $1.69; *c.* $3.95; *d.* $5.07;
e. $1.32; *f.* $1.97; *g.* $1.41; *h.* $.85.

21. *a.* $.30; *b.* $.44; *c.* $1.03; *d.* $1.33;
e. $.34; *f.* $.52; *g.* $.37; *h.* $.22.

22. *a.* $3.27; *b.* $4.90; *c.* $11.43; *d.* $14.69;
e. $3.81; *f.* $5.71; *g.* $4.08; *h.* $2.45.

23. *a.* $4.29; *b.* $6.43; *c.* $15.00; *d.* $19.29;
e. $5.00; *f.* $7.50; *g.* $5.36; *h.* $3.41.

24. *a.* $10.13; *b.* $15.19; *c.* $35.45; *d.* $45.57;
e. $11.82; *f.* $17.72; *g.* $12.66; *h.* $7.60.

26. $224.35.
27. $10.33.
28. $158.27.
29. $.82.
30. $708.13.
31. $11.26.
32. $55.93.
33. $231.39.
34. $14.19.
35. $237.24.

EXERCISE 108. Page 250.

1. *a.* $17.67; *b.* $24.73; *c.* $28.27; *d.* $14.13.
2. *a.* $16.88; *b.* $5.63; *c.* $3.75; *d.* $1.88.
3. *a.* $5.27; *b.* $8.43; *c.* $4.74; *d.* $7.91.
4. *a.* $10.94; *b.* $14.58; *c.* $32.81; *d.* $18.23.
5. *a.* $16.10; *b.* $5.37; *c.* $4.29; *d.* $28.62.
6. *a.* $2.61; *b.* $3.65; *c.* 4.69; *d.* $.78.
7. *a.* $1.88; *b.* $2.50; *c.* $2.81; *d.* $5.00.
8. *a.* $5.10; *b.* $6.56; *c.* $1.46; *d.* $3.28.
9. *a.* $.83; *b.* $.67; *c.* $.75; *d.* $.25.
10. *a.* $6.20; *b.* $2.95; *c.* $4.13; *d.* $5.02.

EXERCISE 109. Pages 250–251.

1. *a.* $25.35; *b.* $18.75; *c.* $17.06.
2. *a.* $68.75; *b.* $75.00; *c.* $53.13.
3. *a.* $130.00; *b.* $110.00; *c.* $80.00.
4. *a.* $11.25; *b.* $13.75; *c.* $15.00.
5. *a.* $19.59; *b.* $22.69; *c.* $24.75.
6. *a.* $3.51; *b.* $4.22; *c.* $5.27.
7. *a.* $2.44; *b.* $3.52; *c.* $4.60.
8. *a.* $10.83; *b.* $11.81; *c.* $7.87.
9. *a.* $7.50; *b.* $4.69; *c.* $9.38.
10. *a.* $9.79; *b.* $22.02; *c.* $36.71.

EXERCISE 110. Page 252.

1. $27.35.
2. $22.27.
3. $273.05.
4. $26.49.
5. $41.52.
6. $18.54.
7. $2.86.
8. $426.56.
9. $13.82.
10. $8.66.

EXERCISE 111. Page 253.

1. By 6 % method, $183.46.
   Exact interest, $183.22.
2. By 6 % method, $31.94.
   Exact interest, $31.85.
3. By 6 % method, $4.58.
   Exact interest, $4.52.
4. $90.11.
5. $76.83.
6. $10.57.
7. $6.04.
8. $54.24.
9. $1451.71.
10. $107.87.

EXERCISE 112. Pages 254–255.

1. 4 %.
2. 5 %.
3. 4 %.
4. 5½ %.
5. 3 %.
6. 4.5 %.
7. 3.5 %.
8. 3 %.
9. 13⅓ %.
10. 28.5 %.
11. 10 %.
12. 6.75 %.
13. 24 %.
14. 6.4 %.
15. 8.27 %.

EXERCISE 113. Page 256.

1. 2 yr. 1 mo. 15 da.
2. 5 yr. 8 mo.
3. 6 mo.
4. 15 yr.
5. 1 yr. 11 mo. 10 da.
6. 1 yr. 5 mo. 12 da.
7. 2 yr. 7 mo. 21 da.
8. 11 mo. 25 da.
9. 8 mo.
10. 2 mo. 3 da.
11. 5 yr. 6 mo. 20 da.
12. 17 yr. 8 mo. 25 da.
13. 7 yr. 7 mo. 7 da.
14. 20 yr.
15. 20 yr.
16. 20 yr.
17. 25 yr.

EXERCISE 114. Page 257.

1. $620.
2. $115.17.
3. $321.
4. $891.60.
5. $392.
6. $2640.
7. $46.
8. $1890.
9. $364.06.
10. $911.
11. $650.
12. $2460.
13. $9600.
14. $3430.
15. $2600.

EXERCISE 115. Pages 258–259.

1. $502.72.
2. $162.36.
3. $607.75.
4. $146.24.
5. $241.37.
6. $5858.30.
7. $3312.24.
8. *a.* $1321.14; *b.* $1200.
9. $1351.39.
10. $1352.19.

EXERCISE 116. Page 261.

1. $302.11.
2. $428.29.
3. $1101.77.

EXERCISE 117. Pages 262–263.

1. *a.* $82.40.
   b. $87.20.
   c. $88.77.
   d. $1.57.
   e. $100.03.
2. *a.* $83.60.
   b. $87.20.
   c. $88.74.
   d. $1.54.
   e. $99.997.
3. *a.* $20.90.
   b. $84.259.
   c. $85.1015.
   d. $88.5569 (1920).
   $92.1525 (1921).
   $95.8941 (1922).
   $99.7876 (1923).

EXERCISE 118. Pages 266–267.

1. *a.* $119.41.
   b. $119.56.
   c. $119.64.
2. *a.* $1216.65.
   b. $1218.99.
   c. $1220.19.
3. *a.* $342.14.
   b. $343.20.
   c. $343.74.
4. *a.* $518.09.
   b. $520.08.
   c. $521.10.
5. $37,221.60.
6. *a.* In column three, the amounts at 1 % for years 20, 19, 18, etc., to 1 inclusive.
   b. $22.24.
   c. $111.20.
7. *a.* In column three, the amounts at 1 % for years 40, 39, 38, etc., to 1 inclusive.
   b. $493.75.
8. $620.23.
9. $808.91.
10. $2297.64.

EXERCISE 119. Page 268.

1. $1041.48.
2. $467.57.
3. $1428.96.
4. $1438.25.
5. $1029.22

EXERCISE 120. Pages 270–271.

1.

Payment				Interest on Payment	Payment Plus Interest	Balance Due
Date	Debt	Due	Time			
5/1/18	$3000	$100 00	—	—	$100.00	$2900
6/1/18	2900	100.00	1 mo.	$ .50	100.50	2800
7/1/18	2800	100.00	2 mo.	1.00	101.00	2700
8/1/18	2700	100.00	3 mo.	1.50	101.50	2600

and so on until thirty payments have been made

2.

Date	Debt	Time	Interest on Balance	Payment Due	Total Due	Balance of Debt
8/1/18	$1500	—	—	$50.00	$50.00	$1450
9/1/18	1450	1 mo.	$7.25	50.00	57.25	1400
10/1/18	1400	1 mo.	7.00	50.00	57.00	1350
11/1/18	1350	1 mo.	6.75	50.00	56.75	1300

and so on until thirty payments have been made

3.

Payment				Interest on Payment	Payment Plus Interest	Balance Due
Date	Debt	Due	Time			
5/15/18	$1325	$125	—	—	$125.00	$1200
6/1/18	1200	100	½ mo.	$ .25	100.25	1100
7/1/18	1100	100	1½ mo.	.75	100.75	1000
8/1/18	1000	100	2½ mo.	1.25	101.25	900

and so on until thirteen payments have been made

4.

Date	Debt	Time	Interest on Balance	Payment Due	Total Due	Balance Due
9/1/15	$650	—	—	—	—	$650
4/1/16	650	7 mo.	$22.75	$65.00	$ 87.75	585
4/1/17	585	1 yr.	35.10	65.00	100.10	520
4/1/18	520	1 yr.	31.20	65.00	96.20	455

and so on until the debt is paid

5.

Date	Debt	Time	Interest on Debt	Payment Made	Applied on Debt	Balance Due
3/5/18	$1000.00	—	—	—	—	$1000.00
9/5/18	1000.00	6 mo.	$30.00	$100	$70.00	930.00
3/5/19	930.00	6 mo.	27.00	100	72.10	857.90
9/5/19	857.90	6 mo.	25.74	100	74.26	783.64
3/5/20	783.64	6 mo.	23.51	100	76.49	707.15

and so on until the debt is paid

6.

Payment				Interest on Payment	Payment Plus Interest	Balance Due
Date	Debt	Due	Time			
5/20/17	$425	$25	—	—	$25.00	$400
6/20/17	400	10	1 mo.	$.05	10.05	390
7/20/17	390	10	2 mo.	.10	10.10	380
8/20/17	380	10	3 mo.	.15	10.15	370

and so on until the debt is paid

7.

Date	Debt	Interest on Debt	Payment on Debt	Total Payment Made	Balance Due
8/15/16	$4000	—	—	—	$4000
2/15/17	4000	$120.00	$300	$420.00	3700
8/15/17	3700	111.00	200	311.00	3500
2/15/18	3500	105.00	—	105.00	3500
8/15/18	3500	105.00	600	705.00	2900
2/15/19	2900	87.00	250	337.00	2650
8/15/19	2650	79.50	250	329.50	2400

and so on until the debt is paid

EXERCISE 121. Pages 274–276.

1. $5.50.
2. $22.40.
3. $12.48.
4. $3.00.
5. $39.60.
6. $19.20.
7. $7.50.
8. $8.32.
9. $18.00.
10. $20.63.
11. *a.* $140; *b.* $21.00; *c.* $119.00.
12. *a.* $165; *b.* $33.00; *c.* A, $52.80; B, $44.00; C, $35.20.
13. *a.* $350; *b.* $52.50; *c.* $297.50.
14. A, $2000; B, $1666.67; C, $1333.33.
15. *a.* $7500; *b.* A, $45.00; B, $22.50; C, $33.75.
c. From A, $6.75; from B, $3.38; from C, $5.07.
d. A, $38.25; B, $19.12; C, $28.68.
e. A, $4444.44; B, 2222.22; C, $3333.34.
16. *a.* $31.25; *b.* $7.81; *c.* $5000.
17. *a.* $31.40; *b.* On house, $8000;
On goods, their value up to $2500.
18. $24.
19. $30.

EXERCISE 122. Pages 278–280.

1. a. $2.80; b. $5.60; c. $10.50; d. $11.90.
2. a. $6.73; b. $11.88; c. $23.76; d. $14.65.
3. a. $.83; b. $3.19; c. $1.47; d. $4.98.
4. a. $2.22; b. $4.45; c. $8.78; d. $11.12.
5. a. $6.18; b. $28.41; c. $40.76; d. $52.49.
6. $7.20; $4.80.
7. $6.60; $2.75.
8. $22.40; $12.44.
9. $14.40; $3.60.
10. $8.78; $2.37.
11. $4.68; $2.53.
12. $28.56; $9.71.
13. $48.40; $16.46.
14. $21.38; $4.70.
15. $12.60.
16. a. $93.75; b. $70.31; c. $56.25.
17. a. $13.75; b. $2.06; c. $11.69; d. $7.15.
18. a. $12.60; b. $4.94.
19. $1.34.
20. a. $6.60; b. $.48.

EXERCISE 123. Pages 280–281.

1. a. $750; b. $1562.50; c. $4062.50.
2. a. $1400; b. $800; c. $4666.67.
3. a. $2100; b. $1200; c. $4000.
4. a. $206.88; b. $1250 on contents, and $364.58 on stores.
5. $583.33.

EXERCISE 125. Pages 283–284.

1. $526.50; $19.74.
2. $471.00; $18.84.
3. $540; $18.90.
4. $1381.50; $27.63.
5. $925; $25.43.
6. $651; $21.16.
7. $1305; $26.10.
8. $1015; $22.84.
9. $825; $22.69.
10. $1345.50; $26.91.
11. $82.50.
12. $250.80.
13. $325.12.
14. $11.25.
15. $62.50.
16. $33.25.
17. $12.38.
18. $105.
19. $18.75.

EXERCISE 126. Page 286.

1. $67.50.
2. $155.50.
3. $123.43.
4. $147.92.
5. $230.90.
6. $171.78.
7. $76.38.
8. $29.93.
9. $224.91.
10. $241.95.
11. $35.78.
12. $82.42.
13. $65.42.
14. $78.40.
15. $122.38.
16. $91.05.
17. $40.48.
18. $15.87.
19. $119.20.
20. $128.24.

21. $17.55.
22. $40.43.
23. $32.09.
24. $38.46.
25. $60.05.
26. $44.66.
27. $19.86.
28. $7.78.
29. $58.48.
30. $62.91.

EXERCISE 127. Pages 289–291.

1. $34.58.
2. $9.90.
3. $326.95.
4. $239.70.
5. $733.25.
6. $82.23.
7. $367.76.
8. $198.90.
9. $307.80.
10. $55.08.
11. 13.13+ or 13.14 mills on $1.
    $111.69.
12. 1.4 %.
13. 2.2 %.
14. $52.78.
15. *a.* 1.24+ % or 1.25 %;
    b. $92.06.
16. *a.* $116; *b.* $100.80.
17. *a.* $4,754,565.12;
    b. $45,157,772.98.
18. Total, $11,196.20.
    State's share, $1679.43.
    Community's share, $9,516.77.
19. 6.45+ %.
20. $72.44.
21. *a.* $411.20; *b.* $34.27; *c.* $68.80; *d.* 4.2+ %; *e.* 10.2 %.
22. $152.33.
23. 1.29+ % or 1.3 %.

EXERCISE 128. Page 293.

1.

INCOME	TAX
$ 1,000	$ 10.00
2,000	22.50
3,000	37.50
4,000	55.00
5,000	75.00
6,000	100.00
7,000	130.00
8,000	165.00
9,000	205.00
10,000	250.00
11,000	300.00
12,000	350.00
13,000	410.00

2. $3.40.
3. *a.* None.
   b. $7.80.
4. $76.88.
5. $983.00.

EXERCISE 129. Page 293.

1. $62.25.
2. $92.50.
3. $118.
4. $184.43.
5. $785.
6. $1941.50.
7. $8384.
8. $19,325.
9. $10,325.
10. $25,775.
11. $31,186.
12. $44,150.

EXERCISE 130. Pages 294–295.

1. *a.*

Wife,	$325.00
Daughter,	180.00
Son,	180.00
Brother,	22.50
Niece	22.50
Aunt,	22.50
Total,	$752.50

b. County's share, $56.44; State's share, $696.06.

2. *a.*

Wife,	$2000.00
Son,	1085.00
Daughter,	1085.00
Sister,	142.50
Nephew,	67.50
Niece,	105.00
Uncle,	97.50
Aunt,	142.50
Total,	$4725.00

County's share, $354.38.

EXERCISE 131. Pages 296–297.

1. $1476.45.
2. $294.33.
3. $68.45.
4. $899.33.
5. $195.41.
6. $146.
7. $521.10.
8. $7300.
9. $5645.25.
10. $10.95.
11. $154.75.
12. $34.74.
13. $9.73.
14. $116.66.
15. $47.50.
16. $18.09.
17. $151.10.
18. $1085.63
19. $37.50.
20. $9.20.

EXERCISE 132. Pages 300–301.

1. $80.
2. $120.
3. $489.
4. $574.
5. $31,190.
6. $2\frac{2}{5}$ %.
7. $2\frac{2}{5}$ %.
8. 5.75 %.
9. 5.74 %.
10. 31.19 %.

EXERCISE 133. Page 302.

1. $350.
2. $1170.
3. $4474.
4. $8400.
5. $11,016.
6. $22,080.
7. $37,500.
8. $44,460.
9. $81,500.
10. $999,500.
11. $2,549,000.
12. $7,604,000.

EXERCISE 134. Page 306.

7. $1262.50.
8. $253.75.
9. $575.
10. $1622.25.

EXERCISE 135. Pages 309–310.

1. $369.86.
2. $340.65.
3. $99.19.
4. $155.79.
5. $799.49.
6. $328.48.
7. $1215.30.
8. $693.98.
9. $2257.40.
10. $1103.76.

EXERCISE 136. Pages 311–312.

1. $1051.06.
2. $806.06.
3. $1303.85.
4. $361.00.
5. $233.33.
6. $508.44.
7. $244.46.
8. $1174.59.
9. $322.48.
10. $774.37.

EXERCISE 137. Pages 314–315.

1. *a.* $6.00; $594.00.
b. $4.70; $595.30.
c. $3.40; $596.60.
2. *a.* $3.75; $246.25.
b. $2.88; $247.12.
c. $1.04; $248.96.
3. *a.* $26.25; $848.75.
b. $16.77; $858.23.
c. $10.50; $864.50.
4. *a.* $5.03; $999.97.
b. $4.19; $1000.81.
c. $3.18; $1001.82.
5. *a.* $5.35; $1599.98.
b. $4.55; $1600.78.
c. $1.87; $1603.46.
6. *a.* $7.22; $2398.78.
b. $6.42; $2399.58.
c. $2.41; $2403.59.
7. *a.* $7.52; $2998.73.
b. $6.01; $3000.24.
c. $3.51; $3002.74.
8. *a.* $5.68; $373.07.
b. $3.66; $375.09.
c. $2.65; $376.10.
9. *a.* $35.62; $557.98.
b. $23.94; $569.66.
c. $16.52; $577.08.
10. *a.* $13.61; $1347.64.
b. $7.71; $1353.54.
c. $2.72; $1358.53.
11. $949.94.
12. $5.42.
13. $6.76.
14. $2982.50.
15. $918.16.
16. $740.31.
17. $1000.53.
18. $475.43.
19. $1253.67.
20. $361.90.
21. $981.34.
22. $125.11.
23. $1452.33.
24. $3523.78.
25. $2679.73.

EXERCISE 138. Page 321.

1. $.03.
2. $.03.
3. $.05.
4. $.10.
5. $.08.
6. $.12.
7. $.05.
8. $.08.
9. $.15.
10. $.25.
11. $.18.
12. $.25.
13. $1.00.
14. $1.75.
15. $2.88.
16. $7.50.
17. $12.50.
18. $9.25.
19. $3.38.
20. $16.00.
21. $6.25.
22. $1.13.
23. $1.93.
24. $3.75.

EXERCISE 139. Page 322.

1. *b.* $2.19.
2. *a.* $27; *b.* $4.91; *c.* $49.40.

EXERCISE 140. Page 324.

1. $.33.
2. $.67.
3. $.84.

EXERCISE 141. Pages 327-328.

1. *c*. The date of maturity is 30 days after the date of acceptance if the draft is "payable 30 days after sight," or else 30 days after the date of the note if it is made "payable 30 days after date."

EXERCISE 142. Page 329.

1. Discouraged when above, and encouraged when below par.
2. $500.63.
3. $283.15.
4. $1972.04.
5. $2521.88.
6. $647.40.

EXERCISE 143. Pages 330-331.

1. *a*. $298.50; *b*. $298.75.
2. *a*. $2807.25; *b*. $2817.70.
3. *a*. $618.75; *b*. $621.35.
4. *a*. $1333.30; *b*. $1334.42.
5. *a*. $928.89; *b*. $930.76.
6. $623.68.
7. $134.48.
8. *a*. $1728.99, $7210.88, $792.00, $2389.74, $9960; *b*. $22,100.13.

EXERCISE 144. Pages 333-334.

1. $1246.45.
2. $333.97.
3. $17,087.44.
4. $828.08.
5. $41,299.38.
6. £514 10s.
7. 5180 fr.
8. $3643.68.
9. £1031.99+, or £1032.
10. 571,500,000 fr.

EXERCISE 145. Pages 336-339.

1. $1260.
2. A, $820;
   B, $1640.
3. A, $870;
   B, $1392;
   C, $2088.
4. A, $2281.81;
   B, $2139.20;
   C, $1853.97.
5. A, $2065.45;
   B, $1678.18;
   C, $1936.36.
6. *a*. B, $2969.09;
   C, $2710.91.
   b. B, $3042.86;
   C, $2637.14.
7. $5680.
8. A, $1800;
   B, $600.
9. $815.
10. A, $991.58;
    B, $638.42.
11. A, $2330;
    B, $2870.
12. A, $3698;
    B, $3632;
    C, $3770.
13. A, $797.54;
    B, $783.92;
    C, $768.54.
14. *a*. $2688.64;
    b. $1448.61;
    c. A, $72;
    B, $198;
    C, $162;
    d. $2498 92;
    e. $832.97.

EXERCISE 146. Page 340.

1. $4000; $12,000.
2. $1500; $1515.
3. $2000; $2360.
4. $500; $725.
5. $2500; $1450.
6. ——; $3175.
7. $1125; $3487.50.
8. $1000; $697.50.
9. $500; $2300.
10. $2000; $3240.

EXERCISE 147. Page 341.

1. 11.62 %.
2. 16.86 %.
3. 9.23 %.
4. 7.5 %.
5. 9.79 %.
6. 6.31 %.
7. 5.55 %.
8. 4.49 %.
9. 6.93 %.
10. 1.69 %.
11. 9.19 %.
12. 6.52 %.
13. 6.25 %.
14. 14.28 %.
15. 10.90 %.

EXERCISE 148. Pages 343–344.

1. $6662.50; $6637.50.
2. $5268.75; $5256.25.
3. $672.50; $667.50.
4. $3640; $3630.
5. $1378.75; $1373.75.
6. $8950; $8900.
7. $7837.51; $7818.75.
8. $3828.13; $3819.37.
9. $6690; $6675.
10. $3340; $3320.
11. Gain, $100.
12. No gain or loss.
13. Gain, $1312.50.
14. Loss, $625.
15. Gain, $3000.
16. 22 shares.

17. 11 shares Beth. Steel Pfd., 11 shares Amer. Smelting Co., and 8 shares General Electric, with a balance of $256.99. With latter, buy one more share of Amer. Smelting Co., and one more share of General Electric.

18. 14 shares.

19. 19 shares U. S. Rubber Pfd., 22 shares Studebaker Common, 47 shares Westinghouse Electric, with a balance of $45.37. With the balance, buy one more share of Studebaker Common.

20. 67 shares U. S. Rubber Pfd., 76 shares Gt. Northern Pfd., and 67 shares Beth. Steel Pfd.; 50 shares Baldwin Loco., 32 shares Gen'l Elec., and 50 shares Republic Iron and Steel; with a balance of $97.12. With the balance, buy one more share of Gt. Northern Pfd.

EXERCISE 149. Pages 347–348.

1. 8.94 % ; $395.32.
2. 6.95 % ; $593.03.
3. 4.84 % ; $836.25.
4. 4.94 % ; $864.67.
5. 5.62 % ; $1078.58.
6. 4.85 % ; $724.07.
7. 5.94 % ; $1039.88.
8. 4.46 % ; $970.08.
9. 4.41 % ; $952.68.
10. 5.31 % ; $946.53.
11. 5.02 % ; $1008.75.
12. 5.17 % ; $874.87.
13. 5.07 % ; $998.75.
14. 5.20 % ; $965.28.
15. 6.16 % ; $829.17.

EXERCISE 150. Page 349.

1. *a.* $1.13 ; *b.* $2.25 ; *c.* $11.25.
2. $49.25 ; $49.13 ; $49.56 ; $48.88.

(Note. These results do not include brokerage to be paid nor the interest due.)

EXERCISE 151. Pages 353–355.

1. 7 mo. and 10 da. after date.
2. 8 mo. and 26 da. after date.
3. 73 da. after date.
4. April 23.
5. October 7.
6. November 2.
7. January 7.
8. June 13.
9. December 24.
10. July 26.
11. 9 mo. after date.
12. October 3.
13. 16 mo. and 3 da. after date.
14. October 11.
15. December 18.